Introduction to the Sets Theory

Exercises

1.

Given

$$A = \left\{ x \in R^1 \mid -1 < x < 3 \right\}$$
$$B = \left\{ x \in R^1 \mid -8 < x \right\}$$
$$C = \left\{ x \in R^1 \mid -8 < x < 1 \right\},$$

1. What are

   $A \cap B, \quad A \cup C, \quad \bar{B}, \quad \bar{A} \cup \bar{C}, \quad \bar{A} \cap \bar{C}, \quad B \setminus A, \quad C \setminus B.$

2. Show the validity of the following relations:

   $$A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$$

   and

   $$A \cap (B \setminus C) \neq (A \cup B) \setminus (A \cap C).$$

2.

Let A, B, C be three sets as shown in the following Venn diagram. For each of the following sets, draw a Venn diagram and shade the area representing the given set:

a. $A \cup B \cup C$
b. $A \cap B \cap C$
c. $A \cup (B \cap C)$
d. $A \setminus (B \cap C)$
e. $A \cup \left(\overline{B \cap C} \right) = A \cup (B \cap C)^C$

3.

Given, find the following sets $A = \{1,2,5\}, B = \{1,2\}$. Find the following sets:

1. AxB
2. BxA
3. A^2
4. B^2

4.

Find the power set of $S = \{2,7,9\}$ and total number of elements.

5.

Find the number of subsets of

$$A := \{x \,|\, x = 4n + \in 1,\ 2 \le n \le 5,\ n \in N\}$$

6.

In a survey of 5000 persons in a town, it was found that 45% of the persons know language A, 25% know language B, 10% know language C, 5% know languages A and B, 4% know languages B and C, and 4% know languages A and C.
If 3% of the persons know all the three languages, find the number of persons who know only language A.
Solve the problem by

1. property of cardinality
2. Venn diagram

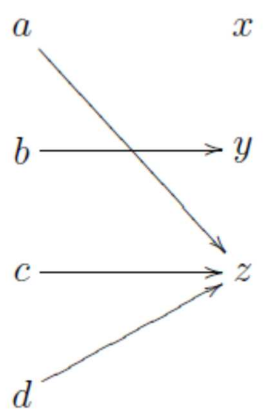
7.

We have four flavors of ice cream: **banana, chocolate, lemon, and strawberry**. How many different ways can we have them?

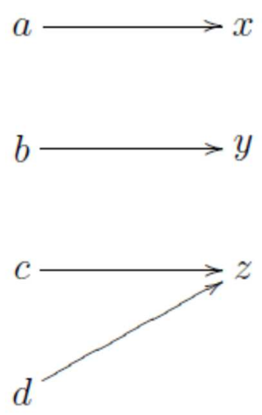
8.

Check the injectivity, surjectivity, and bijectivity of the following functions:

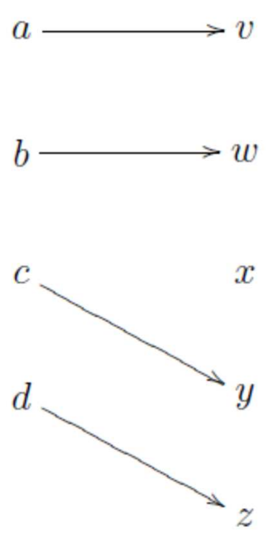
1.
Given $f: A \to B$ where $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$



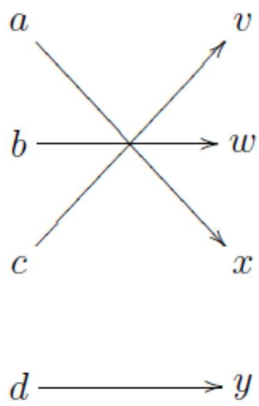
2.
Given $f: A \to B$ where $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$



3.
Given $f: A \to B$ where $A = \{a, b, c, d\}$ and $B = \{v, w, x, y, z\}$



4.
Given $f: A \to B$ where $A = \{a, b, c, d\}$ and $B = \{v, w, x, y\}$



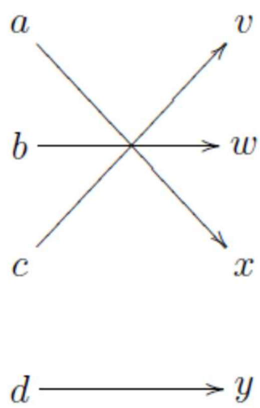
5.
Given $f: [0, \infty[\to [0, \infty[$ defined by $f(x) = \sqrt{x}$.

6.
Given $f(x) = x^2$ with both domain and codomain sets of real numbers.

9.

Find the inverse of the following functions:

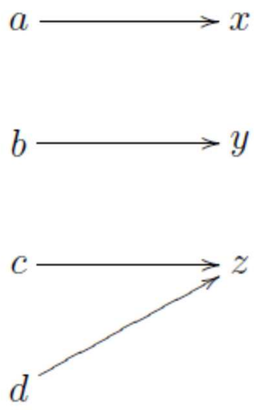
1.
Given $f: A \to B$ where $A = \{a, b, c, d\}$ and $B = \{v, w, x, y\}$



2.
Given $f: R \setminus \{2\} \to R \setminus \{1\}$ defined by $f(x) = \dfrac{x}{x-2}$.

10.

Given $f : A \to B$ where and $B = \{x, y, z\}$ defined as follows



find the corresponding inverses.

11.

The binary relation

$$R = \{(a,a),(a,b),(a,c),(b,b),(b,c),(c,c),(d,d)\}$$

Is defined on the set $A = \{a,b,c,d\}$.
Determine whether R is

1. reflexive
2. symmetric
3. antisymmetric
4. transitive

12.

Are these equivalence relations on $\{0,1,2\}$?

1. $\{(0,0),(1,1),(0,1),(1,0)\}$
2. $\{(0,0),(1,1),(2,2),(0,1),(1,2)\}$
3. $\{(0,0),(1,1),(2,2),(0,1),(1,2),(1,0),(2,1)\}$
4. $\{(0,0),(1,1),(2,2),(0,1),(0,2),(1,0),(1,2),(2,0),(2,1)\}$
5. $\{(0,0),(1,1),(2,2)\}$

(Last updated: 30.11.20)