On the Optimal Reform of Income Support for Single Parents

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Abstract

We characterize the optimal reform of U.S. income support for low-income single parents using a life-cycle heterogeneous agent model with idiosyncratic risk and incomplete asset markets. We use the U.S. tax-transfer system as the benchmark policy and a sample of single mothers drawn from the CPS to assess reforms that maximize average expected utility among single mothers-to-be. When policy cannot be tagged by the age of the children, the optimal reform calls for an increase in out-of-work income support by about 15 percent, and a decrease in earnings subsidies to low-wage workers by roughly 50 percent. This reform delivers substantial welfare gains. Tagging policy by the age of the children makes the government's trade-off between providing insurance to single mothers with children of pre-school age, on the one hand, and providing work incentives to those with school-age children, on the other hand, more favorable, thus increasing their scope for smoothing marginal utility throughout the life cycle. With tagging, mothers of pre-school age children get a substantial increase in out-of-work income support and no earnings subsidies. Tagging brings additional welfare gains.

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1 Introduction

In most developed economies, income support for low-income families with children consists of a mix of a guaranteed minimum income and earnings subsidies. In the U.S., the Temporary Assistance for Needy Families program (TANF) and the Supplemental Nutrition Assistance Program (SNAP) provide a guaranteed income (out-of-work income support) to eligible families with children that is phased out as their income increases.[1] On the other hand, the Earned Income Tax Credit (EITC), and to a lesser extent the Child Tax Credit (CTC), provide earnings subsidies to eligible workers. This paper is concerned with the optimal reform of income support for low-income, single-parent families, with special attention to the optimal mix of a guaranteed minimum income and earnings subsidies. Specifically, we take the U.S. tax-transfer system as a benchmark—accounting for all the non-convexities in taxes and income support—and then obtain the optimal reform that maximizes average ex-ante expected welfare of single parents-to-be. We find that the optimal reform implies substantial welfare gains. Our main motivation to focus on single-parent families is their higher at-risk-of-poverty rate, compared to two-parent families. For instance, in 2018 the poverty rate for single-mother families was 34 percent, against 6 percent for married families. Also, single mothers represent more than 90 percent of all the families receiving TANF cash benefits, and 60 percent of SNAP households with children (U.S. Census Bureau). The over-representation of single mothers among the poor and among those on welfare warrants an assessment of the existing anti-poverty policy for this demographic group.[2]

To carry out our analysis we use a dynamic structural model of consumption/savings and labor supply for single parents. Labor supply includes both a participation decision (there are fixed costs of work), and a work intensity decision (number of hours). In order to account for the dynamics and uncertainty inherent in child rearing, we model the life cycle of single parents from the moment they enter the economy with young children, up to their deaths. Single parents, who are ex-ante heterogeneous in their education levels, face three types of idiosyncratic risk. First, while, on average, child care needs are higher for pre-school age children than for children of school age, single parents face idiosyncratic risk about these needs. Second, they face idiosyncratic earnings risk, both from shocks to their labor productivity and, in a model's extension, from human capital accumulation through learning by doing. Third, there is fertility risk. Single parents can partially

[1] While SNAP provides food benefits and not cash payments to eligible families, we follow most of the literature in considering this in-kind assistance as income.

[2] It should also be noted that a two-parent family's behavioral response to policy involves trade-offs that are not available to single-parent families. For instance, two-parent families engage in within-household risk sharing, and adjust family labor supply to individual earnings shocks (the added worker effect). We hence follow a growing literature focusing on single parents (a review of key contributions to this literature is provided below).

self-insure against all these risks by saving in a risk-free asset, and by managing labor supply (e.g. working longer hours when their labor productivity is high and their child care needs are low). Precautionary savings play an essential role in our analysis. First, the asset limits imposed by income support programs distort the saving decisions of low-income parents with assets close to those limits, yielding bunching in asset holdings.[3] This results in less self-insurance, thus shaping optimal income support, particularly the optimal mix of guaranteed income and earnings subsidies. Second, the ability to self-insure through savings increases over the life cycle. Young single parents are less likely to have built a buffer stock of savings to use as self-insurance, compared to older parents with school-age children. This differential in the access to self-insurance through savings over the life cycle also shapes the optimal mix of guaranteed income and earnings subsidies.

We calibrate our model to the tax-transfer treatment of single parents in the U.S., and to match key moments from a sample of single mothers drawn from the Annual Social and Economic (ASEC) Supplement of the Current Population Survey (CPS). Our calibrated model implies labor supply elasticities that differ both across the two margins of response and across the number and age of the children. Extensive margin elasticities are higher than intensive margin elasticities; also, single mothers of children of pre-school age have higher extensive margin elasticities than single mothers of school-age children.

The calibrated model is used to characterize the optimal reform of income support for single parents. We restrict our analysis to reforms within the same parametric family of the current programs, subject to a government budget constraint. Specifically, we impose that the amount of income redistributed from the rest of the economy to the population of single mothers is kept fixed. The restriction to the current parametric family does not impose apparent limitations to the set of feasible reforms, in the sense that we can generate and assess reforms that yield a rich array of combinations of out-of-work income support and earnings subsidy rates, and their resulting marginal and participation tax rates. Our focus is on the shape of these profiles at low levels of income.

We find that the guaranteed minimum income under the benchmark tax-transfer system is too low. Without tagging by the age of the children, the optimal reform prescribes an increase in guaranteed income by 13 percent for single mothers with one child, and by 19 percent for single

[3]For early work showing how asset-based social insurance distorts savings see Hubbard et al. (1995). More recently, Ortigueira and Siassi (2022) use a quantitative macro model and show that the savings distortions are especially large among low-productive workers. Findings from the empirical literature are mixed. Hurst and Ziliak (2006) estimate the effects on household savings of the looser limits on liquid assets and vehicle wealth holdings implemented as part of the 1996 welfare reform. Focusing on female-headed households with children they find a positive effect on vehicle ownership, but only a negligible impact on other asset holdings.

mothers with two children. However, earnings subsidy rates for low earners are reduced by 50 percent. As a result, marginal and especially participation tax rates increase with the optimal reform, both of them becoming positive even at very low levels of earnings. Succinctly, these results are explained by the particularly marked mismatch between the evolution of means and needs over the single mothers' life cycle. Single mothers at the beginning of their life cycle are more likely to be wealth-poor, to have children of pre-school age and, hence, face high child care needs if they work. Therefore, these mothers are more likely to hit the borrowing constraint and, as a result, to have relatively higher marginal valuations of consumption insurance. As single mothers move along their life cycle and their children start schooling, child care needs drop and their ability to self-insure, both through savings and labor supply, improves. Although some mothers still continue to be subjected to fertility risk, on average, as they become less likely to be credit constrained, they are more willing to trade some insurance for earnings subsidies. The prescribed shift in the policy mix toward a higher out-of-work income support allows single mothers at the beginning of their life cycle to ease credit constraints, and thus to more efficiently allocate labor supply. This enables single mothers to further smooth marginal utility across their life cycle.

The trade-off between guaranteed income and earnings subsidies faced by the government eases off when policy can be made contingent on the age of the children. Under tagging, the optimal reform prescribes a larger increase in guaranteed income and very small earnings subsidies for single mothers of pre-school age children. However, single mothers of children of school age get a lower increase in guaranteed income and still receive sizable earnings subsidies, providing them with incentives to work. That is, the optimal reform under tagging increases further the scope for intertemporal marginal utility smoothing: Insurance for mothers of pre-school children, and incentives to work for mothers of children in school age. Finally, we also consider an extension of our model with human capital accumulation by learning by doing. In this version of the model a new force shaping the optimal policy reform comes into play. Namely, since human capital accumulation has long-lasting effects on labor productivity and wages, there are positive social benefits from incentivizing single mothers to work (and thus to accumulate human capital). This tension between insurance and incentives to work to accumulate human capital is resolved in the optimal reform by reducing slightly the guaranteed income (with respect to the optimal policy in the benchmark model without learning by doing), and by increasing the earnings subsidy rates in a now shorter phase-in region. Overall, this exercise shows that the policy prescribed by our benchmark model is robust to the introduction of human capital accumulation by leaning by doing.

Related literature. Our work in this paper is related to several strands of the literature on optimal income taxation. One such strand approaches the design of the optimal tax system using quantitative macroeconomic models that emphasize the role of uninsurable idiosyncratic risks under incomplete asset markets and borrowing constraints. Since progressive taxation provides income insurance at the cost of distorting savings and labor supply, these models contain the key margins for a normative analysis of the tax code. In an influential paper, Conesa and Krueger (2006) find that the optimal earnings tax code is roughly a flat rate with a fixed deduction. Krueger and Ludwig (2016) extend the analysis by introducing a trade-off between tax progressivity and education subsidies and find that the optimal tax code is far less progressive, becoming close to a proportional system. Heathcote et al. (2017) study optimal tax progressivity in a framework that allows for analytical solutions and present arguments suggesting that the optimal progressivity is close to the one in the U.S. tax code. Karabarbounis (2016) studies age-dependent optimal earnings taxation and finds that tax distortions should be hump shaped in age. Heathcote et al. (2020) allow the progressivity of the tax code to depend on age and find that (1) progressivity should be U-shaped in age, and (2) the average marginal tax rate should be increasing and concave in age. Our work in this paper departs from the above-mentioned studies in two important ways. First, while their focus is on the joint characterization of the optimal earnings tax code for all households across the earnings distribution, our focus is restricted to low-income households, taking the tax code for high incomes as given. We fix the level of redistribution from high-income to low-income households and do not characterize the optimal tax code for all income levels jointly. Second, in the above-mentioned papers the tax-transfer system is approximated using a smooth function of earnings that depends on three parameters. The optimal system is then found by choosing the values of these parameters that maximize a welfare criterion. By contrast, we model taxes and transfers with all their kinks and non-convexities, and then search for the optimal system within this parametric family. Beyond the value of this approach in terms of readiness of the results for practical implementation, there are two other main advantages. On the one hand, using a smooth function to approximate net taxes greatly reduces the nature and scope of the reforms that can be considered. Since our focus is only on the optimal tax-transfer system to low-income households, by adopting the parametric family of the current system we retain the ability to assess the optimality of thresholds and the phasing in and out of transfers. On the other hand, the smooth functions used in the literature to approximate net taxes are not defined at zero earnings for all values of the parameters. As our population of interest is made up of single mothers with dependent children—a population that can face potentially high child care costs if they work—it is important to assess tax-transfer systems that imply zero earnings for a non-negligible fraction of households, thus rendering the use of these smooth approximations less suitable for our purposes.

Our work is also related to a strand of the literature focusing on single mothers. Blundell and Shephard (2012) are concerned with the optimal taxation of earnings of single mothers of one child.

Using a static model and abstracting from a consumption/savings decision and from the dynamics introduced by child aging, these authors find that the optimal tax schedule is characterized by positive out-of-work income support and by a non-monotonic profile of marginal tax rates. They also find a welfare improving role for tagging according to the age of the child. Mullins (2022) develops a dynamic model of labor supply that includes maternal investment in the child's skill development. His model, however, abstracts both from investment in financial assets—and hence from the self-insurance role of savings—and from child care costs. Restricting the analysis to the family of continuous, one-kink policies, Mullins finds that the optimal policy is characterized by a positive out-of-work income support that is phased out as earnings increase. We complement these studies by introducing new trade-offs in the design of the optimal tax-transfer system arising from the uninsurability of earnings, fertility and child care cost risks faced by single mothers. By accounting for the variation in access to self-insurance over the single mothers' life cycle and for child aging, we show that these new trade-offs have important implications for the taxation of this population group.

A different strand of literature tries to answer whether the optimal marginal and participation tax rates are positive or negative at low levels of income (Saez 2002, Jacquet et al. 2013, Hansen 2021). This debate has followed a mostly theoretical approach establishing necessary and sufficient conditions for each case. Jacquet et al. (2013) provide a sufficient condition for these tax rates to be positive, although, as acknowledged by the authors, the empirical plausibility of this condition cannot be immediately assessed. In our quantitative model, the optimal policy mix (guaranteed income versus earnings subsidies) yields positive marginal and participation tax rates at the lower end of the earnings distribution. The robustness of this result leads us to conclude that the negative marginal and participation tax rates stemming from the current tax-transfer system for single parents are not optimal.

Finally, our study in this paper also relates to and builds upon recent positive analyses on taxes and female labor supply. For instance, using a life-cycle model where individuals face no risks after entering the economy, Guner et al. (2020) compare the effects on female labor supply of expansions in child care credits versus expansions in child credits. They find that the former lead to a large increase in married female labor market participation, while the latter have a negative effect. Both expansions, however, generate welfare gains for newborn households. (For other related studies see Kaygusuz 2010, Guner et al. 2012, and Ortigueira and Siassi 2022, to name a few.)

The remainder of the paper is organized as follows. Section 2 presents our structural model of consumption/savings and labor supply for the analysis of optimal transfers. The data, the calibration of the model, and the benchmark solution are presented in Section 3. Section 4 contains the analysis of the optimal reform of the tax-transfer system, both without and with tagging by child age. Section 5 presents an extension of our model that includes returns to experience. Concluding remarks are offered in Section 6. There are five appendices.

2 The Model

Demographics. Our population of interest is made up by single mothers, whose life cycle is modeled as follows. Single mothers enter the economy at age 23 with one or two children (age 23 corresponds to the model age $s = 1$). Among those entering with one child, the child's age at the time they enter ranges from 0 (a newborn) to 3 years; these mothers face a risk of having a second child until they reach the biological age of 35. That is, in model ages $1 \le s \le 12$ mothers who entered with only one child face a per-period probability χ of having a second child. The age of a mother's first child is denoted by k_1, and, if a second child is born, its age is denoted by k_2 (if a second child is not present, $k_2 = \emptyset$). For single mothers entering the economy with two children, the distribution of children's ages, i.e. the distribution over (k_1, k_2) at entry, is specified below. Children live with their mothers until they turn 18 (i.e. until k reaches 18), at which moment they leave the household and can no longer be claimed as dependents. The model assumes mandatory retirement at age 67 ($s = 45$) and that mothers spend 15 years in retirement until they die at age 82 ($s = 59$).

Preferences. Preferences are described by a per-period utility function, $U(c/\psi(k_1, k_2), h)$, and by a discount factor β. Household consumption is denoted by c; and the equivalence scale, $\psi(k_1, k_2)$, accounts for the change in size and composition of the household.[4] The time endowment is normalized to 1; hours worked are denoted by h and the remaining $1-h$ hours are allocated to non-market activities, which may include leisure, time spent with the children, etc. We do not model the split of non-working hours across these alternative uses, and, hence, our utility function should be interpreted as aggregating the mother's utility from the time devoted to all these uses.

Labor productivity, earnings, income, and assets. The productivity of time devoted to market work depends on a deterministic, age-specific component, ε, as well as on an idiosyncratic stochastic component, z. Thus, a single mother that supplies h hours to market work receives labor income $e \equiv \varepsilon z h w$ before taxes and transfer payments, where w is the wage per efficiency unit of labor. When we extend the model to include human capital accumulation, the age-specific component ε will be replaced by a process of learning-by-doing, according to which the allocation of

[4] Note that in our model there is a direct correspondence between the age of the children k_1, k_2 and the number of dependent children, which we denote by n.

hours to market work can increase productivity. The idiosyncratic stochastic component z evolves according to a random walk with innovation ϵ,

$$\ln z' = \ln z + \epsilon, \qquad \text{with } \epsilon \sim N(0, \sigma_\epsilon^2). \tag{2.1}$$

Upon entering, single mothers draw their initial idiosyncratic productivity level, z_0, from a log-normal distribution $LN\big(0, \sigma_{\epsilon_0}^2\big)$.[5] Mothers enter the economy without assets, face a borrowing constraint, but can save in a one-period risk-free bond. Asset holdings are denoted by a, and the capital income derived from these assets by ra, where r is the risk-free interest rate. All single mothers with dependent children also receive child support, $\vartheta(k_1, k_2)$, from their children's father. Hence, pre-tax-and-transfer income is $\varepsilon zhw + ra + \vartheta(k_1, k_2)$. (Note that child support is not considered as income by the IRS.) Retired single mothers' income is made up of income from their savings plus Social Security benefits, denoted by b.

Child care costs. Single mothers with children at home may incur child care costs while working. Child care costs are assumed to depend on the children's age (k_1, k_2), on the number of hours worked (h), and on idiosyncratic characteristics of the mother (η). We think of the latter as characteristics determined by the mother's social network, which may provide a number of hours of free child care when the mother is at work (family members that do not live in the household, friends, neighbors, a church, etc.). For instance, a neighbor takes care of the child for, say, one hour a day; or the child is with her father a number of hours per week. Entering single mothers draw a value for η from a log-normal distribution. The value of η then evolves according to a law of motion denoted by $H(\eta', \eta, k_1, k_2)$ which is specified below. We denote a working mother's child care cost function by $\Gamma(h, k_1, k_2, \eta)$.

Taxes and transfers. Our model embeds the following tax-transfer programs: individual income and payroll taxes; the Earned Income Tax Credit (EITC); the Child Tax Credit (CTC); Temporary Assistance for Needy Families (TANF); and the Supplemental Nutrition Assistance Program (SNAP). We model these tax and transfer programs quite closely to the actual programs, including all their kinks and non-convexities. Specifically, the income tax scheme in the model contains the seven tax brackets, the tax deduction, and the personal exemptions. The payroll tax is a flat rate with a tax cap. The CTC has two tranches, one refundable and one non-refundable. The EITC is refundable and has both earnings and investment income limits for eligibility. TANF and SNAP are assistance programs that provide a guaranteed income to eligible applicants. We model their actual eligibility criteria (including the asset limits), out-of-work income support, and phase-outs.[6] Ap-

[5] The parameter values for the productivity process and the initial productivity distribution will be set to match wage dispersion at two different stages of the single mothers' life cycle. By setting these values internally, instead of relying on external estimates, we follow an approach similar to Guner et al. (2021).

[6] TANF programs vary widely across states in terms of generosity, deductions, earnings disregards, and the rules

pendix A presents detailed descriptions of these tax and transfer programs, and provides an account of the non-convexities created by each program on the budget constraints of single mothers.

We denote the net transfer received by a single mother (tax credits and assistance transfers minus income and payroll taxes paid) by $TT(a, e, h, k_1, k_2, \eta)$, that is

$$\begin{aligned} TT(a, e, h, k_1, k_2, \eta) \; &= \underbrace{\Big(EITC(a, e, k_1, k_2) + CTC(a, e, k_1, k_2)}_{\text{tax credits}} \\ &+ \underbrace{TANF(a, e, h, , k_1, k_2, \eta) + SNAP(a, e, h, k_1, k_2, \eta)\Big)}_{\text{assistance transfers}} \\ &- \underbrace{\Big(T(a, e, k_1, k_2) + T_p(e)\Big)}_{\text{income and payroll taxes}}. \end{aligned}$$

Note that net transfers depend on the number of dependent children as (i) mothers with children under the age of 18 file as head of household, while mothers with grown-up children file as single, (ii) the EITC rates and income thresholds depend on the number of dependents, (iii) mothers with grown-up children are not eligible for the CTC and TANF, and (iv) SNAP eligibility and benefits depend on the size of the assistance unit. It must also be noted that, conditional on having dependent children, net transfers depend on the age of the children because of the child care deductions used in the calculation of net income for the purpose of determining TANF and SNAP eligibility and benefits. These child care deductions also make net transfers depend on h and η. (See Appendix B for an illustration of the net transfer function.)

Bellman equations. We now write down the problems solved by single mothers in our economy.

The maximization problem of single mothers in working age. As stated above, single mothers enter the economy at age 23 with either one or two children. Specifically, a fraction q of mothers enter with two children, and the remaining fraction, $1 - q$, enter with one.[7] The Bellman equation of a working-age mother (i.e. in model periods $1 \leq s \leq 44$) is

$$v^s(z, a, k_1, k_2, \eta) = \max_{c,h,a'} \left\{ U\left(\frac{c}{\psi(k_1, k_2)}, h\right) + \beta \mathbb{E}\left[v^{s+1}(z', a', k_1', k_2', \eta')\right] \right\}, \tag{2.2}$$

subject to the budget constraint

$$c + \Gamma(h, k_1, k_2, \eta) + a' = e + (1 + r)a + \vartheta(k_1, k_2) + TT(a, e, h, k_1, k_2, \eta), \tag{2.3}$$

governing eligibility. Since implementing all this variation in our structural model is not tractable (some states also have different rules for urban and rural areas), we implement a TANF scheme that falls around the median generosity of TANF programs across the nation. We conduct a number of exercises below and show that our results are robust to variations in key TANF rules. We also assume full take-up of transfers.

[7] As explained below, the value of q is set to match the fraction of 23-year old single mothers with two children in our sample.

and to the laws of motion and feasibility restrictions

$$k_1' = \min(k_1 + 1,\ 18), \tag{2.4}$$

$$k_2' = \begin{cases} \min(k_2 + 1,\ 18) & \text{if } k_2 \geq 0, \\ 0 & \text{with probability } \chi \text{ if } k_2 = \emptyset \text{ and } s \leq 12, \\ \emptyset & \text{otherwise,} \end{cases} \tag{2.5}$$

$$z' \text{ evolves according to (2.1)}, \tag{2.6}$$

$$\eta' \text{ evolves according to law of motion } H(\eta', \eta, k_1, k_2), \tag{2.7}$$

$$0 \leq h \leq 1 \quad \text{and} \quad a' \geq 0. \tag{2.8}$$

The budget constraint, (2.3), is self-explanatory (recall that $e \equiv \varepsilon z h w$ denotes pre-tax-and-credits labor earnings). Equations (2.4) and (2.5) correspond to the two laws of motion for the children's age. Mothers of two can no longer claim their children as dependents as soon as both k_1 and k_2 reach value 18. Single mothers that reach model period $s = 13$ with only one child cannot claim a dependent beyond $s = 15$. The distributions of children's ages for entering mothers are denoted by F_1 and F_2, respectively for mothers entering with one and two children.

The maximization problem of single mothers in retirement age. In model periods $45 \leq s \leq 59$, mothers are retired. Their income is made up of savings income, ra, and Social Security benefits, b. In the last period of life, $s = 59$, they consume any remaining wealth. The Bellman equation of retired mothers is

$$v^s(a) = \max_{c,a'} \left\{ U(c) + \beta v^{s+1}(a') \right\} \tag{2.9}$$

$$\text{s.t.} \qquad c + a' = b + (1+r)a, \qquad a' \geq 0,$$

where $v^{59}(a) = U(b + (1+r)a)$.

It must be noted that the model just outlined introduces a number of restrictions which are not met by all U.S. single parent households. For instance, we limit the sources of household income to own labor earnings, investment income, child support, and government transfers, omitting private transfers, income from business, unemployment income, etc. We assume that upon entering the economy single mothers' education level remains unchanged throughout their lives, thus omitting those who become educated later on in life. We also restrict the age at which women become first-time single mothers—i.e., the age at which they have their first child—to the age range 20 to 23 years.[8]

[8]Using this age range, and the distribution of children' age obtained from the data, the model yields a median age

Before moving on to the analysis of the optimal reforms, we find it useful to stress that we will keep fixed the tax-transfer system to single mothers without dependents (single mothers whose children are 18 or older). We will conduct our analysis both without tagging policy by child age, as under the current system (the parameters in the income transfer programs are not allowed to depend on the age of the children) and with tagging by child age (where transfer program parameters can vary with the age of the children).

2.1 Parameterization

Preferences. Per-period utility is represented by an additively separable utility function in consumption and non-market time, extended to include a quasi-fixed cost of labor market participation. This participation cost is assumed to take on three different values, depending on whether the single mother chooses to work part time (PT), full time (FT), or extra time (ET).[9]

We adopt the following functional form

$$U\left(\frac{c}{\psi(k_1,k_2)},h\right) \;=\; \frac{(c/\psi(k_1,k_2))^{1-\sigma}-1}{1-\sigma} + \varphi\frac{(1-h)^{1-\zeta}-1}{1-\zeta} - \mathbb{1}_{h>0}\times\begin{cases}\nu_{PT} & \text{if } 0<h\leq\bar{h}_1\\ \nu_{FT} & \text{if } \bar{h}_1<h\leq\bar{h}_2\\ \nu_{ET} & \text{if } h>\bar{h}_2,\end{cases} \tag{2.10}$$

where σ is the coefficient of relative risk aversion, φ is a utility weight on non-market time, and $\zeta>0$ controls the Frisch elasticity of labor supply. The last term in (2.10) is the utility cost of labor market participation. The hours thresholds $\bar{h}_1$ and $\bar{h}_2$ define part-, full- and extra-time employment. Part-time work corresponds to working less or equal than 1,300 hours per year; full-time work corresponds to working between 1,301 and 2,080 hours; and extra-time work to more than 2,080 hours.

at which women become first-time single mothers of 22 years, which is very close to the median age of 23 years among the U.S. population of single mothers (calculated from a sample of 16-53 year-old single mothers drawn from ASEC). By construction, this modeling strategy does not capture single mothers on the two tails of the distribution. For instance, the 25 and 75 percentiles of the variable "age at which women become first-time single mothers" generated by the model are, respectively, 21 and 23 years, and the ones from the data are 20 and 27 years.

[9] Our assumption of a quasi-fixed cost of working generalizes the standard model of labor supply by introducing non-convexities also along the intensive margin, and thus discontinuous hours decisions. This is especially important for the population of interest in our study, single mothers. For instance, a mother working part-time, 5 hours a day say, is likely to face a discrete (non-continuous) change in the utility cost of working if she decides to work one more hour as she may be unable to share meals with the children, may miss school drop-off or pick-up, or miss the children's wake-up or bedtime hours, etc. For related generalizations of the standard labor supply model that also imply discrete behavior along the intensive margin see, for instance, Bick et al. (2022) who develop a model where the fixed cost of working varies across sectors. Saez (2002) also uses a model with discrete occupational choice.

Child care costs. The child care cost function is parameterized as follows

$$\Gamma(h, k_1, k_2, \eta) = \max\left\{\gamma_j + \eta \times h\ ,\ 0\right\} \qquad \text{for} \;\; j = y, o, \tag{2.11}$$

where $j = y$ corresponds to single mothers with at least one child of pre-school age (0-4 years); and $j = o$ to single mothers with only school-age children (5-17 years). γ_y, $\gamma_o < 0$ are parameters, and η is a persistent idiosyncratic component. Single mothers draw a value of η upon entering the economy from the log-normal distribution $LN(\mu_y^\eta, \sigma_y^\eta)$; after that the law of motion for η is specified as follows. Mothers draw a new value for η in two instances: (i) if a second child is born, in which case the new value for η is drawn from the same distribution; and (ii) when a child in the household turns 5, in which case the new value for η is drawn from $LN(\mu_o^\eta, \sigma_o^\eta)$ if she has no child under 5, or from $LN(\mu_y^\eta, \sigma_y^\eta)$ if she has another child who is under 5. That is, conditioning on either (i) or (ii) occurring, if $j = y$ the new value for η is drawn from a distribution with parameter vector $(\gamma_y, \mu_y^\eta, \sigma_y^\eta)$. If $j = o$, the new η is drawn with parameter vector $(\gamma_o, \mu_o^\eta, \sigma_o^\eta)$. In periods in which none of these two conditions applies, mothers retain the same value for η as in the previous period. Lastly, if there are no children in the household, child care costs are zero. That is,

$$\Gamma(h, k_1, k_2, \eta) = \begin{cases} \max\left\{\gamma_y + \eta \times h\ ,\ 0\right\} & \text{if} \;\; k_1 \leq 4 \;\vee\; k_2 \leq 4, \\ \max\left\{\gamma_o + \eta \times h\ ,\ 0\right\} & \text{if} \;\; (5 \leq k_1 \leq 17 \;\wedge\; k_2 = \emptyset) \;\vee\; (5 \leq k_2 \leq 17), \\ 0 & \text{otherwise.} \end{cases}$$

Note that the idiosyncratic component η determines (i) the number of hours worked at which the mother needs to start purchasing child care, and (ii) the cost of child care per hour. To the left of the intercept with the hours-axis, child care costs are zero (e.g., because someone from the mother's social network can look after her children for a few hours a day), thus rationalizing the empirical observation that many working single mothers do not pay child care while working. At the same time, the model delivers heterogeneity in paid child care costs across mothers with the same number of hours worked, which is also in accordance with what we observe in the data.

3 Data, Calibration and Model Fit

3.1 Data and Summary Statistics

We calibrate the model to a sample of single mothers with one and two dependent children drawn from the 2013-2018 Annual Social and Economic (ASEC) Supplements of the Current Population Survey (CPS). For a single mother to be in our sample she needs to meet all the following conditions:

(1) She is between 23 and 53 years of age and has never been married;[10] (2) She lives with her children who are under 18 years of age; (3) Neither the father of the children nor a mother's partner live with them in the same dwelling; (4) The single mother is the reference person in a *female reference person family*, and has no family member, other than her children, living in the same dwelling; (5) She is not in the armed forces; (6) She did not receive any income from: business and/or farm activities, disability, retirement, social security, unemployment, veterans income, and survivors' benefits.

Our final sample of single mothers with dependent children contains 5,207 households, including 7,316 children. Table 3 (panel B) below presents summary statistics of labor market variables (employment rates, annual hours worked, and earnings) for non-college-educated single mothers in our sample.[11] Single mothers with one child have a higher employment rate than single mothers with two children, 83.39 versus 78.43 percent. Among single mothers of one child, the employment rate is higher for those with a school-age child (5-17 years) at 84.72 percent, compared to an employment rate of 80.88 percent for those with a pre-school age child (0-4 years). A similar pattern is found among single mothers with two children. Conditional on working, single mothers of one child supply more hours to work than mothers of two: $1,741$ annual hours versus $1,662$. Average annual earnings, conditional on working, are also higher among single mothers of one child: $\$24,384$ versus $\$21,414$ (in dollars of 2017).

3.2 Calibration

Parameters calibrated exogenously. A period in the model is one year. The coefficient of relative risk aversion is set at $\sigma = 1.5$, which is the midpoint value of the interval estimated by Chetty (2006). The real interest rate is set at $r = 0.02$. We apply the equivalence scales used by the CPS to calculate per-adult equivalent household expenses and set $\psi_1 = 1, \psi_2 = (1 + 0.8)^{0.7} = 1.509$, and $\psi_3 = (1 + 0.8 + 0.5)^{0.7} = 1.791$. The preference parameter ζ affects the Frisch elasticity of labor supply. In models like ours where budget constraints contain kinks and preferences are discontinuous (fixed costs of labor market participation), this parameter is difficult to pin down. We first set ζ to 3 so that the Frisch elasticity at average hours of work equals 0.74. As a sensitivity analysis we then increase ζ to 4, which is associated with a Frisch elasticity of 0.55. The hours thresholds for

[10] To calibrate the extension of the model in Section 5 (learning by doing) we will include single mothers aged 54-66 years with no dependent children. Since divorced mothers may receive part of the joint assets saved while married and also income from alimony, we do not include them in our sample.

[11] Since a large portion of our sample is made up of non-college-educated single mothers—69.6 percent—and since transfer-program reforms affect these mothers' behavior and outcomes relatively more than those of the college educated, we relegate summary statistics for the latter group to Appendix C. Although our analysis of the optimal reforms includes both college and non-college-educated single mothers, our discussion throughout will focus mainly on the non-college educated, unless otherwise noted.

part-, full-, and extra-time work are, respectively, 0.237 and 0.376.

We calibrate the fertility process separately for mothers with and without a college degree, so that the demographic structures of these two groups match the ones from the data. Among the non-college educated (nc), the fraction of mothers that enter with two children is set at $q_{nc} = 0.40$, and the annual probability of having a second child for those who entered with one is set at $\chi_{nc} = 0.025$. For mothers with a college degree (c), we set $q_c = 0.25$ and $\chi_c = 0.01$. These numbers generate age profiles for the percentage of mothers with two children similar to those in the data (see Table C1 in Appendix C for the empirical and model-implied age profiles). We specify the children's age distributions for entering mothers, $\digamma_1$ and $\digamma_2$, using the empirical distributions for mothers aged 23 in our sample (Appendix C contains further details, and Table C2 reports the parameter estimates). Finally, we set child support using the average values received by mothers in our sample: for non-college-educated mothers of one (two), child support is $\vartheta_{1,nc} = 3,343$ ($\vartheta_{2,nc} = 3,786$), and for college-educated mothers, child support is $\vartheta_{1,c} = 4,890$ and $\vartheta_{2,c} = 5,976$, respectively.[12]

Table 1 presents the parameter values set outside of the model in our benchmark economy. (For the values of the parameters in the tax-transfer programs and their sources, see Appendix B.)

TABLE 1– PARAMETERS SET EXOGENOUSLY

Description	Param.	Value	Description	Param.	Value
Risk aversion	σ	1.5	Child support (1 child)	$\vartheta_{1,nc}$	$3,343$
Real interest rate	r	0.020	Child support (1 child)	$\vartheta_{1,c}$	$4,890$
Curv. non-market time	ζ	3	Child support (2 children)	$\vartheta_{2,nc}$	$3,786$
Equivalence scale	ψ_1	1	Child support (2 children)	$\vartheta_{2,c}$	$5,976$
Equivalence scale	ψ_2	1.509	Prob. entering with 2 children	q_{nc}	0.40
Equivalence scale	ψ_3	1.791	Prob. entering with 2 children	q_c	0.25
Hours threshold	$\bar{h}_1$	0.237	Annual prob. second child	χ_{nc}	0.025
Hours threshold	$\bar{h}_2$	0.376	Annual prob. second child	χ_c	0.010

Parameters calibrated endogenously. The remaining parameters are calibrated internally so that the model matches a set of empirical moments obtained from our sub-samples of college and non-college-educated single mothers.[13] The empirical targets calculated from our sub-sample of non-college-educated single mothers are listed below (in parenthesis we indicate the parameter that

[12]While there is dispersion in child support received by single mothers in our sample, the model abstracts from both heterogeneity and risk in child support. There are several reasons for this modeling choice. It is not uncommon that custodian parents have informal agreements with noncustodial parents concerning child support payments which may lead to misreporting. Also, liquidity-constrained noncustodial parents that missed payments may pay out more than one year of child support in a single installment. Since child support risk for lone mothers is not well understood we use the mean values from the sample.

[13]The discount factor, β, is assumed to be common to all single mothers, and the targeted empirical moment is

influences each moment the most). Table C3 in the Appendix reports the empirical targets and the calibrated parameter values for college-educated mothers.

1. Average hours worked, conditional on working, represent 31.2 percent of the time endowment. (φ)

2-4. The employment rate is 81.2 percent. The fractions of working mothers who work part-time, full-time and extra-time are 26.3, 66.2 and 7.5 percent respectively. ($\nu_{PT}, \nu_{FT}, \nu_{ET}$)

5. Average hourly earnings over the mothers' life cycle. We match average hourly earnings by setting the deterministic component of the wage rate. (ε_s for $1 \leq s \leq 44$)

6-7. The interquartile range of hourly earnings for working mothers aged 23-26 is \$5.74. The interquartile range twenty years later, that is, for mothers aged 43-46 is \$9.50. (σ_{ϵ_0}, σ_ϵ)

8-9. The fraction of working mothers paying child care is 38.2 percent (among those with children aged 0-4 years), and 22.6 percent (among those with children aged 5-17 years). (γ_y, γ_o)

10-13. The average and standard deviation of child care expenditures, conditioning on being positive, paid by working mothers with children aged 0-4 years are \$3,163 and \$2,856, respectively; and \$2,940 and \$2,721 for those with children aged 5-17 years. ($\mu_y^\eta, \sigma_y^\eta, \mu_o^\eta, \sigma_o^\eta$)

14. Average financial wealth among single mothers with one and two children is \$11,386 (Survey of Consumer Finances 2019). (β)

15. We use the Social Security's Average Indexed Monthly Earnings (AIME) to set retirement benefits, b.

Table 2 reports the parameter values that match these moments.

3.3 Benchmark Solution and Model Fit

Summary statistics for labor market outcomes obtained from the model are shown in panel A of Table 3; their empirical counterparts are shown in panel B. The model fits well moments that were not used as targets. The employment rates for single mothers with one and two children—which were not calibration targets—match well those in the data. Annual hours worked, total earnings,

the sample-wide value for average financial wealth. It must be noted, however, that the model fits average financial wealth by education relatively well, despite the restriction to a common discount factor.

TABLE 2– PARAMETERS CALIBRATED ENDOGENOUSLY (NON-COLLEGE EDUCATED)

Description	Param.	Value	Moment	Target	Model
Weight non-market time	φ	0.118	Avg hours worked	0.312	0.312
Participation cost PT	ν_{PT}	0.137	Employment rate	0.812	0.813
Participation cost FT	ν_{FT}	0.142	Fraction full-time	0.662	0.658
Participation cost ET	ν_{ET}	0.143	Fraction extra-time	0.075	0.077
Log-normal distribution	μ_y^η	0.90	Avg. child care paid	3,163	3,152
Log-normal distribution	μ_o^η	0.30	Avg. child care paid	2,940	2,837
Log-normal distribution	σ_y^η	2.80	Std. child care paid	2,856	2,795
Log-normal distribution	σ_o^η	1.60	Std. child care paid	2,721	2,796
Child care intercept	γ_y	-1.58	Frac. paid child care	0.382	0.379
Child care intercept	γ_o	-2.32	Frac. paid child care	0.226	0.219
Dispersion initial prod.	σ_{ϵ_0}	0.48	P75-P25 wages 23-26	5.74	5.79
Random walk innov.	σ_ϵ	0.09	P75-P25 wages 43-46	9.50	9.69
Discount factor	β	0.977	Avg. wealth	8,514	9,833
Age-specific productivity	ε_s	cf. text	Hourly earnings by age		
Retirement benefit	b	14.54	AIME formula		

Notes: Endogenously calibrated parameter values and empirical targets from the sub-sample of non-college-educated single mothers. Table C3 in the Appendix reports the parameter values for college-educated mothers along with the corresponding empirical targets.

and hourly earnings by number of children are also close to their empirical values. The model also matches well the profile of single mothers' financial asset holding. For instance, the model yields average financial assets among mothers aged 23 to 32 years equal to \$4,115, and among those aged 33-42 equal to \$24,788; the corresponding empirical averages using data from the SCF are \$5,642 and \$24,697, respectively. The model matches fairly well the correlation between child care expenditures per hour and hourly wages in our sample, another moment that was not a calibration target. Among all working single mothers the model generates a correlation of 0.101, which is close to its empirical counterpart, 0.08. When conditioning on working mothers with a child under 5 years of age the model yields a correlation of 0.2, against an empirical correlation of 0.162. Restricting further to working mothers with strictly positive child care expenditures the correlation in the model is 0.233, compared to 0.164 in the data. To account for outliers, we computed this latter correlation in logarithms and obtained a correlation of 0.175 in the model and 0.24 in the data.

The labor supply responses to changes in the wage rate under our benchmark parameterization are presented in Table 4. We report the extensive, intensive, and total hours elasticities for different subsamples of single mothers and holding fixed the distribution of wealth, which implies that the reported elasticities should be interpreted as short-run elasticities. Extensive margin elasticities

TABLE 3– SUMMARY STATISTICS: MODEL VS DATA (NON-COLLEGE EDUCATED)

	Single mothers children aged 0-17		Single mothers of one child		Single mothers of two children		
	# of children		age of child		age of children		
	1	2	0-4	5-17	0-4	mixed	5-17
A. BENCHMARK SOLUTION							
Employment rate (%)	85.22	76.21	76.52	87.29	68.79	70.57	79.34
Part-time (%)	16.78	40.30	31.28	13.76	48.74	44.51	37.72
Full-time (%)	73.50	54.92	65.88	75.09	50.05	53.09	56.27
Extra-time (%)	9.72	4.77	2.85	11.15	1.21	2.41	6.01
Annual hours worked†							
Average	1,798	1,576	1,605	1,821	1,505	1,521	1,603
p25	1,696	1,281	1,281	1,779	1,281	1,281	1,281
p75	2,027	1,820	1,862	2,048	1,737	1,758	1,882
Annual earnings† ($)							
Average	25,616	23,367	18,739	27,048	17,928	20,818	24,989
p25	14,452	13,549	9,820	16,658	13,400	13,476	13,833
p75	29,588	20,815	22,191	30,790	17,628	18,806	30,524
B. DATA							
Employment rate (%)	83.39	78.43	80.88	84.72	70.98	78.57	80.19
Part-time (%)	23.48	29.87	32.25	16.27	44.29	31.68	25.62
Full-time (%)	69.06	62.60	62.72	72.26	51.50	61.62	65.61
Extra-time (%)	7.45	7.52	5.02	8.68	4.19	6.69	8.75
Annual hours worked†							
Average	1,741	1,662	1,602	1,811	1,414	1,633	1,733
p25	1,440	1,248	1,040	1,600	720	1,152	1,300
p75	2,080	2,080	2,080	2,080	2,080	2,080	2,080
Annual earnings† ($)							
Average	24,384	21,414	19,770	26,711	15,544	19,689	23,739
p25	13,727	10,444	10,679	15,511	7,238	9,931	12,041
p75	31,552	29,902	26,294	34,958	22,459	26,967	31,961

Notes: †Conditional on working. In dollars of 2017.

are computed as the percentage change in employment rates after a one percent change in wages. Single mothers with two children have higher extensive margin elasticity: 0.21 for mothers of one child, and 0.56 for mothers of two. Intensive margin elasticities are computed as the percentage change in hours worked among working mothers (i.e. those working both before and after the

TABLE 3– CONTINUATION

	Single mothers children aged 0-17		Single mothers of one child		Single mothers of two children		
	# of children		age of child		age of children		
	1	2	0-4	5-17	0-4	mixed	5-17
A. BENCHMARK SOLUTION							
Hourly earnings† ($)							
Average	13.58	14.34	11.35	14.05	12.03	13.50	14.97
p25	8.79	9.50	7.69	8.79	8.97	8.97	9.50
p75	16.24	17.11	14.22	17.11	14.22	16.24	17.56
B. DATA							
Hourly earnings† ($)							
Average	13.97	12.51	12.51	14.70	10.94	11.78	13.29
p25	9.24	7.92	8.16	9.70	7.46	7.70	8.13
p75	17.17	15.36	14.66	18.03	12.61	14.93	16.68

Notes: †Conditional on working. In dollars of 2017.

change in the wage rate). For single mothers with one child the intensive margin elasticity is 0.12; the value for single mothers of two is similar. Total hours elasticities are 0.36 for mothers with one child, and 0.77 for mothers of two. Mothers with two children where at least one is aged 0-4 years have the highest elasticity at 0.92. In sum, under the benchmark policy and parameter values the participation margin is more responsive than the intensive margin. While there are no direct empirical counterparts against which these responses can be compared, they are in line with the simulated wage elasticities for single mothers obtained from the models used by other authors (e.g. Blundell and Shephard 2012, Blundell et al. 2016).

In the benchmark economy, net transfers paid to single mothers with dependent children (i.e. total transfers minus total tax revenues collected from them) amount to $1,035 per household. This corresponds to the amount of income redistributed from the rest of the economy to our population of interest. Net transfers to single mothers with one child amount to -$1,113 (i.e. they are net contributors), also on a per-household basis, and to $4,537 to single mothers with two children. As will become clearer in Section 4, we assume throughout this study that the amount of redistribution towards our population of interest remains constant.

TABLE 4– ELASTICITIES OF LABOR SUPPLY (SINGLE MOTHERS AGED 23-53)

	Single mothers children aged 0-17		Single mothers of one child		Single mothers of two children		
	# of children		age of child		age of children		
	1	2	0-4	5-17	0-4	mixed	5-17
Extensive margin	0.21	0.56	0.25	0.20	0.51	0.61	0.55
Intensive margin	0.12	0.12	0.43	0.06	0.29	0.17	0.08
Total hours	0.36	0.77	0.72	0.28	0.92	0.92	0.71

3.4 Guaranteed Income and Work Incentives in the U.S. Tax-Transfer System

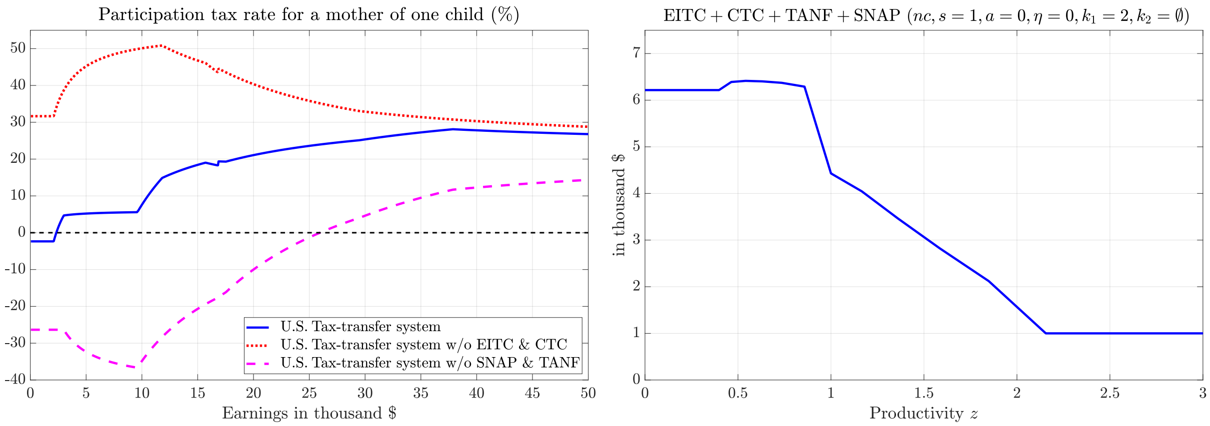
Before moving to the analysis of the optimal reform of income support, we find it useful to start with an illustration of how the benchmark tax-transfer system trades off a guaranteed income (TANF and SNAP) with subsidies to work (EITC and CTC). The left panel of Figure 1 plots the labor market participation tax rates at different levels of earnings that emerge from the benchmark tax-transfer system (solid blue line), and from two counterfactuals. For the sake of expositional clarity, the participation tax rates in Figure 1 correspond to single mothers of one with no wealth and no child care expenses while working.[14] Participation tax rates are computed as the difference between net transfers if working and net transfers if not working, as a percentage of earnings

$$\tau^P(e) = -\frac{TT(e) - TT(0)}{e}. \tag{3.1}$$

As is apparent from Figure 1, the current tax-transfer programs yield negative participation tax rates of about -2 percent at low levels of earnings, and then increase to reach values of about 27 percent at earnings over \$35,000. The average participation tax rate among all working mothers with dependent children in our benchmark economy is 10.5 percent. (Note that mothers with assets above the TANF and SNAP asset limit face lower rates than mothers with no assets.) Participation tax rates depend critically on the level of guaranteed income and how quickly it phases out, as well as on the earnings subsidies to low-wage workers. It is useful to look at the two polar cases that emerge under no guaranteed income, on the one hand, and under no earnings subsidies, on the other hand. The dashed purple line in Figure 1 shows the participation tax rates in the first case (i.e. if TANF and SNAP were to be removed). In this case, working mothers earning up to \$26,000 would face negative participation tax rates. In fact, single mothers earning between five and ten thousand dollars would get a participation subsidy amounting to more than one third of their earnings. The

[14]Participation tax rates for mothers of two are similar to the ones depicted in Figure 1.

FIGURE 1– PARTICIPATION TAX RATES AND INCOME FROM EITC, CTC, TANF, SNAP



Notes: Left panel: Labor market participation tax rates for a single mother with no assets and no child care costs. Right panel: Sum of income collected by a single mother with no assets and no child care costs from the EITC, CTC, TANF and SNAP as a function of labor productivity.

dotted red line shows the participation tax rates in the second polar case (i.e. if EITC and CTC were to be removed). Single mothers would face participation tax rates higher than 40 percent at earnings levels between \$3,000 and \$20,000, and higher than 30 percent up to earnings of \$40,000. The participation tax rates under the benchmark policy lie roughly in the middle of those generated by these two polar cases.

We also use the policy function for hours worked obtained from the solution of the model to illustrate how the benchmark mix of guaranteed income and work subsidies shapes the relationship between labor productivity, labor supply decisions, and income transfers. The right panel of Figure 1 displays the sum of income collected by single mothers from the EITC, CTC, TANF and SNAP as a function of their labor productivity. Specifically, we use the optimal policy of a newly-entered single mother of one child with no child care costs while working. (Using the optimal policy function of mothers of two would yield qualitatively similar results.) The flat segment at \$6,216 (which is the guaranteed income for mothers of one child) shows that mothers with labor productivity below 0.45—which is about 41 percent of the average productivity—choose not to work. Above this productivity threshold they supply hours to market work, and the sum of the transfers they collect from these programs does not decline monotonically with labor productivity. The mix of guaranteed income and work incentives affects both the productivity threshold for participation and the level of hours worked conditional on participation. In the next section we assess the optimality of this mix in the benchmark tax-transfer system.

4 The Optimal Reform of the Tax-Transfer System

This section describes the quantitative exercises and presents our findings characterizing the optimal reform of the tax-transfer system. As explained above, we assess reforms within the parametric family of the actual system. Since the tax-transfer system in our model includes two tax codes (income and payroll taxes), two tax credits (EITC and CTC) and two income welfare programs (TANF and SNAP), and since each contains a large number of parameters, we must necessarily specialize our analysis to a subset of these parameters. Notwithstanding this restriction, we retain enough flexibility to generate and assess a large set of feasible reforms that fully reshape the current system, especially in terms of the generated trade-off between a guaranteed income and work incentives to low-wage earners. The reforms assessed in this paper also maintain the asset-eligibility conditions of the current system. Specifically, we retain the asset-income limit of the EITC, and the asset limits of TANF and SNAP (we discuss below the role of asset limits).

The set of feasible reforms. For the sake of clarity, let us re-write the EITC equation for eligible workers with earnings e and n dependent children as (see Appendix A for details)

$$EITC(e,n) = \begin{cases} \pi_0^n + \pi_1^n e & \text{if} \quad 0 \leq e < \mathrm{e}_{I_1}^n \\ \pi_0^n + \pi_1^n \mathrm{e}_{I_1}^n & \text{if} \quad \mathrm{e}_{I_1}^n \leq e < \mathrm{e}_{I_2}^n \\ \max\{\pi_0^n + \pi_1^n \mathrm{e}_{I_1}^n - \pi_2^n (e - \mathrm{e}_{I_2}^n), 0\} & \text{if} \quad e \geq \mathrm{e}_{I_2}^n. \end{cases}$$

By definition of an earned income tax credit, the values of π_0^n, for $n = 0, 1, 2$, in the current EITC schedule are equal to zero; π_1^n are the earnings subsidy rates in the phase-in region, and π_2^n are the phase-out rates. The thresholds, $\mathrm{e}_{I_1}^n$ and $\mathrm{e}_{I_2}^n$, mark the end of the phase-in region and the beginning of the phase-out region, respectively. In the region between these thresholds, the credit plateaus at its maximum value $\pi_0^n + \pi_1^n \mathrm{e}_{I_1}^n$.

We consider reforms that change the values of π_0^n, π_1^n and $e_{I_1}^n$ for $n = 1, 2$, and that are revenue neutral (i.e., the government's net transfer to the population of single mothers with one and two dependent children is kept constant at the level of the benchmark economy). We let the government maximize over these parameters, leaving one of them free to meet the revenue neutrality restriction. Note that with only these parameters we can generate a continuum of reforms, spanning the following two extreme policies. On the one hand, the EITC can be transformed into a TANF-like program ($\pi_0^n > 0$ and $\pi_1^n = 0$), which would increase the guaranteed income and remove work subsidies. On the other hand, the sum of TANF, SNAP and EITC can be transformed into an EITC-like program ($\pi_0^n < 0$ and $\pi_1^n > 0$), which would reduce the guaranteed income and increase work subsidies. Our parametric class of reforms is relatively flexible, enabling us to assess a large set of income-transfer systems featuring a rich array of marginal and participation tax rate profiles at low income levels.

Welfare criterion. We compute the reform of the transfer system that maximizes the average of entering single mothers' ex-ante expected utility, subject to the revenue-neutrality condition. By ex ante we mean before the entering single mother draws her initial level of productivity, z, the idiosyncratic characteristic, η, which determines her child care needs and cost if she works, and before knowing whether she will have one or two children. Specifically, the government chooses the policy, $\{\pi_0^n, \pi_1^n, e_{I_1}^n\}_{n=1}^2$, that maximizes the average of ex-ante expected utilities of entering non-college and college-educated single mothers

$$\sum_{j=c,nc} m_j \sum_{i=1,2} q_{ji} \sum_{(k_1,k_2)\in K_i} F_i(k_1,k_2) \int_z \int_\eta v_j^1\big(z, a=0, k_1, k_2, \eta; \pi_0^n, \pi_1^n, e_{I_1}^n\big) f_j(z) g_j(\eta) dz d\eta, \quad (4.1)$$

where m_j is the measure of mothers of education level j; $q_{j1} = 1 - q_j$ and $q_{j2} = q_j$ are the fractions of mothers of education level j entering with one and two children, respectively. The supports of the children's age distributions at the time single mothers enter the economy (K_1 for mothers entering with one child and K_2 for those entering with two) are specified in Appendix C. Functions v_j^1, for $j = c, nc$, are the value functions of college- and non-college-educated entering single mothers under policy parameters π_0^n, π_1^n and $e_{I_1}^n$. Functions f_j are the densities of log-normal distributions with parameters 0 and $\sigma_{\epsilon_0,j}$; functions g_j are the densities of log-normal distributions with parameters $\mu_{y,j}^\eta$ and $\sigma_{y,j}^\eta$.

We also assess the support of incumbent single mothers for the optimal reform (i.e. among single mothers who are already in the economy at the time the reform is implemented). To this end, we use the distribution of single mothers from the model under the benchmark policy and then compute their continuation values using the policy functions under the optimal reform.

4.1 The Optimal Reform without Tagging by Age of Child

We start with the case where policy is not contingent on the age of the children. That is, the policy parameters that define the set of policy reforms, $(\pi_0^n, \pi_1^n, e_{I_1}^n)$, for $n = 1, 2$, are not allowed to vary with the children's age, as is the case in the benchmark policy. The government gives equal treatment to mothers with children under 5 years of age and to mothers with children aged 5-17 years. Under this restriction, the optimal policy must trade off the provision of insurance to single mothers of preschool-age children against the provision of work incentives to mothers of school-age children. On the one hand, since single mothers enter the economy with one or two young children, no assets, face relatively higher child care needs if they work, and a lower market wage rate, they are more likely to be credit constrained at the beginning of their life cycle than later on when the children are of school age. As working longer hours is the only means for credit-constrained mothers to smooth consumption, the provision of insurance via a higher guaranteed income allows them to more efficiently allocate labor over the life cycle, thus furthering their ability to smooth marginal utility. On the other hand, the provision of work incentives helps to retain mothers of school-age children in the labor force, which contributes to alleviate the budget constraint of the government. To make the computation of the optimal reform tractable, we impose the following three conditions on the six parameters defining the set of reforms: EITC payments to non-working mothers with two children are twice the payments to non-working mothers with one child, i.e. $\pi_0^2 = 2\pi_0^1$; credit rates in the phase-in region, relative to their benchmark values, must be the same for mothers with one and two dependent children, that is, $\pi_1^1/\pi_{1b}^1 = \pi_1^2/\pi_{1b}^2$, where subscript b is for benchmark; the maximum credits, relative to their benchmark values, must also be the same for mothers with one and two dependent children, $(\pi_0^1 + \pi_1^1 \mathrm{e}_{I_1}^1)/\pi_{1b}^1 \mathrm{e}_{I_1 b}^1 = (\pi_0^2 + \pi_1^2 \mathrm{e}_{I_1}^2)/\pi_{1b}^2 \mathrm{e}_{I_1 b}^2$. (Note that under these conditions the optimal reform preserves some features of the benchmark schedule concerning the relative distribution of transfers to mothers of one and two children.)

The Welfare-maximizing Income-transfer System.

The values of the parameters $(\pi_0^n,\ \pi_1^n,\ e_{I_1}^n)$, for $n = 1, 2$, at the optimal policy are presented in Table 5. Compared to the values of the benchmark policy, the optimal reform amounts to a partial transformation of the EITC towards a TANF-like program. Namely, π_0^n increase from \$0 to \$880 for mothers with one child and to \$1,760 for mothers with two children, which, added to income from TANF and SNAP, yield an increase in their guaranteed incomes from \$6,216 to \$7,096, and from \$8,563 to \$10,323, respectively.[15] The earnings subsidy rates in the phase-in region, (π_1^1, π_1^2), decrease from the benchmark values of $(0.340, 0.400)$ to $(0.187, 0.220)$; and the income thresholds defining the end of the phase-in regions, $(e_{I_1}^1, e_{I_1}^2)$, decrease from (\$9,559, \$13,430) to (\$9,190, \$11,524), resulting in a lower maximum tax credit and a longer plateau region.

Figure 2 displays the benchmark and the optimal EITC schemes (top panels), and the sum of income transfers from the EITC, CTC, TANF and SNAP collected by single mothers with no assets as a function of their earnings (bottom panels). It is readily apparent from this figure that the shift from work subsidies to guaranteed income brought about by the optimal policy comes at the cost of positive marginal tax rates at low earnings. It should be noted that the additional guaranteed

[15] Recall that the EITC has an investment income limit of \$3,300 but no asset limit for eligibility. Hence, the additional guaranteed incomes prescribed by the optimal policy are paid to all non-working mothers with investment income below \$3,300. Some of these mothers, however, may not be eligible for TANF and SNAP as they may fail the asset limit test for these latter programs. In short, the \$880 and \$1,760 prescribed by the optimal EITC schedule should be regarded as child allowances to single mothers of one and two children, respectively, with no labor earnings and with investment income below \$3,300 per year.

TABLE 5– BENCHMARK AND OPTIMAL POLICY WITHOUT TAGGING

	π_0^n		π_1^n		$e_{I_1}^n$	
	$n=1$	$n=2$	$n=1$	$n=2$	$n=1$	$n=2$
Benchmark policy	\$0	\$0	34.0%	40.0%	\$9,559	\$13,430
Optimal policy w/o tagging	\$880	\$1,760	18.7%	22.0%	\$9,190	\$11,524

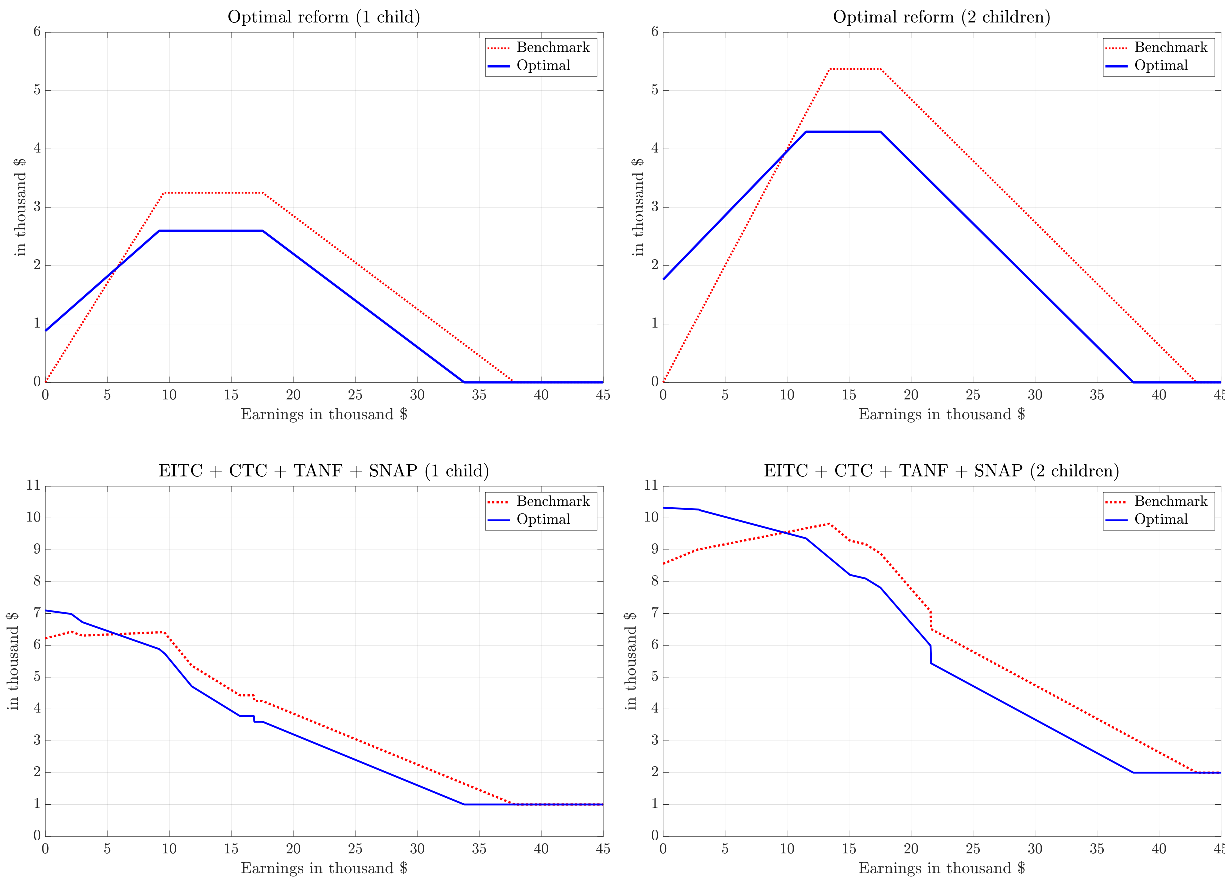
income provided by the optimal reform is phased out from the first dollar of earnings.

The shift towards a higher guaranteed income also increases labor market participation tax rates. The top panels of Figure 3 show the participation tax rates faced by single mothers of one and two children with no assets and no child care costs. The optimal policy yields positive participation tax rates even at very low earnings, increasing benchmark tax rates by as much as 15 percentage points up to earnings of about \$10,000 for mothers with one child, and by almost 20 percentage points for mothers with two children. As a result, labor supply drops significantly under the optimal policy, especially among low-productive mothers. The productivity thresholds for labor market participation increase from 0.40 to 0.54 for mothers with one child, and from 0.54 to 0.86 for mothers of two. This is illustrated in the bottom panels of Figure 3, where we plot total income collected from EITC, CTC, TANF and SNAP as a function of labor productivity. These panels also show that mothers who work under the optimal policy collect less income from these programs than they would under the benchmark policy if they had also worked.

Model summary statistics under the optimal policy are presented in column [3] of Table 6. In line with the discussion above, employment rates decline. Namely, the employment rate of mothers with one child declines from 88.0 percent under the benchmark policy to 73.5 percent under the optimal; for mothers with two children, employment falls from 79.1 to 50.4 percent. The decline in the employment rate is more pronounced among single mothers with children aged 0-4 years (24.1 percent from the benchmark) than among mothers with children aged 5-17 (18.8 percent from benchmark). However, conditional on working, average hours worked increase. This effect follows from the change in the composition of working mothers: Low-productive mothers drop out of the labor market under the optimal policy, hence increasing average hours. Among the mothers that work both under the benchmark and the optimal policy, average hours worked remain mostly unchanged.

Average annual earnings, conditional on working, increase by more than \$4,000 among mothers of one child, and by more than \$8,000 for mothers of two. Again, this is explained by the fact that low-productive mothers that are pulled into the labor market by the high work subsidies of the benchmark policy leave the market under the optimal policy as participation tax rates increase.

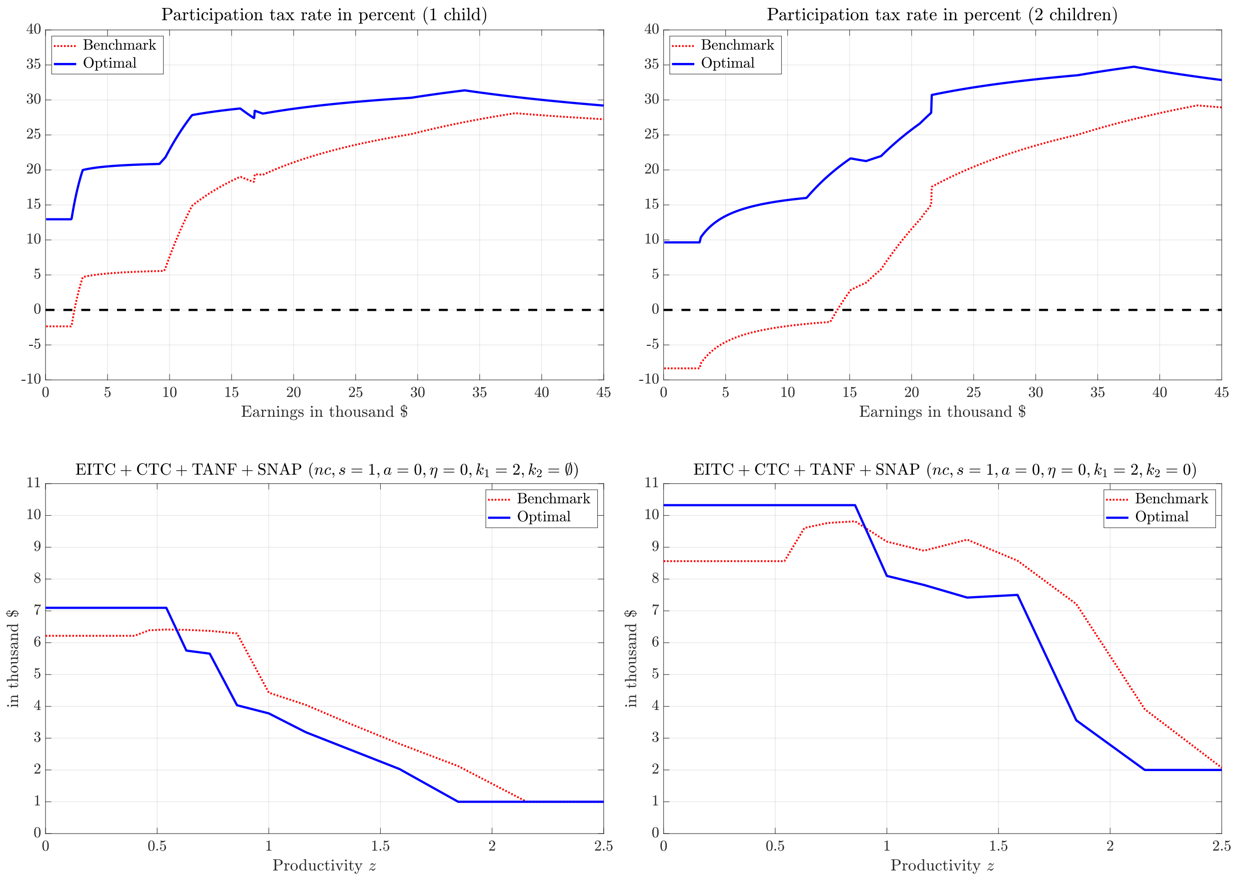
FIGURE 2– BENCHMARK AND OPTIMAL POLICY WITHOUT TAGGING



Notes: Mothers with one child (left panels); mothers with two children (right panels).

Specifically, the average participation tax rate among mothers with one child increases from 13.9 percent under the benchmark to 22.6 percent under the optimal policy; and from 4.6 to 21.5 percent among mothers with two children. The decomposition of total net transfers between working and non-working mothers reveals the relative shift away from work subsidies under the optimal policy: Net transfers to working mothers decrease by almost $3,000 compared to the benchmark policy; in contrast, net transfers to non-working mothers increase by more than $1,500. When we decompose net transfers by the number of children, there is little variation relative to the benchmark: Net transfers to mothers of one decline by $80; and increase by $140 for mothers of two. A similar result is found when we compare net transfers according to the age of children: mothers with a child under 5 years of age get about $100 dollars less under the optimal policy than under the benchmark; while mothers with children aged 5-17 years see an increase in net transfers of about $40. (It will become clearer below that this latter result will not hold when we allow for tagging, as the government will use policy to change the allocation of consumption and hours worked according to the age of the

FIGURE 3– BENCHMARK AND OPTIMAL POLICY WITHOUT TAGGING



Notes: Mothers with one child (left panels); mothers with two children (right panels).

children.) Average disposable income declines. Most of this decline is accounted for by mothers in the second and third quintiles, whose disposable incomes fall by $7,000 and $2,000 respectively. These are mainly low- and medium-productive mothers that move from employment under the benchmark policy to non-employment under the optimal reform. Mothers in the fifth quintile see a slight increase in disposable income with the reform. This stems from high productive mothers increasing hours worked because of the reduction in the earnings threshold for EITC eligibility.

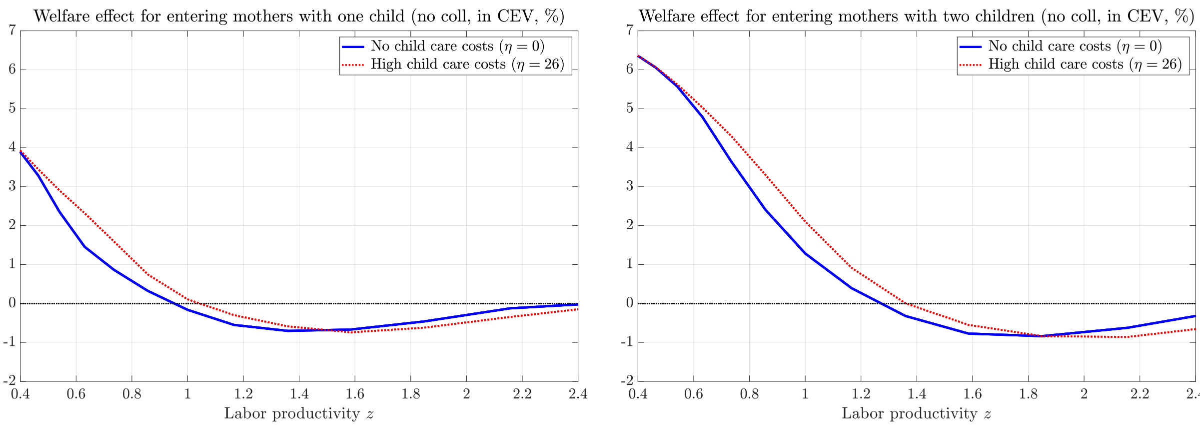
The optimal policy increases the efficiency of the allocation of hours across productivity realizations. The correlation between hours worked and labor productivity among mothers with one child increases from 0.31 under the benchmark policy to 0.37 under the optimal. For mothers with two children, the correlation increases from 0.44 to 0.56. The increase in this correlation is somewhat higher among single mothers with two children aged 0-4 years and with no assets: from 0.43 under the benchmark to 0.63 under the optimal reform. This is because the increased insurance of the optimal policy especially helps these mothers to smooth consumption without having to work long

TABLE 6– SUMMARY STATISTICS: BENCHMARK AND OPTIMAL POLICY

Description	Sample	Benchmark policy	Optimal policy w/o tagging	Optimal policy w. tagging
	[1]	[2]	[3]	[4]
Employment rate (%)	All mothers	84.5	64.4	64.7
	$n = 1$	88.0	73.5	73.8
	$n = 2$	79.1	50.4	50.7
Part-time work (%)	All mothers	22.1	12.7	11.8
Full-time work (%)	All mothers	68.3	70.9	71.4
Extra-time work (%)	All mothers	9.6	16.4	16.8
Annual hours worked*	All mothers	$1,783$	$1,911$	$1,927$
	$n = 1$	$1,863$	$1,976$	$1,991$
	$n = 2$	$1,643$	$1,762$	$1,781$
Annual earnings* ($)	All mothers	$30,650$	$36,475$	$36,480$
	$n = 1$	$32,275$	$36,683$	$36,727$
	$n = 2$	$27,845$	$36,005$	$35,921$
Hourly earnings* ($)	All mothers	16.22	18.23	18.10
	$n = 1$	16.34	17.80	17.72
	$n = 2$	16.02	19.23	18.95
Total net transfer ($)	All mothers	$1,035$	$1,035$	$1,035$
	Working	90	$-2,806$	$-3,019$
	Non working	$6,619$	$8,187$	$8,676$
	$n = 1$	$-1,113$	$-1,196$	$-1,332$
	$n = 2$	$4,537$	$4,674$	$4,888$
Disposable income ($)	All mothers	$29,917$	$27,721$	$27,876$
Quintile 1		$12,263$	$11,338$	$10,658$
Quintile 2		$21,845$	$14,944$	$15,955$
Quintile 3		$26,870$	$24,703$	$24,386$
Quintile 4		$32,124$	$31,536$	$31,503$
Quintile 5		$56,494$	$56,714$	$56,913$
Corr(hours,productivity)	$n = 1$	0.31	0.37	0.35
	$n = 2$	0.44	0.56	0.50
Wealth ($)	All mothers	$11,415$	$11,290$	$11,535$
Frac. with no assets	All mothers	33.1	37.1	33.4
Frac. with $\leq \$2,000$	All mothers	73.6	74.0	73.8
Welfare	Entering mother	–	+ 0.94%	+ 1.21%

Notes: Model summary statistics under benchmark and optimal policy. *Conditional on working.

FIGURE 4– WELFARE EFFECTS FROM OPTIMAL REFORM



hours in periods of low labor productivity. To see this, average annual hours worked (without conditioning on working) by this latter group of mothers is 1,177 hours under the benchmark, against 671 under the optimal policy.

The increase in insurance brought about by the higher guaranteed income of the optimal policy crowds out self-insurance, and, hence, reduces wealth accumulation. Average wealth falls slightly from \$11,415 to \$11,290. The percentage of mothers with no assets increases from 33.1 percent under the benchmark policy to 37.1 percent under the optimal. Likewise, the percentage of mothers holding less than two thousand dollars in wealth increases from 73.6 to 74.0 percent.

The welfare gain from the optimal reform and its decomposition. The welfare gain for a single mother-to-be from entering the economy under the optimal policy, relative to entering under the benchmark, amounts to an increase by 0.94 percent in consumption in every period and state of nature. To get a better understanding of this consumption-equivalent welfare gain, we first look at the distribution of welfare effects across entering mothers upon drawing a productivity level (see Figure 4). As expected, mothers that draw a low productivity value win the most, especially if they also draw a high child care cost parameter, η. For mothers entering with one child and no child care costs, welfare gains become negative at productivity level equal to 0.95, to then approach zero for high-productive mothers. For mothers entering with two children and no child care costs, welfare gains become negative at productivity 1.3. In summary, the welfare gain from our behind-the-veil-of-ignorance optimal policy reform stems mainly from the large gains accruing to low-productive entering mothers. The relatively low welfare losses accruing to more productive mothers are traded-off for insurance against low productivity and high child care cost realizations.

A decomposition of the aggregate welfare gain along the lines suggested in Conesa et al. (2009) pins down the contributions from changes in consumption and non-market time. Each of these two sources of welfare gains can in turn be decomposed into a level component and a volatility component.[16] The breakdown of the aggregate welfare gain across all these components (see Table 7) reveals that the gains stem from the increase in the average level of non-market time and the more efficient allocation of hours across labor productivity and child care cost realizations.

TABLE 7– DECOMPOSITION OF WELFARE GAIN

Total change		0.94
Consumption	Total	−4.47
	Level	−2.11
	Dispersion	−2.41
Non-market time	Total	5.66
	Level	2.80
	Dispersion	2.75

Notes: All numbers in percent.

Reform support among mothers in the benchmark solution. The optimal reform increases average welfare among incumbent mothers (i.e. among those populating the economy at the moment the reform is implemented). However, the fraction of these mothers winning from the reform is 38.4 percent, which falls short of majority support. (The support rate among the non-college educated is 43.6 percent, compared to 25.9 percent among those with a college degree.) The highest support is found among single mothers with a young child, at 50.7 percent, followed by mothers of two, 48.5 percent, and by mothers under 30 years of age, 47.3 percent. This is explained by the higher demand for insurance among younger mothers and mothers of young children, making them the population groups with more to gain from the higher guaranteed income provided by the reform.

We addressed the question of whether the optimal reform can be modified to attain majority support among incumbents mothers while still improving on ex-ante welfare to single mothers-to-be relative to the benchmark. We find that a relatively small reduction of the guaranteed income and an increase of the maximum tax credit to all mothers would bring enough support from mothers with older children to secure majority support for the reform. Compared to the benchmark policy, welfare for entering single mothers would increase by 0.64 percent under this modification of the optimal reform (recall that the welfare gain under the optimal reform is 0.94 percent). This shows that the benchmark policy can be reformed gathering majority support from incumbents mothers while retaining most of the welfare gains for entering mothers from the optimal reform. The values of π_0^n, for $n = 1, 2$, that would attain majority support are \$500 and \$1,000, respectively. (Recall

[16]See Appendix E for a detailed explanation of how all these components are calculated in our model.

that these values are zero under the benchmark policy.)

4.1.1 Robustness

We now assess the robustness of the prescribed increase in the guaranteed income to single mothers with respect to key parameter values, to TANF time limits, marital risk, and deviations from a utilitarian government. The robustness exercises and the discussions below suggest that the prescription of a higher guaranteed income is not only robust, but also that the particular value prescribed by our model could be interpreted as a lower bound.

Robustness with respect to preference parameter ζ. This parameter controls the curvature of the utility function with respect to non-market time and is linked to the intensive-margin Frisch elasticity of labor supply. As discussed above, the kinks and non-convexities in the budget constraints make this parameter difficult to pin down. To investigate the robustness of our results with respect to this parameter, we increase its value from 3 to 4, which would amount to a reduction in the Frisch elasticity from 0.77 to 0.58.[17] All other parameters are consequently re-calibrated so that the model continues to match the empirical moment conditions. The optimal policy reform is now characterized by $\pi_0^1 = \$880$, $\pi_0^2 = \$1,760$, $\pi_1^1 = 0.180$, $\pi_1^1 = 0.212$, $e_{I_1}^1 = \$9,085$, and $e_{I_1}^2 = \$11,324$, values that are very close to those obtained under the benchmark calibration. Aggregate variables are also robust to changes in the value of ζ.

Guaranteed income versus asset limits for welfare eligibility. Increasing the TANF and SNAP asset limit would lessen the disincentives to save and, hence, would increase single mothers' ability to self-insure through savings.[18] This leads to the question whether increasing self insurance via the asset limit is more efficient than the prescribed increase in guaranteed income. To answer this question we carried out a number of exercises where the government can increase the asset limit and reduce the guaranteed income to balance the budget. Specifically, we increased the asset limit to \$5,000 and found a reduction in ex-ante welfare. The reason is that increasing the asset limit does not help when insurance is more needed, which is when single mothers have no or little assets. Expanding welfare eligibility by increasing the asset limit provides extra insurance to mothers when they already have some ability to self-insure through savings, but not to those without assets. In other words, since building a buffer stock of savings takes time and is especially costly for single

[17] Recall that this would be the intensive-margin Frisch elasticity of a single mother working the number of hours equal to the sample average and whose decision is not at a kink.

[18] It should be recalled that countable assets for the purpose of the TANF and SNAP asset tests include cash, deposits, certificate of deposits, stocks and bonds. The household's home and personal property are typically excluded. Most states also exclude one vehicle per household. Since in our model we define assets as financial wealth, there is a close correspondence between the model and the TANF and SNAP definitions.

mothers at the beginning of their life cycle, increasing the asset limit does not benefit those who are in the highest need of insurance. One way to illustrate this is to compare the correlation between labor productivity and hours worked under our optimal policy to that under a policy with a higher asset limit (and lower guaranteed income). Among single mothers aged 23 to 28 these correlations are, respectively, 0.5 and 0.42, which indicates an efficiency loss in the allocation of hours worked from the increase in the asset limit (compared to our optimal policy). Consistent with this reasoning, we have found the opposite effect when the asset limit is reduced (and the guaranteed income increased to balance the budget). We reduced that asset limit to $1,000 and found an increase in ex-ante welfare and in the correlation between labor productivity and hours worked (from 0.5 to 0.53).

TANF time limits. Although states cannot use federal TANF funds to provide cash assistance to families that include an adult recipient for more than 5 years, they have discretion to set their own time limit policies when using their own funds. Our model does not impose a limit to the number of years eligible single mothers can collect TANF, which implies that our modeling of the benchmark policy overstates the amount of insurance (guaranteed income) provided by TANF to single mothers. Since we find that the optimal reform prescribes an increase in the guaranteed income, introducing time limits would only exacerbate this finding. To see this we conducted two exercises limiting the number of years single mothers can get TANF cash assistance. The first exercise introduces intermittent time limits (about 10 states have some form of intermittent limits), where eligible single mothers can get TANF for up to 4 years, after which they become ineligible for other 4 years before they can apply again. The increase in the guaranteed income prescribed by the optimal reform is about 51 percent higher than the one we obtained above under no TANF time limits. In the second exercise we restrict eligible single mothers from TANF cash assistance according to their age. Regardless which age group is restricted, the optimal guaranteed income is always higher than the one under no restrictions. Notably, the guaranteed income is highest when the youngest mothers are the ones being restricted, a result that follows quite straightforwardly from our discussion above on single mothers' ability to self-insure over their life cycle.

In Appendix D we explore two additional deviations from our benchmark TANF. We implement California's TANF, which is one of the most generous in the U.S., and show that our results are robust. We also explore incomplete TANF take-up and again find that our results remain robust. It should be noted, however, that the size of the welfare gain from adopting the optimal reform when we implement these deviations to our benchmark may be higher or lower than the one reported above in Table 7. For instance, when we implement California's TANF as the benchmark the welfare gain is 0.68 percent in consumption (compared to the 0.94 percent found above). This reduction follows from the higher generosity offered by the new benchmark policy.

Inequality aversion. We have assumed that the government is utilitarian. Deviations from utilitarianism, for instance by assuming individual weights that decrease with labor productivity, would imply an optimal policy reform with higher guaranteed income. This follows quite straightforwardly from Figure 4 above showing the welfare gains from our reform as a function of labor productivity. The welfare gains from the increased guaranteed income are highest for low-productive single mothers; increasing the weight of these mothers' utility in the social welfare function would lead to an optimal policy with an even higher guaranteed income. However, assuming weights that depend on the number of children would not have a significant effect on our results. Finally, it should be noted that our results do not hinge on single mothers' education. To see this we let the government weight individual utilities according to their level of education (instead of using the sum of individual utilities). As expected, the guaranteed income prescribed by the optimal policy increases with the weight assigned to non-college-educated single mothers. More importantly, when this weight is set equal to zero (so that the government cares only about the college educated) the optimal policy continues to prescribe an increase in the guaranteed income (relative to the benchmark).

More comprehensive policy reforms. The main objective of this study is to characterize the optimal policy mix of guaranteed income and earnings subsidies for low-wage single mothers. Through this we aim to gain insights into the optimal marginal and participation tax rates at the lower end of the earnings distribution and, ultimately, into the optimal labor market participation of single mothers at different stages of their life cycle. Considering more comprehensive policy reforms—for instance by including the income thresholds $e^n_{I_2}$ or the phase-out rates π^n_2 in the set of maximizing parameters—would critically impact our ability to reliably compute the optimal policy (see Appendix E for details). Instead, we conducted some exercises to gauge whether the government would trade off an extension of the EITC income eligibility range for guaranteed income. Specifically, we assessed the welfare implications of increasing $e^n_{I_2}$ by \$3,000 and reducing π^n_0 to balance the government budget. We find that this would result in a welfare loss, showing that the government would not reduce guaranteed income in order to increase EITC eligibility to higher earners. We repeated the exercise for different increases in $e^n_{I_2}$ and obtained similar results.

Marriage and divorce. Our model abstracts from transitions to marriage, and from marriage to divorce. How would marital risk affect the prescribed optimal tax-transfer policy for single mothers? We argue that the overall effect of modeling these transitions would work in the direction towards a policy mix with an even higher guaranteed income. First, transitions to marriage would introduce attrition: the population of single mothers at the beginning of their life cycle would be higher than that of single mothers at more advanced stages of the life cycle. This would yield an increase in the demand for insurance relative to earnings subsidies, which would lead the government to provide more guaranteed income. Second, transitions from marriage to single motherhood would have only modest effects, if any, on the key channels driving our results. Let us start by noting two trends in U.S divorce rates: (1) the overall divorce rate has declined quite markedly in the last decades; (2) age-specific divorce rates show that this decline is driven by younger women. By contrast, the divorce rate among older women doubled from 1990 to 2019 (Cohen 2019). This implies an increasing share of transitions from marriage to single motherhood at more advanced stages of the mother's life cycle, when the children are likely of school age. As most of these new single mothers enter the economy later in life, rather than at the beginning of their life cycle when insurance is needed, the optimal reform will increase the guaranteed income to insure these latter mothers. (These effects will become more apparent in section 4.2 below where we study the optimal policy reform under tagging by the age of the children.)

4.2 The Optimal Reform with Tagging by Age of Child

In this section we examine reforms that allow for tagging according to the age of the children. The merit of tagging, as pointed out in a seminal paper by Akerlof (1978), is that it makes the trade-off between guaranteed income and distortionary taxation more favorable. As is apparent from the optimal reform without tagging presented above, the government increases the guaranteed income at the cost of lowering work subsidies. Tagging eases this trade-off because changes to the guaranteed income and work subsidies can be made contingent on the age of the children. Namely, the government will now increase—relative to the optimal policy without tagging—the guaranteed income to mothers with pre-school age children, and will reduce it to mothers with all their children in school age. Credit rates in the phase-in region to mothers of pre-school age children are drastically reduced. Since our model is dynamic, a transfer policy that depends on the age of the children enables the government to ease the lack of insurance faced by mothers of young children, while at the same time provide single mothers with incentives to work when their children enter school age, when both their child care needs and demand for insurance are lower. This significantly improves the allocation of work and consumption across the mothers' life cycle.

Under tagging, the policy parameters that define the set of policy reforms are indexed by the number and age of the children, i.e, $(\pi_0^{nj}, \pi_1^{nj}, e_{I_1}^{nj})$, where $j = y, o$ if $n = 1$, and $j = y, m, o$ if $n = 2$. (Recall that y stands for young children, 0-4; o for older children, 5-17; and m for mixed ages, i.e., one child aged 0-4 and the other 5-17.) This implies searching for the values of 15 parameters in order to maximize the ex-ante utility of entering single mothers, subject to the revenue neutrality restriction. As we did above, we introduce a number of conditions on these parameters so that the search for the optimal reform is tractable. Specifically, $\pi_0^{2y} = 2\pi_0^{1y}$, $\pi_0^{2m} = \pi_0^{1y} + \pi_0^{1o}$, and $\pi_0^{2o} = 2\pi_0^{1o}$, which are the counterparts of the condition above that EITC payments to non-working mothers of two are twice the payments to non-working mothers of one child. Conditions on the phase-in credit rates and the maximum credits are also similar to those imposed above. Note that feasible reforms allow for different levels of guaranteed income depending on both the number and the age of the children, as well as for different work subsidy profiles.

The optimal policy is shown in Table 8. π_0^{1y} is equal to \$1,920, which implies a guaranteed income of \$8,136 for mothers with one child aged 0-4 years, amounting to a 15 percent increase with respect to the optimal policy without tagging. For mothers with one child aged 5-17 years, π_0^{1o} equals \$360, yielding a guaranteed income of \$6,576 for these mothers, a reduction of 7.3 percent. Mothers with two children aged 0-4 and mothers with two children of mixed ages see an increase in their guaranteed income by 20.2 and 5.0 percent, respectively. In contrast, mothers of two school-age children, aged 5-17, lose 10.1 percent of their guaranteed income, relative to the policy without tagging.

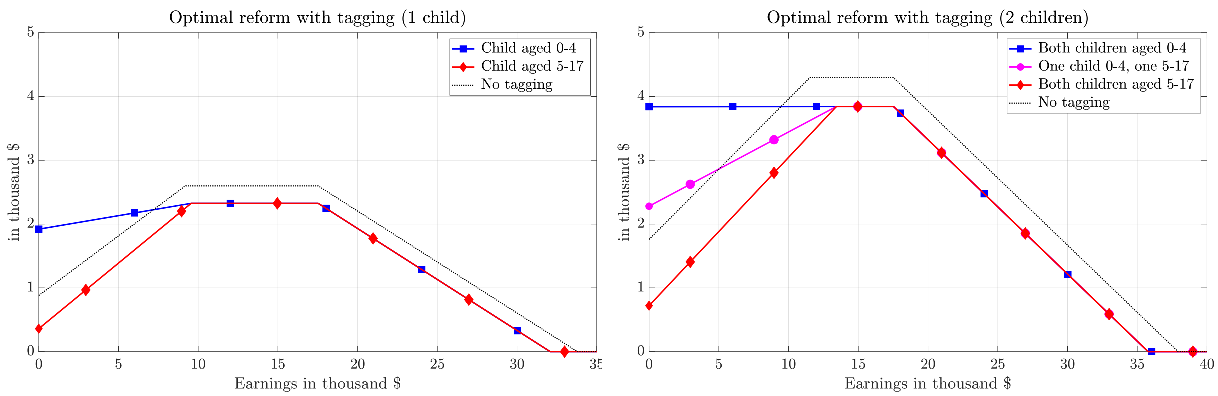
TABLE 8– OPTIMAL POLICY WITH TAGGING BY AGE OF CHILD

	Number of children				
	$n = 1$ age of child		$n = 2$ age of children		
	0-4	5-17	0-4	mixed	5-17
π_0	\$1,920	\$360	\$3,840	\$2,280	\$720
π_1	4.2%	20.6%	0.1%	11.6%	23.3%
e_{I_1}	\$9,559	\$9,559	\$13,430	\$13,430	\$13,430

The optimal reform prescribes a phase-in credit rate for working mothers of two young children very close to zero, $\pi_1^{2y} = 0.1\%$, practically transforming the EITC schedule for these mothers into a TANF-like program (see Figure 5). For mothers of one young child, the phase-in credit rate is also very small, $\pi_1^{1y} = 4.2\%$, making their EITC schedule also close to a TANF-like program. The earnings threshold marking the end of the phase-in region for mothers of one increases from \$9,190 to \$9,559 under tagging, thus shortening the plateau region. For mothers of two, this earnings threshold increases to \$13,430. In sum, the optimal policy with tagging increases guaranteed income and reduces work incentives for mothers with pre-school age children; for mothers of one and two children of school age the government reduces their guaranteed income.

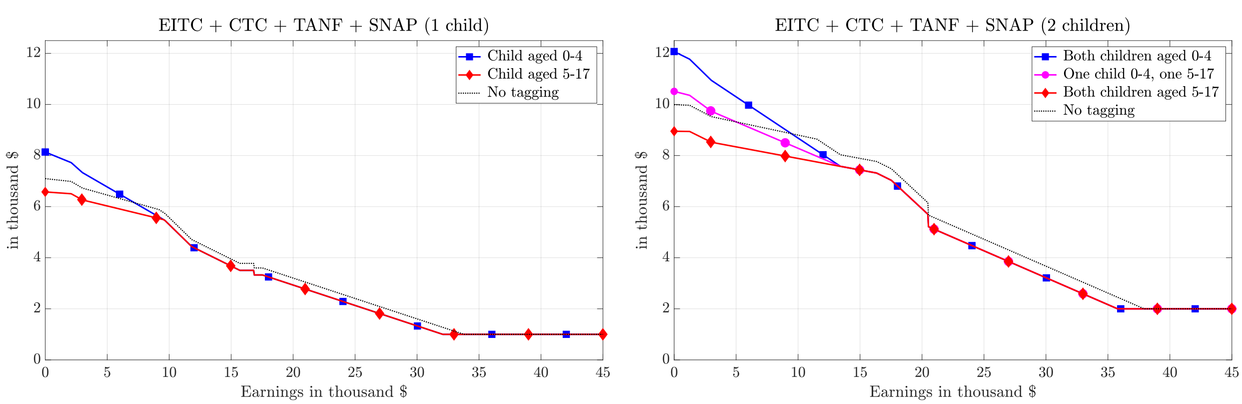
Total income transfers (the sum of EITC, CTC, TANF and SNAP collected by a single mother)

FIGURE 5– OPTIMAL POLICY WITH TAGGING BY AGE OF CHILD



under the optimal reform with tagging are plotted in Figure 6 along with those we obtained under no tagging. The left chart displays total income transfers for mothers of one child and no assets as a function of earnings. The right chart displays total income transfers for mothers of two. The consequences of tagging are apparent from this Figure: Compared to the optimal reform without tagging, non-working mothers with one pre-school age child get an increase in total transfers by more than $1,000; this increase begins to taper off as they join the workforce and start earning labor income. At earnings between $10,000 and $35,000, total transfers are lower than those under the optimal policy without tagging. In contrast, mothers with one school-age child collect less income transfers independently of their labor supply decision. A similar pattern is found among single mothers of two.

FIGURE 6– INCOME FROM EITC, CTC, TANF, SNAP (TAGGING BY CHILD AGE)



Summary statistics under the optimal policy are presented in column [4] of Table 6. The drop in employment rates for mothers with one and two children are similar to those stemming from the optimal policy without tagging. Conditional on working, average hours worked and average earnings increase. Again, this is due to a composition effect: The increased guaranteed income for mothers of young children leaves more low-productive mothers out of the labor force. Total net transfers to non-working mothers increase to \$8,676 (compared to the \$8,187 under the optimal policy without tagging). Working mothers continue to be net tax payers, and actually see their net tax liability (taxes minus transfers) go up to \$3,019, on a per-household basis, compared to the \$2,806 they would pay under the optimal policy without tagging. Both average disposable income and its distribution remain roughly unchanged.

We find additional welfare gains from tagging. Specifically, the welfare gain from this reform for entering mothers amounts to an increase by 1.21 percent in consumption in every period and state of nature, higher than the 0.94 percent gain obtained above without tagging. When the welfare gain is computed conditioning on the level of productivity drawn at entry, we find larger gains at low productivity levels.

In sum, our model provides two reasons to explain why making taxes and transfers contingent on child age is valuable. The first one is that single-parent households with children of pre-school age face, on average, higher child care needs than similar households with school-age children. Second, single parents with pre-school age children are more likely to be young, and hence to have less access to self insurance from precautionary savings. If the government can tag its transfer policy by child age, the optimal policy prescribes that households with pre-school age children get a larger guaranteed income and lower earnings subsidies than households with school-age children. Since child age and parent age are correlated, both features matter for the result. It should be noted, however, that a transfer policy tagged by the age of the parent, instead of by the age of the children, would not yield identical quantitative results, as it would shift some weight away from child care needs to precautionary savings. Since the compulsory education starting age is 5 or 6, tagging the transfer policy for single parents by child age seems a more clear-cut option than tagging by the parent age.

5 Returns to Experience: Learning by Doing

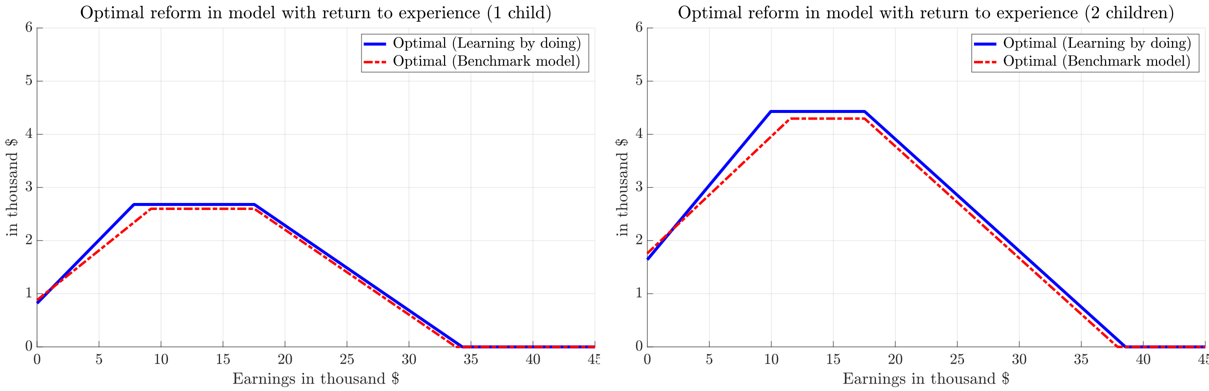
Here we extend the baseline model by introducing learning by doing. More specifically, we assume that working mothers may accumulate skills that increase their future average productivity. Even though evidence on the role of on-the-job learning for female labor supply is mixed (Olivetti 2006, Attanasio et al. 2008), our aim here is simply to assess whether the optimal transfer policy found in our baseline model is substantially altered if there are productivity gains from working, and if so how. The way we model learning-by-doing is rather crude—no human capital depreciation, and returns to experience are the only determinant of the increase in the average wage—and, hence, our quantitative findings in this section are better interpreted as an upper bound. Of special interest to us is the robustness of our previous finding supporting a reform that increases the guaranteed income for single mothers.

We consider two different levels of human capital, which are denoted by L (low) and H (high). Single mothers enter the economy with low human capital, and it is only by working that they can upgrade their human capital to level H. More precisely, we assume that mothers working full- and extra-time face a probability p of permanently increasing their human capital from L to H (no human capital depreciation).[19] With probability $1 - p$ they do not increase their human capital and start next period again with low human capital. The modeling of human capital accumulation as a stochastic process may be justified, for instance, by the different opportunities to learn offered across occupations. The value of parameter p, and the two wage rates, w^L and w^H, are set to match the life-cycle profile of hourly wages for single mothers aged 23-66 years. This profile of average hourly wages is obtained from the sample of mothers described above, and from a sample of single mothers aged 54-66 years with no dependent children (i.e. all their children are 18 and older). The values for these three parameters (p, w^L, w^H) are (0.05, 8.69, 14.35) for non-college-educated single mothers, and (0.03, 12.97, 29.18) for the college educated. All other parameters are recalibrated to match the same set of moments as in the baseline model.

Before presenting our results for the optimal policy reform, it is useful to discuss the new trade-offs that emerge under learning by doing. As in the baseline model, increasing the guaranteed income allows single mothers to further smooth marginal utility. But since now allocating time to market work can increase single mothers' human capital—which would make them more productive, and more likely to earn higher wages and remain employed in the future—a new force emerges: In order to incentivize single mothers to accumulate human capital, the government wants to reduce the guaranteed income and increase work subsidies. The way in which these conflicting forces are resolved in the optimal policy is shown in Figure 7. The values of π_0^n for $n = 1, 2$ are \$820 and \$1,640, respectively (recall that these parameters are equal to zero under the benchmark policy). That is, despite the potential productivity gains from working, it is still optimal to increase the guaranteed income with respect to the benchmark policy. However, the increase is slightly lower than the one prescribed by the optimal reform obtained in the model without human capital accumulation. The optimal earnings subsidy rates in the phase-in region, π_1^n, are now 23.8 and 28 percent for

[19] Our assumption that part-time work does not increase human capital is consistent with the findings by Blundell et al. (2016) for the U.K. We have experimented with an alternative assumption where part-time working mothers may increase their human capital as well and found our conclusions to be unaffected.

FIGURE 7– OPTIMAL REFORM IN MODEL WITH LEARNING-BY-DOING



$n = 1, 2$, respectively, which are slightly higher than in the model without human capital. These higher work subsidies are, however, applied to earnings levels in shorter phase-in regions: the value of $e^1_{I_1}$ declines from \$9,190 to \$7,814, and $e^2_{I_1}$ declines from \$11,524 to \$9,962. In sum, compared to the optimal policy prescribed by the benchmark model without learning by doing, the optimal policy now combines a more moderate increase in the guaranteed income, so that wealth-poor mothers with very low productivity can smooth consumption, with higher incentives to work, so that medium-low productivity mothers join the labor force and get a chance of increasing human capital. Figure 7 plots the optimal EITC schedules in the two models (with and without human capital accumulation). These results show that the new trade-off introduced by learning by doing shapes the optimal EITC schedule but does not reverse our findings from the benchmark model of Section 2. The consumption-equivalent welfare gain for entering single mothers of adopting this policy is 1.24 percent.

The effects of adopting the optimal reform on labor market variables, earnings, and net transfers are qualitatively similar to those obtained in the benchmark model of Section 2. Quantitatively, there are some differences: The implied reduction in employment rates is smaller in the model with learning-by-doing. For instance, the employment rate among mothers with one child drops now to 74.6 percent, compared to the 73.5 percent employment rate implied by the optimal reform in the benchmark model. A similar result is found among mothers with two children. Also, the magnitude of the reduction in net transfers to working mothers implied by the optimal reform is substantially smaller now. These mothers would pay on average \$2,350, while in the optimal reform obtained with the model of Section 2 the net tax paid is \$2,806, a difference that reflects the added incentives to work and accumulate human capital. As a consequence, the correlations between hours worked and productivity—a measure of the efficiency in the allocation of hours—would increase to 0.46 and

0.63 for mothers of one and two, respectively, which are slightly higher compared to those obtained before.

6 Concluding Remarks

This paper presents a normative analysis of the tax-transfer system to single mothers. Using the U.S. system of taxes and income transfers as our baseline, we develop a dynamic model of consumption/savings and labor supply to quantitatively characterize the optimal reform of this tax-transfer system. Our analytical framework is a standard model of heterogeneous agents, incomplete markets and borrowing constraints, extended to include fixed costs of working, child care cost risk, and child aging. We embed into this model two tax schemes (income and payroll taxes), two tax credits (the Earned Income Tax Credit and the Child Tax Credit), and two income assistance programs (the Temporary Assistance for Needy Families and the Supplemental Nutrition Assistance Program). By modeling an endogenous consumption/savings decision and uninsurable earnings and child care risks, the optimal tax policy problem in our study also assesses the differing needs for insurance of single mothers depending on the age of their child. (Single mothers of pre-school age children are more likely to be credit constrained and hence more willing, compared to single mothers of older children, to trade off less out-of-work income support and higher work subsidies in the future for more out-of-work support and lower subsidies today).

We find that the optimal out-of-work income support is higher than the one provided by the benchmark policy. When policy cannot be made dependent on the age of the children, the optimal reform prescribes an increase in out-of-work income support and a reduction in the subsidy rate to low-wage earners. When policy can depend on the age of the children, the optimal reform provides even more insurance to single mothers of pre-school age children by increasing their out-of-work income support, but eliminates work subsidies for them. Our results highlight the important role of savings for the optimal taxation of single-parent households. While our model embeds the key margins needed for a normative analysis of the tax-transfer system to single mothers—consumption/savings, the extensive and intensive margins of labor supply, child care costs, child aging—there are a number of extensions that may be worth exploring in future work. First, we have assumed that the wage rate is exogenous, thus neglecting the effects of taxes and transfers to single mothers on the market wage rate. Second, in our model fertility is exogenous and therefore we do not assess the effects that taxes and transfers may have on the fertility decision of single women. While the empirical literature has found no or little effects of taxes and transfers on fertility (Baughman and Dickert-Conlin 2009, Crump et al. 2011), it may be worth disentangling quantitatively the different mechanisms at play from taxes and transfers to fertility. Third, we have not modeled marriage and divorce, meaning that our model is silent about the effects of taxes and transfers on household formation and dissolution.

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APPENDICES

There are five appendices. Appendix A presents a description of the tax and transfer programs modeled in this paper. Appendix B presents the parameter values of these tax and transfer programs used to compute the solution of the model. Appendix C contains further details on the calibration and the model fit for college-educated mothers. Appendix D presents some robustness exercises with respect to the modeling of TANF. Appendix E describes the decomposition of the welfare gains from implementing the optimal policy reform.

Appendix A

Taxes, Tax Credits, and Income Assistance Programs

In this appendix we describe the U.S. federal individual income tax scheme, the payroll tax, two tax credits and two income transfer programs to assist low-income households. We embed these tax and transfer schemes into our model as they are described here, including all the discontinuities and kinks.

INCOME AND PAYROLL TAXES

Single mothers in our model file with the Internal Revenue Service (IRS) using two filling statuses, j: head of household, ($\hbar$), for single mothers with dependent children; and single, (s), for single mothers with grown-up children.

A single mother's income is made up of earnings, e, and capital income, ra, where r is the return on investment and a is the level of assets. Income taxes before credits owed by a tax filer under filing status $j = \hbar, s$, income $y = e + ra$, and n dependent children are given by

$$T^j(y,n) \;=\; \sum_{i=1}^{7} \tau_y^{j,i} \max\{\min\{y - d_T^j - \xi_T^{jn}, b^{j,i}\} - b^{j,i-1}, 0\},$$

where $b^{j,i} \geq 0$ are parameters characterizing the seven income brackets in the federal individual income tax code, and $\tau_y^{j,i}$ are the corresponding tax rates. The upper bound for the last bracket, $b^{j,7}$, is set to a very large value such that taxable income for any household is below this limit. The remaining values, $b^{j,i}$ for i from 1 to 6, are the break points between the different income brackets. The income tax deduction is denoted by d_T^j and personal exemptions by ξ_T^{jn}.

Payroll taxes are denoted by $T_p(e) = \tau_p \min\{e, \bar{e}\}$, where $\tau_p = \tau_{p,\mathrm{SS}} + \tau_{p,\mathrm{ME}}$ is the employee's tax rate (the sum of social security and medicare tax rates), and $\bar{e}$ is the payroll tax cap.

TAX CREDITS

The Earned Income Tax Credit (EITC). The Earned Income Tax Credit is a refundable credit. Eligibility for single mothers filing as *single* or as *head of household* is determined by the following conditions: (i) Investment income, ra, cannot exceed a level, say $\bar{\mathrm{ra}}_I$; (ii) Income (earned plus non-earned income) cannot exceed a level, say y_I^n, which depends on the number of dependents. The EITC-eligibility set of a single mother with $n = 0, 1, 2$ children is

$$EES \equiv \{ra \leq \bar{\mathrm{ra}}_I\} \cap \{e + ra \leq \mathrm{y}_I^n\}. \tag{6.1}$$

The amount of the credit accruing to a tax filer with assets a, earned income e and n children is given by

$$EITC(a, e, n) = \begin{cases} \pi_1^n e & \text{if} \quad 0 \leq e \leq \mathrm{e}_{I_1}^n \text{ and } a \in EES \\ \pi_1^n \mathrm{e}_{I_1}^n & \text{if} \quad \mathrm{e}_{I_1}^n \leq e \leq \mathrm{e}_{I_2}^n \text{ and } a \in EES \\ \max\{\pi_1^n \mathrm{e}_{I_1}^{jn} - \pi_2^n (e - \mathrm{e}_{I_2}^n), 0\} & \text{if} \quad e \geq \mathrm{e}_{I_2}^n \text{ and } a \in EES, \end{cases}$$

and EITC(a,e,n) = 0 if $(a, e) \notin EES$. Parameters π_1^n are the earnings subsidy rates in the phase-in region, and π_2^n are the phase-out rates. The thresholds, $\mathrm{e}_{I_1}^n$ and $\mathrm{e}_{I_2}^n$, mark the end of the phase-in region and the beginning of the phase-out region, respectively. In the region between these two thresholds, the credit is constant at its maximum value $\pi_1^n \mathrm{e}_{I_1}^n$. Note that both the credit rates and the earnings thresholds depend on the number of qualifying children. However, the maximum level of investment income for program eligibility, $\bar{\mathrm{ra}}_I$, does not depend on the number of children. (Figure A1 below displays the EITC schedule for the 2013 tax returns.)

The Child Tax Credit (CTC). The non-refundable component of the child tax credit for a single mother with n dependent children (filling under status $j = \hbar$) and income y is

$$CTC(y, n) = \begin{cases} \theta n & \text{if} \qquad y \leq \mathrm{y}_{CTC} \\ \max\{\theta n - \varrho(y - \mathrm{y}_{CTC}),\ 0\} & \text{if} \qquad y > \mathrm{y}_{CTC}, \end{cases}$$

where θ is the subsidy per child and y_{CTC} is the income level at which the child tax credit starts being phased out. Parameter ϱ characterizes the child tax credit phase-out rate.

If the non-refundable component of the child tax credit, $CTC(y, n)$, is lower than the tax liability, $T^{\hbar}(y, n)$, then this liability is reduced by the amount of the child tax credit. If the child tax credit is higher than the liability, then the liability is reduced to zero and the filer can apply for the refundable component of the Child Tax Credit, i.e. the Additional Child Tax Credit (ACTC). The additional child tax credit for a mother with one or two eligible children is

$$ACTC(y,e,n) = \min\left\{ CTC(y,n) - T^{\hbar}(y,n),\ \max\{\phi(e-\delta),0\}\right\} \tag{6.2}$$

where ϕ and δ are parameters. (Figure A1 below displays the CTC schedule for the 2013 tax returns.) With some abuse of notation, we refer to the sum of the two components (the non-refundable and the refundable) as the Child Tax Credit and write this sum as $CTC(a,e,n)$.

INCOME ASSISTANCE PROGRAMS

Temporary Assistance for Needy Families (TANF). This is a program to assist families with dependent children. Despite variation across states, many features of the program are common across most states. Eligibility and benefits are determined by categorical and quantitative variables of the assistance unit on a monthly basis. For the sake of our analysis, we consider assistance units made up of a single mother with one and two dependent children. Financial eligibility requirements include: (i) Assets cannot exceed a certain limit, say a_B.[20] (ii) Gross family income cannot exceed $\mathrm{y}^n_{B_1}$, say. Gross income includes earned and non-earned income, such as interests and child support income. (iii) Net family income cannot exceed $\mathrm{y}^n_{B_2}$. Net income for the purpose of determining TANF eligibility is computed as

$$\iota_B(a,e,h,n,\eta) = \left(e - d_{B_1}\mathbb{1}_{\{h>0\}} - d_{B_2}\Gamma(h,\eta) - d_{B_3}\right)\sigma_B + ra + \vartheta(k_1,k_2), \tag{6.3}$$

where $\sigma_B < 1$ is a parameter that introduces an earned income disregard; d_{B_1} is a work deduction, $\mathbb{1}_{\{h>0\}}$ is an indicator function which takes value 1 if hours worked are strictly positive; d_{B_2} is a child care deduction, which is set as a fraction of child care costs incurred while working, Γ; and d_{B_3} is a fixed deduction. Parameter ϑ is child support.

These three financial requirements define the TANF-eligibility set of a single mothers with dependent children as

$$TES \equiv \{a \leq \mathrm{a}_B\} \cap \{e + ra + \vartheta(k_1,k_2) \leq \mathrm{y}^n_{B_1}\} \cap \{\iota_B(a,e,h,n,\eta) \leq \mathrm{y}^n_{B_2}\}. \tag{6.4}$$

If eligible, the income transfer is determined by a standard of need and net family income, with a maximum payment set by a payment standard. That is, an eligible single mother with dependents is entitled to TANF benefits

$$TANF(a,e,h,n,\eta) = \min\left\{ \bar{B}^n,\ \max\{[S^n - \iota_B(a,e,h,n,\eta)] \times \varsigma, 0\}\right\}, \tag{6.5}$$

[20] Eight states have eliminated TANF asset limits (Ohio, Louisiana, Colorado, Hawaii, Illinois, Virginia, Alabama and Maryland). Other states do not impose limits on certain assets, such as retirement and education accounts and vehicles.

where $\bar{B}^n$ is the maximum transfer for a single-mother family with n dependent children; S^n is the standard of need for that family, which is set as a percentage of the federal poverty level; $\iota_B(a,e,h,n,\eta)$ is net income as defined above; and ς is a parameter that controls when, and the rate at which, transfers are phased out. (Figure A1 below displays the 2013 TANF schedule.)

TANF has work requirements and time limits, typically of 60 months, to receive TANF benefits. However, the extent of enforceability of these limits varies widely across states. Besides a number of exemptions from time limits, states are allowed to extend assistance beyond these limits to up to 20% of their caseload.

Supplemental Nutrition Assistance Program (SNAP). While this is a federal in-kind transfer program, we follow many studies in considering the food coupons near-cash transfers. For SNAP, an assistance unit is an individual or a group of individuals who live together and purchase and prepare meals together. In our model there are three distinct types of assistance units according to the number of dependent children, $0, 1, 2$. For a single mother with n dependents eligibility is determined by (i) a resource limit, a_F, which is independent of household size; (ii) a gross income limit, $\mathrm{y}^n_{F_1}$, where gross income is defined to include earned and non-earned income, such as investment income, child support and income received from TANF; and (iii) a net income limit, $\mathrm{y}^n_{F_2}$. Net income is computed as gross income minus an earned income disregard, a child care deduction when needed for work, d_{F_1}, and a standard deduction

$$\iota_F(a,e,h,n,\eta) = e \cdot \sigma_F + ra + \vartheta(k_1,k_2) + TANF(a,e,h,n,\eta) - d_{F_1}\Gamma(h,\eta) - d_{F_2}, \tag{6.6}$$

where $1-\sigma_F$ is the earned income disregard.

In sum, the SNAP-eligibility set of an assistance unit with three children is

$$SES \equiv \{a \leq \mathrm{a}_F\} \cap \{e + ra + \vartheta(k_1,k_2) + TANF(a,e,h,n,\eta) \leq \mathrm{y}^n_{F_1}\} \cap \{\iota_F(a,e,h,n,\eta) \leq \mathrm{y}^n_{F_2}\}. \tag{6.7}$$

If a single mother receives TANF income she does not need to pass the income tests, and is immediately entitled to SNAP transfers provided she meets the resource test.

SNAP benefits are calculated by subtracting the household's expected contribution towards food, i.e. υ times net income, from a maximum allotment for the household. That is, an eligible single mother with n children is entitled to SNAP benefits

$$SNAP(a,e,h,n,\eta) = \max\left\{\bar{F}^n - \upsilon \cdot \iota_F(a,e,h,n,\eta), \underline{F}^n\right\}, \tag{6.8}$$

where $\bar{F}^n$ is the maximum allotment a single mother with n dependents can receive from SNAP, and $\underline{F}^n$ is the minimum benefit.

Appendix B

Tax-transfer parameter values

This Appendix presents the parameter values of the 2013 federal income tax schedule, payroll taxes, and the four transfer programs in our model (the Earned Income Tax Credit, the Child Tax Credit, the Temporary Assistance for Needy Families and the Supplemental Nutrition Assistance Program).

Income and Payroll Taxes

Table B1 presents the income tax brackets for tax filers under the *single* status and for tax filers under the *head of household* status.

TABLE B1—INCOME BRACKETS (ALL VALUES IN $)

Bracket	Parameter	Single ($j = s$)	Head of household ($j = \hbar$)
1	$b^{j,0}$	0	0
2	$b^{j,1}$	8,925	12,750
3	$b^{j,2}$	36,250	48,600
4	$b^{j,3}$	87,850	125,450
5	$b^{j,4}$	183,250	203,150
6	$b^{j,5}$	398,350	398,350
7	$b^{j,6}$	400,000	425,000

Source: 2013 income brackets for federal income taxes, from IRS website.

Table B2 presents the standard deduction, the personal exemption, and the marginal tax rates in the seven income tax brackets. The table also presents the payroll tax rates and the tax cap.

TABLE B2—INCOME AND PAYROLL TAX RATES

Description	Comment	Parameter	Value
Standard deduction (in $)	Single	d_T^s	6,100
Standard deduction (in $)	Head of household	$d_T^{\hbar}$	8,950
Personal exemption (in $)	Per person	ξ_T	3,900
Marginal tax rate	Bracket 1	τ_y^1	0.10
Marginal tax rate	Bracket 2	τ_y^2	0.15
Marginal tax rate	Bracket 3	τ_y^3	0.25
Marginal tax rate	Bracket 4	τ_y^4	0.28
Marginal tax rate	Bracket 5	τ_y^5	0.33
Marginal tax rate	Bracket 6	τ_y^6	0.35
Marginal tax rate	Bracket 7	τ_y^7	0.396
Social Security tax	Employee's share	$\tau_{p,SS}$	0.0620
Medicare tax	Employee's share	$\tau_{p,MA}$	0.0145
Social Security cap (in $)	Earnings cap	$\bar{e}$	113,700

Source: 2013 standard deductions, federal income tax rates and payroll taxes, from IRS website.

The Earned Income Tax Credit (EITC)

EITC eligibility is determined by wealth and income. Table B3 below presents the eligibility thresholds for single mothers with one and two dependent children and for single mothers without dependents.

TABLE B3—EARNED INCOME TAX CREDIT: ELIGIBILITY

	Max. investment income, $\mathrm{r\bar{a}}_I$ (\$)	Max. total income, y_I^n (\$)
No children, $n = 0$	$3,300$	$14,340$
One child, $n = 1$	$3,300$	$37,870$
Two children, $n = 2$	$3,300$	$43,038$

Source: Investment and total income limits for 2013 EITC eligibility, from IRS website.

The amount of the credit is determined by the level of earnings and the number of dependents. Table B4 presents the credit rates and the earning threshold that determine the three EITC regions (phase-in, plateau, and phase-out).

TABLE B4—EARNED INCOME TAX CREDIT: CREDIT RATES AND EARNINGS THRESHOLDS

	Phase-in rate, π_1^n(%)	Earnings end phase-in, $e_{I_1}^n$($)	Earnings beginning phase-out, $e_{I_2}^n$($)	Phase-out rate, π_2^n (%)
No children, $n = 0$	7.65	6,350	8,000	7.65
One child, $n = 1$	34.0	9,559	17,550	15.9
Two children, $n = 2$	40.0	13,430	17,550	21.0

Source: Subsidy rates and earnings thresholds for 2013 EITC, from IRS website.

The Child Tax Credit and the Additional Child Tax Credit

Within the context of our model, only single mothers with dependent children are eligible for the (non-refundable) CTC and the (refundable) ACTC. Table B5 presents the parameters determining eligibility and the amount of the credit.

TABLE B5—CHILD TAX CREDIT: CREDIT RATES & INCOME AND EARNINGS THRESHOLDS

Description	Parameter	Value
Credit per child	θ	1,000
Phase-out income threshold	y_{CTC}	75,000
Phase-out rate	ϱ	5%
Earnings limit (ACTC)	δ	3,000
Weight on earnings gap (ACTC)	ϕ	0.15

Source: Credit rates and income thresholds for 2013 CTC and ACTC, from IRS website.

Temporary Assistance for Needy Families (TANF)

Within the context of our model, only single mothers with dependent children are eligible for TANF. Table B6 presents the parameters determining eligibility and benefits from TANF.

TABLE B6—TEMPORARY ASSISTANCE FOR NEEDY FAMILIES (TANF)

Description	Parameter	Size assistance unit		
		1 person	2 persons	3 persons
Standard of need	S^n	638	855	1,073
Work deduction (per worker)	d_{B1}	90	90	90
Child care deduction	d_{B2}	0.5	0.5	0.5
General deduction	d_{B3}	30	30	30
Maximum grant	$\bar{B}^n$	201	270	338
Gross income test	y_{B1}^n	1,180	1,581	1,985
Net income test	y_{B2}^n	638	855	1,073
Asset test	a_B	2,000	2,000	2,000
Generosity	ς	0.5	0.5	0.5
Earned income disregard	σ_B	2/3	2/3	2/3

Source: Income and asset limits, deductions and benefits for 2013 TANF, from the state of Delaware's website. TANF is a monthly program and the dollar amounts in this table are monthly values. Since the length of a period in our model is one year, we annualize these values to fit our model.

Supplemental Nutrition Assistance Program (SNAP)

Table B7 presents the parameters determining eligibility and benefits from SNAP.

TABLE B7—SUPPLEMENTAL NUTRITION ASSISTANCE PROGRAM (SNAP)

Description	Parameter	Size assistance unit		
		1 person	2 persons	3 persons
Asset test	a_F	2,000	2,000	2,000
Gross income test	y_{F1}^n	1,245	1,681	2,116
Net income test	y_{F2}^n	958	1,293	1,628
Child care deduction	d_{F1}	0.5	0.5	0.5
Standard deduction	d_{F2}	152	152	152
Earned income disregard	σ_F	0.8	0.8	0.8
Maximum allotment	$\bar{F}^n$	200	367	526
Weight on net income	υ	0.3	0.3	0.3
Minimum benefit	$\underline{F}^n$	15	15	0

Source: Income and asset limits, deductions and benefits for 2013 SNAP, from the U.S. Department of Agriculture, Food and Nutrition Service's website. SNAP is a monthly program and the dollar amounts in this table are monthly values. Since the length of a period in our model is one year, we annualize these values to fit our model.

In Figure B1 we display the EITC, CTC, TANF and SNAP for an eligible single mother with one child as a function of her annual earnings. The figure is meant to illustrate the kinks generated by these programs in the budget constraints of single mothers.

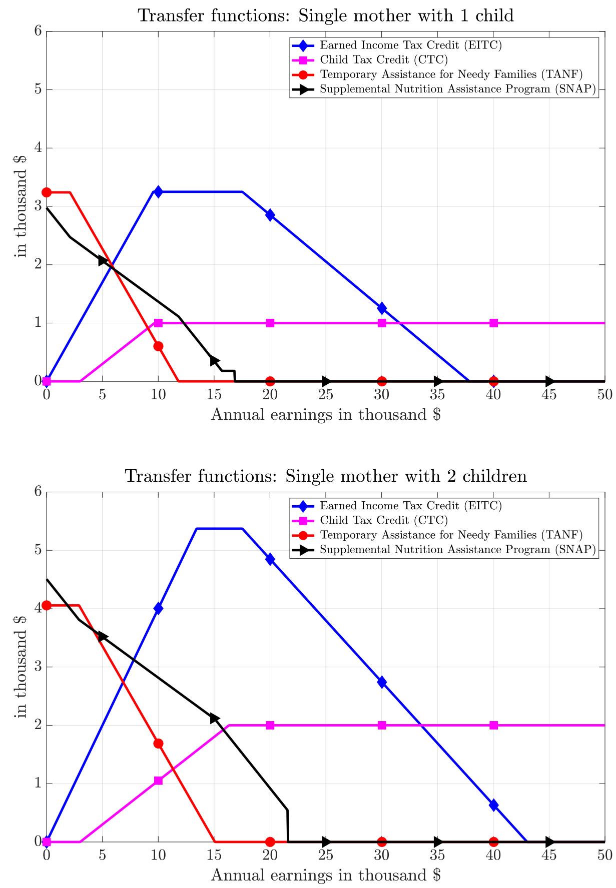

FIGURE B1. *Notes:* Income tranfers from EITC, CTC, TANF and SNAP collected by single mothers with no assets and no child care costs as a function of annual earnings.

Figure B2 displays th assets and no child care costs.

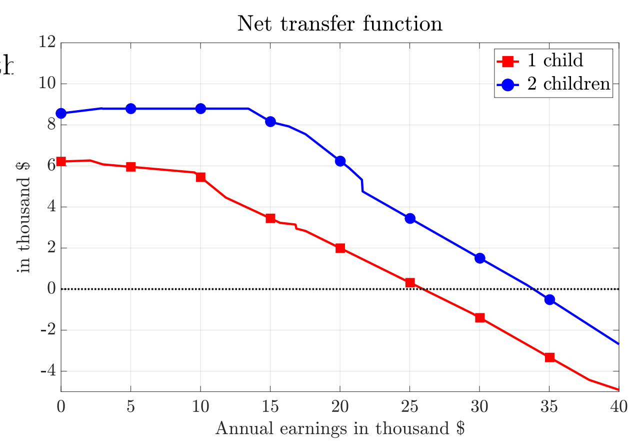

FIGURE B2. *Notes:* Net transfer functions for single mothers of one and two children with no assets and no child care costs.

Appendix C

Fertility process. Table C1 below reports the empirical and model-implied age profiles for the share of mothers with two children for both education groups.

TABLE C1 – SHARE OF MOTHERS WITH TWO CHILDREN

Mother's age	23-26	27-30	31-34	35-38
Non-college educated				
Data	43.00	51.97	54.22	51.22
Model	42.21	47.78	52.81	55.72
College educated				
Data	35.58	30.10	31.19	33.46
Model	26.12	29.03	31.83	33.52

Notes: All numbers in percent.

The distribution of children's age among entering mothers. We consider the population of 23-year old mothers with one or two children. Since our sample size for this age group is not large enough to estimate the children's age distributions separately by the mothers' level of education, our estimates are from the combined sample. Moreover, for mothers of one child, we restrict attention to those whose child is between 0 (newborn) and 3 years of age; for mothers of two, we restrict to those where the first child is between 1 and 3 years old, and the second child is either 0 or 1 year old. Table C2 presents the empirical children's age distributions that we feed into the model.

TABLE C2 – INITIAL DISTRIBUTION OF CHILDREN'S AGES

	Age first child				
	0	1	2	3	Cumul.
Mothers with one child	21.2	17.2	35.1	26.5	100
Mothers with two children					
Age second child: 0	–	7.5	32.0	18.3	57.8
Age second child: 1	–	–	16.7	25.5	42.2

Notes: All numbers in percent.

Parameters calibrated endogenously for mothers with a college degree. In the following, we list the set of empirical targets for the sample of college-educated mothers. Table C3 below reports the parameter values that match these moments.

1. Average hours worked, conditional on working, represent 35.5 percent of the time endowment. (φ_c)

2-4. The employment rate is 92.3 percent. The fractions of working mothers who work part-time, full-time and extra-time are 12.9, 73.3 and 13.9 percent respectively. $(\nu_{PT,c}, \nu_{FT,c}, \nu_{ET,c})$

5. Average hourly earnings over the mothers' life-cycle. We match average hourly earnings by setting the deterministic component of the wage rate. $(\varepsilon_c(s)$ for $1 \leq s \leq 44)$

6-7. The interquartile range of hourly earnings for working mothers aged 23-26 is \$11.61. The interquartile range twenty years later, that is, for mothers aged 43-46 is \$17.24. $(\sigma_{\epsilon_0,c},\ \sigma_{\epsilon,c})$

8-9. The fraction of working mothers paying child care is 49.2 percent (among those with children aged 0-4 years), and 31.8 percent (among those with children aged 5-17 years). $(\gamma_{y,c}, \gamma_{o,c})$

10-13. The average and standard deviation of child care expenditures, conditioning on being positive, paid by working mothers is \$6,000 and \$3,453, respectively (among those with children aged 0-4 years), and \$5,449 and \$3,217 (among those with children aged 5-17 years). $(\mu^{\eta}_{y,c}, \sigma^{\eta}_{y,c}, \mu^{\eta}_{o,c}, \sigma^{\eta}_{o,c})$

14. We use the Social Security's Average Indexed Monthly Earnings (AIME) to set retirement benefits, b_c.

Summary statistics for mothers with a college degree. Table C4 below compares model-generated statistics with their empirical counterparts for the sample of college-educated mothers. As for the case of single mothers without a college education, the model fits well moments that were not used as targets obtained from our sample of college-educated mothers. For instance, the employment rates for single mothers with one and two children match well those from the data. The model also matches well annual hours worked, total earnings, and hourly earnings by number of children.

TABLE C3– PARAMETERS CALIBRATED ENDOGENOUSLY (COLLEGE EDUCATED)

Description	Param.	Value	Moment	Target	Model
Weight non-market time	φ_c	0.077	Avg hours worked	0.355	0.355
Participation cost PT	$\nu_{PT,c}$	0.108	Employment rate	0.923	0.923
Participation cost FT	$\nu_{FT,c}$	0.111	Fraction full-time	0.732	0.734
Participation cost ET	$\nu_{ET,c}$	0.115	Fraction extra-time	0.139	0.136
Log-normal distribution	$\mu^{\eta}_{y,c}$	1.90	Avg. child care paid	6,000	5,962
Log-normal distribution	$\mu^{\eta}_{o,c}$	1.40	Avg. child care paid	3,453	3,490
Log-normal distribution	$\sigma^{\eta}_{y,c}$	2.60	Std. child care paid	5,449	5,280
Log-normal distribution	$\sigma^{\eta}_{o,c}$	1.50	Std. child care paid	3,217	3,369
Child care intercept	$\gamma_{y,c}$	-2.15	Frac. paid child care	0.492	0.491
Child care intercept	$\gamma_{o,c}$	-3.05	Frac. paid child care	0.316	0.314
Dispersion initial prod.	$\sigma_{\epsilon_0,c}$	0.57	P75-P25 wages 23-26	11.61	11.60
Random walk innov.	$\sigma_{\epsilon,c}$	0.10	P75-P25 wages 43-46	17.24	17.59
Age-specific productivity	$\varepsilon_c(s)$	cf. text	Hourly earnings by age		
Retirement benefit	b_c	23.35	AIME formula		

Appendix D

TANF rules vary across states and contain, to different extents, features not modeled in this paper. In this Appendix we carry out a number of exercises to assess the robustness of our findings with respect to some of the cross-state variation. Variation across states includes benefit generosity, deductions, and earned income disregards. Since the TANF we model in this study corresponds to one with about median generosity, and since our results suggest that the government should provide more insurance (especially to single mothers of pre-school age children), it is useful to inquire whether our findings would still hold under a more generous TANF. Another element in the modeling of TANF that deserves attention is our assumption of a 100 percent take-up rate. Even though we are not aware of estimated take-up rates for the population of single mothers, there exists evidence indicating that take-up rates for the entire population can be as low as 30 percent. In this Appendix, we address these issues and show that our results are robust.

TANF generosity, deductions, and income disregards: California

The TANF rules of California place this state among the most generous states, especially among the most populous ones (Urban Institute, 2019). To test the sensitivity of our results to TANF rules,

TABLE C4– SUMMARY STATISTICS: MODEL VS DATA (COLLEGE EDUCATED)

	Single mothers children aged 0-17		Single mothers of one child		Single mothers of two children		
	# of children		age of child		age of children		
	1	2	0-4	5-17	0-4	mixed	5-17
A. BENCHMARK SOLUTION							
Employment rate (%)	93.18	89.96	92.35	93.38	90.67	87.27	90.53
Part-time (%)	10.91	18.80	17.41	9.33	21.58	19.71	17.97
Full-time (%)	75.33	68.09	70.05	76.61	62.90	68.32	69.13
Extra-time (%)	13.76	13.11	12.55	14.06	15.52	11.97	12.90
Annual hours worked†							
Average	1,974	1,859	1,879	1,971	1,826	1,837	1,871
p25	2,027	1,716	1,944	2,048	1,613	1,716	1,716
p75	2,069	2,069	2,069	2,069	2,069	2,069	2,069
Annual earnings† ($)							
Average	43,562	42,152	34,929	45,664	34,550	37,466	44,981
p25	19,169	17,424	15,093	20,841	16,439	17,239	17,447
p75	54,318	54,318	51,030	58,402	50,164	51,463	58,402
B. DATA							
Employment rate (%)	93.44	90.76	89.83	94.75	90.47	93.22	89.43
Part-time (%)	10.77	15.54	17.31	5.95	22.69	15.82	14.62
Full-time (%)	72.92	73.66	69.48	74.11	72.13	73.76	73.76
Extra-time (%)	16.29	10.78	13.20	17.35	5.17	10.41	11.60
Annual hours worked†							
Average	2,003	1,868	1,863	2,052	1,639	1,837	1,911
p25	1,924	1,820	1,768	2,080	1,320	1,680	1,820
p75	2,080	2,080	2,080	2,080	2,080	2,080	2,080
Annual earnings† ($)							
Average	45,144	35,912	41,620	46,356	33,628	34,639	36,890
p25	25,630	19,223	20,417	26,698	13,960	19,223	20,417
p75	57,992	46,601	57,168	58,737	45,921	41,543	46,944

Notes: †Conditional on working. In dollars of 2017.

we implemented California's TANF (generosity, deductions, income disregards) in our benchmark model, recalibrated all the other parameters values in order to match the sample moments, and computed the optimal reform. Our results are shown in Table D1.

The guaranteed income prescribed by the optimal policy under California's TANF is about the

TABLE 3– CONTINUATION

	Single mothers children aged 0-17		Single mothers of one child		Single mothers of two children		
	# of children		age of child		age of children		
	1	2	0-4	5-17	0-4	mixed	5-17
A. BENCHMARK SOLUTION							
Hourly earnings† ($)							
Average	21.02	21.37	17.84	21.79	18.14	19.46	22.55
p25	11.25	12.03	10.01	11.25	10.18	12.03	12.23
p75	25.54	25.54	20.90	26.26	20.90	23.49	26.26
B. DATA							
Hourly earnings† ($)							
Average	22.10	18.61	21.54	22.29	19.32	18.57	18.57
p25	13.44	12.48	12.48	13.67	8.72	11.43	10.62
p75	28.00	28.33	18.33	27.93	23.16	25.28	22.57

Notes: †Conditional on working. In dollars of 2017.

TABLE D1– OPTIMAL POLICY WITH CALIFORNIA'S TANF AND WITH INCOMPLETE TAKE-UP

	π_0^n		π_1^n		$e_{I_1}^n$	
	$n=1$	$n=2$	$n=1$	$n=2$	$n=1$	$n=2$
Optimal policy (benchmark model)	$880	$1,760	18.7%	22.0%	$9,190	$13,430
Optimal policy (model w/ California's TANF)	$900	$1,800	19.7%	23.2%	$8,287	$10,297
Optimal policy (model w/ incompl. take-up)	$870	$1,740	18.0%	21.2%	$9,504	$11,928

same as the one obtained under the TANF modeled in our benchmark model. Mothers with one child would received $20 more in guaranteed income, and mothers with two children would receive $40 more. Earnings thresholds are however lower: the earnings threshold for the end of the phase-in region goes down by $900, and the earnings threshold for the beginning of the phase-out region declines by a bit more than $3,000. Overall, the welfare gain for an entering mother would now be 0.68 percent in consumption equivalent units, which is a bit lower than the one obtained under our benchmark model.

Incomplete take-up rates

Our model assumes full welfare and tax credit participation rates provided eligibility. However, there is some evidence that take-up rates may be lower than 100 percent, especially for TANF. To assess the sensitivity of our findings to a lower TANF take up rate, we solved the model imposing

TABLE D2– DECOMPOSITION OF WELFARE GAIN FOR DIFFERENT MODELS

		Benchmark	Calif. TANF	Incompl. take-up
Total change		0.94	0.68	0.85
Consumption	Total	−4.47	−4.37	−4.28
	Level	−2.11	−2.09	−1.85
	Dispersion	−2.41	−2.33	−2.48
Non-market time	Total	5.66	5.28	5.36
	Level	2.80	2.79	2.73
	Dispersion	2.75	2.39	2.53

Notes: All numbers in percent.

a minimum TANF benefit amount below which mothers do not claim it. The typical argument to explain incomplete take-up rates is that there are costs associated with collecting welfare benefits (stigma, the cost of filing forms and conducting interviews, etc.). Our assumption that only single mothers entitled to benefits above a threshold will claim it, captures the existence of such costs. Specifically, we targeted a TANF take-up rate of 60 percent (that is, only 60 percent of eligible single mothers claim their TANF payments), recalibrated all parameter values to match the sample moments, and computed the optimal reform. Our results are shown in Table D1.

References

Urban Institute. 2019. "Welfare Rules Databook: State TANF Policies as of July 2018." OPRE Report 2019-83.

Urban Institute. 2019. "What Was the TANF Participation Rate in 2016?" Report, July 2019.

Appendix E

In this appendix we describe our numerical approach to solve the model, and we explain how we decompose the welfare effects into various subcomponents.

Numerical Solution

Bellman equation. To solve the maximization problem solved by single mothers we use a discrete-state value function iteration approach, as the kinks and non-differentiabilities in the budget constraints generated by the tax-transfer system render Euler equation methods inapplicable. Because of the presence of these kinks and non-convexities, it is crucial for us to capture as accurately as possible how the hours choice responds to taxes and transfers. To this end, we need to discretize the labor supply choice on a much finer grid than the ones typically used when taxes and transfers are approximated by a smooth function (otherwise we miss the effects of the kinks). Specifically, we let single mothers choose annual hours of work from a fine, quasi-continuous grid, where two adjacent nodes lie only 20 hours apart from each other. Given an annual time endowment of 5,475 hours, this implies an hours choice set with almost 300 different options. Then, for each point in the state space, and for each value of hours worked in the grid—including zero—we compute the exact value of taxes, credits and transfers. Next we compute the optimal consumption and savings choice for each combination of hours and taxes/transfers; finally, we compute the optimal value for hours worked.

Asset holdings are discretized on a 50-node grid from zero to 1 million dollars, with considerably more nodes placed at the lower end of the domain. In addition, we allow the optimal decision rule for saving to lie off the grid by using piecewise-linear interpolation between grid points. The labor productivity process and the distribution of child care needs are discretized using 21 nodes and 6 nodes, respectively. We further keep track of the age of all children until they leave the household at the age of 18, which yields 59 different age groups. Overall, this implies that the state space for single mothers of a given education group consists of $(50 \times 21 \times 6 \times 19 \times 19 \times 59) \approx 134{,}000{,}000$ grid points (since we solve the model for two education groups, the state space is twice as large). We have experimented with increasing even further the number of total grid points, but we found no differences in our results. As is standard for this class of models, we solve the life-cycle problem by backward recursion, iterating backwards through all age groups.

Distribution. The distribution of single mothers is approximated as a density across the state space. We use the transition functions constructed from all exogenous processes and the optimal decision rules obtained from the solution of the Bellman equation. Given the very large dimensionality of our state space, this method is highly superior to Monte Carlo-type methods that would suffer from sampling variation (this is important, because to find the welfare-maximizing, revenue-neutral tax-transfer system we need to solve the model many times for different parameter configurations).

Welfare decomposition

We partially follow Conesa, Kitao and Krueger (2009) who describe a similar procedure in footnote 18. It should be noted, however, that their Cobb-Douglas specification for the utility function allows them to compute consumption equivalent variations in closed form. Conversely, with our additively separable utility function, this is not possible and we must solve nonlinear equations numerically. Let (c_0, h_0) denote the consumption-labor allocation in the benchmark economy, and let (c_*, h_*) denote the consumption-labor allocation under the optimal reform. Then, the total welfare gain from implementing the optimal reform, measured in terms of a consumption equivalent variation CEV, is defined as

$$W(c_*, h_*) \;=\; W(c_0(1 + CEV), h_0).$$

The solution to this equation, CEV, is the aggregate welfare gain for an entering mother under the optimal reform. We proceed by decomposing this welfare gain into a component stemming from the change in consumption from c_0 to c_*, and another component stemming from the change in hours worked from h_0 to h_*. These two components are denoted by CEV_C and CEV_H, respectively, and they are defined as:

$$\begin{aligned} W(c_*, h_0) \;&=\; W(c_0(1 + CEV_C), h_0) \\ W(c_*, h_*) \;&=\; W(c_*(1 + CEV_H), h_0). \end{aligned}$$

Note that, in contrast to Conesa, Kitao and Krueger (2009), with our additively separable utility function, the relation $1 + CEV = (1 + CEV_C)(1 + CEV_H)$ generally does not hold. We then decompose the welfare effect stemming from the change in consumption, CEV_C, into a level effect CEV_{CL} and a distribution effect CEV_{CD}, defined as follows:

$$W(\hat{c}_0, h_0) \;=\; W(c_0(1 + CEV_{CL}), h_0) \tag{6.9}$$

$$W(c_*, h_0) \;=\; W(\hat{c}_0(1 + CEV_{CD}), h_0), \tag{6.10}$$

where $\hat{c}_0 = (C_*/C_0)c_0$ is the consumption allocation resulting from scaling the allocation c_0 by the change in aggregate consumption. Similarly, the welfare effect stemming from the change in hours worked, CEV_H, can be decomposed into CEV_{HL} and CEV_{HD}:

$$W(c_*, \hat{h}_0) \;=\; W(c_*(1 + CEV_{HL}), h_0) \tag{6.11}$$

$$W(c_*, h_*) \;=\; W(c_*(1 + CEV_{HD}), \hat{h}_0), \tag{6.12}$$

where $\hat{h}_0 = (H_*/H_0)h_0$ is the non-market time allocation resulting from scaling the allocation h_0 by the change in aggregate hours worked.