

Elementary Mathematical Logic

Solutions

1.

1. Proposition (True)
2. Proposition (False)
3. No proposition
4. Proposition Truth unknown value (known as The Goldbach Conjecture).

2.

1.

- a) $p_1 \vee p_2 \vee p_3$
- b) $\neg(p_1 \wedge p_2) \Rightarrow \neg p_3$
- c) $(p_1 \vee \neg p_3) \Rightarrow \neg p_2$

2.

$$p := (p_1 \vee p_2 \vee p_3) \wedge (\neg(p_1 \wedge p_2) \Rightarrow \neg p_3) \wedge ((p_1 \vee \neg p_3) \Rightarrow \neg p_2)$$

3.

$$p_1 \vee p_2 \vee p_3:$$

p_1	p_2	p_3	$p_1 \vee p_2 \vee p_3$
T	T	T	T
T	T	F	T
T	F	T	T
F	T	T	T
T	F	F	T
F	T	F	T
F	F	T	T
F	F	F	F

$$\neg(p_1 \wedge p_2) \Rightarrow \neg p_3$$

p_1	p_2	p_3	$p_1 \wedge p_2$	$\neg(p_1 \wedge p_2)$	$\neg p_3$	$\neg(p_1 \wedge p_2) \Rightarrow \neg p_3$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	F	F
F	T	T	F	T	F	F
T	F	F	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	F	F
F	F	F	F	T	T	T

$$(p_1 \vee \neg p_3) \Rightarrow \neg p_2$$

p_1	p_2	p_3	$\neg p_3$	$p_1 \vee \neg p_3$	$\neg p_2$	$(p_1 \vee \neg p_3) \Rightarrow \neg p_2$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	T	T	T
F	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	F	T	T
F	F	F	T	F	T	T

p_1	p_2	p_3	$p_1 \vee p_2 \vee p_3$	$\neg(p_1 \wedge p_2) \Rightarrow \neg p_3$	$(p_1 \vee \neg p_3) \Rightarrow \neg p_2$	p
T	T	T	T	T	F	F
T	T	F	T	T	F	F
T	F	T	T	F	T	F
F	T	T	T	F	T	F
T	F	F	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	F	T	F
F	F	F	F	T	T	F

$\therefore C_1$ is responsible.

3.

1. Necessary: $e^x \geq 2 \Rightarrow x \geq 0$
 Not sufficient $x = 0 \Rightarrow e^x < 2$;
2. Sufficient : $x > 4 \Rightarrow e^x \geq 2$
 Not necessary: $x = 2 \Rightarrow e^x \geq 2$;

3. Necessary and sufficient: $e^x \geq 2 \Leftrightarrow x \geq \ln 2$;

4. Neither sufficient nor sufficient: $x = -10$ does not imply $e^x \geq 2$,

$$x = 2.5 \Rightarrow e^x \geq 2.$$

(Last updated: 30.09.2011)