Chapter IX

Series of Functions

<u>D. 9. 1.</u> (Pointwise Convergence)
Let $\{f(n)\}, n \in \mathbb{N}$, be a sequence of functions, all defined on the same domain D. The sequence *converges pointwise* towards a function D, defined on D too, if

$$\forall x \in D, \forall \varepsilon > 0, \exists N_0 : n \geq N_0 \Rightarrow \geq \left| f(n(x)) - F(x) \right| < \varepsilon$$

<u>R. 9. 1.</u>
N_0 depends not only on ε, but on the point x too.

<u>D. 9. 2.</u>
Let $\{f(n)\}, n \in \mathbb{N}$, be a sequence of functions. The series of functions $\sum_{i=1}^{\infty} f_i(n)$ is pointwise convergent if the sequence of partial sums $\sum_{i=1}^{n} f_i(n)$ is pointwise convergent.

<u>D. 9. 3.</u> (Uniform Convergence)
Let in D. 9. 1. N_0 be independent of the point x. The sequence $\{f(n)\}, n \in \mathbb{N}$, is then said to be *uniformly convergent*.

<u>R. 9. 2.</u>
Similarly, we define a *uniformly convergent series of functions*.

<u>D. 9. 4.</u> (Power Series)
A *power series* is a series of the form

$$\sum_{i=1}^{+\infty} c_i (x - x_0)^n$$

where x is a real variable, x_0 is a fixed real number called the *center*.

The sequence of real numbers c_i is called the *sequence of coefficients of the series.*

<u>T. 9. 1.</u>
1.

$$\sum_{i=1}^{+\infty} c_i x^i \text{ is convergent for } x = x_0$$

$$\Rightarrow$$

$$\sum_{i=1}^{+\infty} c_i x^i \text{ is absolutely convergent for any } x \text{ such that } |x| > x_l$$

2.

$$\sum_{i=1}^{+\infty} c_i x^i \text{ is divergent for } x = x_l$$

$$\Rightarrow$$

$$\sum_{i=1}^{+\infty} c_i x^i \text{ is divergent convergent for any } x \text{ such that } |x| > x_l$$

(To be continued.)