Introduction to the Sets Theory

Solutions

1.

1.

$A \cap B = A$,

$A \cup C = \{x \in R^1 \mid -8 < x < 3\}$,

$\bar{B} = \{x \in R^1 \mid x \leq -8\}$,

$\bar{A} \cup \bar{C} = \{x \in R^1 \mid x \leq -1 \vee x \geq 1\}$, $\quad \bar{A} \cap \bar{C} = \{x \in R^1 \mid x \leq -8 \vee x \geq 3\}$,

$B \setminus A = \{x \in R^1 \mid -8 < x \leq -1 \vee x \geq 3\}$,

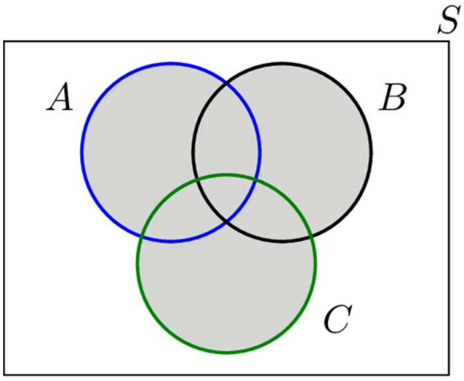
$C \setminus B = \varnothing$

2.

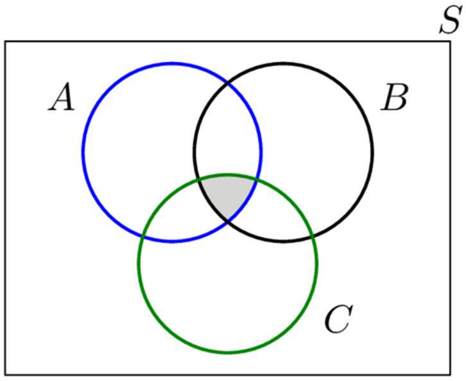
$A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A) = \{x \in R^1 \mid -1 < x\}$

$A \cap (B \setminus C) = \{x \in R^1 \mid 1 \leq x < 3\} \neq \{x \in R^1 \mid -8 < x \leq -1 \vee x \geq 1\} = (A \cup B) \setminus (A \cap C)$

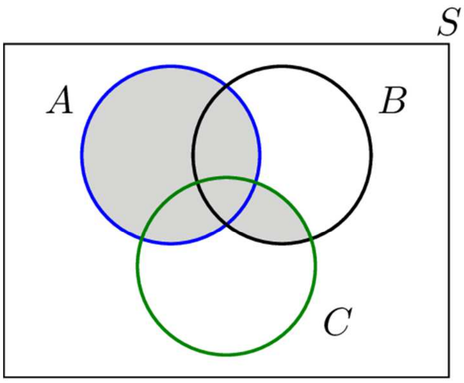
2.



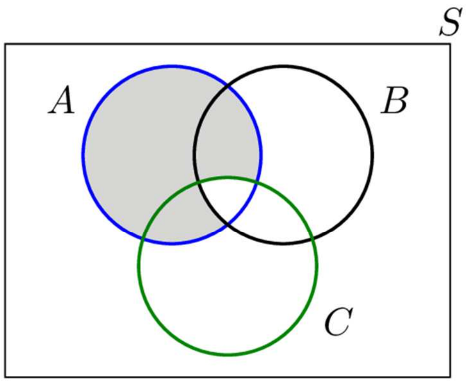
$A \cup B \cup C$



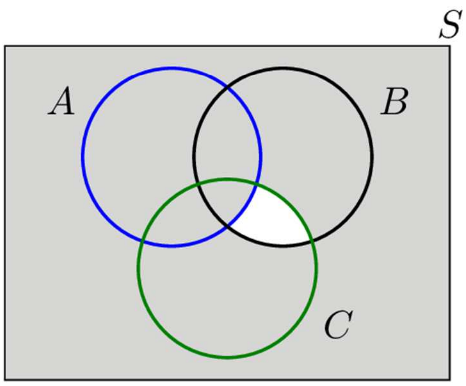
$A \cap B \cap C$



$A \cup (B \cap C)$



$A - (B \cap C)$



$A \cup (B \cap C)^C$

3.

1.

$$AxB = \{1,2,5\}\,x\{1,2\} = \{(1,1),(1,2),(2,1),(2,2),(5,1),(5,2\}$$

2.

$$BxA = \{1,2\}\,x\{1,2,5\} = \{(1,1),(1,2),(1,5),(2,1),(2,2),(2,5\}$$

3.

$$A^2 = AxA = \{1,2,5\}\,x\{1,2,5\} = \{(1,1),(1,2),(1,5),(2,1),(2,2),(2,5),(5,1),(5,2),(5,5)\}$$

4.

$$B^2 = BxB = \{1,2\}\,x\{1,2\} = \{(1,1),(1,2),(2,1),(2,2)\}$$

4.

$$2^{|3|} = 8$$

$$2^S = \{\{\ \},\{2\},\{7\},\{9\},\{2,7\},\{2,9\},\{7,9\},\{2,7,9\}\}$$

5

$$A = \{9,13,17,21\}, \qquad 2^{|a|} = 2^4 = 16$$

6.

1.

$n(A) = 5000 \cdot 0.45 = 2250$

$n(B) = 5000 \cdot 0.25 = 1250$

$n(C) = 5000 \cdot 0.10 = 500$

$n(A \cap B) = 5000 \cdot 0.05 = 250$

$n(B \cap C) = 5000 \cdot 0.04 = 200$

$n(A \cap C) = 5000 \cdot 0.04 = 200$

$n(A \cap B \cap C) = 5000 \cdot 0.03 = 150$

The number of persons who know only language A is

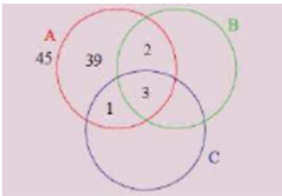
$$n(A\cap\bar{B}\cap\bar{C})=n\left\{A\cap\left(\overline{B\cup N}\right)\right\}=n(A)-n\{A\cap(B\cup C)\}$$

$$=n(A)-n(A\cap B)-n(A\cap C)+n\left(A\cap B\cap C\right)$$

$$=2250-250-200+150=1950.$$

2.

We draw the Venn diagram using percentage:



From the above figure the percentage of persons who know only language A is 39. Therefore, the required number persons is $5000\cdot 0.39=1950.$

7.

Let's use letters for the flavors: $\{b,c,l,s\}$.

$$P=\{\{\ \},\{b\},\{c\},\{l\},\{s\},\{b,c\},\{b,l\},\{b,s\},\{c,l\},\{c,s\},\{l,s\},\{b,c,l\},\{b,c,s\}\{b,l,s\}\{c,l,s\},\{b,c,l,s\}\}$$

8.

1.
Neither injective nor surjective.

2.
Surjective but not injective.

3.
Injective but not surjective.

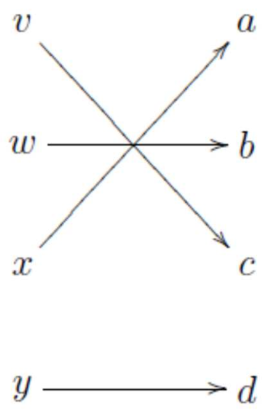
4.
Both injective and surjective, and therefore bijective. Notice that for a function to be bijective, the domain and codomain must have the same cardinality.

5.
The function is both injective and surjective and therefore bijective.

6.
The function is neither surjective nor injective. It is not surjective because the range is not equal to the codomain. For example, there is no number in the domain with image -1 which is an element of the codomain. It is not an injection since more than one distinct element in the domain is mapped to the same element in the codomain. For example, $f(-1) = f(1)$ but $-1 \neq +1$.

9.

1.
The inverse function is $f^{-1}: B \to A$ defined as follows:



2.
The inverse is

$$f^{-1}(x) = \frac{2x}{x-1}$$

10.

- $f^{-1}(\{z\}) = \{c, d\}$
- $f^{-1}(\{x, y\}) = \{a, b\}$

11.

The relation R is

1. *reflexive* since it contains all 4 pairs $(a,a),(b,b),(c,c),(d,d)$.
2. *not symmetric*. For example, $(a,b) \in R$, but $(b,a) \notin R$.
3. *antisymmetric*. It contains 3 non-reflexive elements: $(a,b),(a,c),$ and (b,c). For each of the elements the reverse does not belong to R.
4. *transitive*. There is only one non-reflexive overlapping pair: (a,b) and (b,c). We see that $(a,b),(b,c) \in R$, and $(a,c) \in R$

12.

1.
R is not reflexive: $(2,2) \notin R$. Thus, by definition, R is *not* an equivalence relation.

2.
R is not symmetric: $(1,2) \in R$ but $(2,1) \notin R$. Thus, R is *not* an equivalence relation.

3.
R is not transitive: $(0,1),(1,2) \in R$ but $(0,2) \notin R$. Thus, R is *not* an equivalence relation.

4.
R is reflexive, symmetric, and transitive. Thus, R *is* an equivalence relation.

5.
R is reflexive, symmetric, and transitive. Thus, R *is* an equivalence relation.

(Last updated: 30.11.2020)