Sequences

Exercises (Solution)

1.

1.

$$a_1 = \frac{1}{2}, \quad a_2 = \frac{1}{4}, \quad a_3 = \frac{1}{8}, \quad a_4 = \frac{1}{16}, \quad a_5 = \frac{1}{32}.$$

2.

$$a_1 = 2 \quad a_2 = 6, \quad a_3 = 10, \quad a_4 = 14, \quad a_5 = 18.$$

2.

1. $a_n = \dfrac{1}{n^2}$;
2. $a_n = \dfrac{1}{3n}$;
3. $a_n = \dfrac{1}{4n-1}$.

3.

$$a_{n+1} = 2(n+1) + (-1)^{n+1}; \qquad a_n = 2n + (-1)^n;$$

$$a_{n+1} - a_n = 2(n+1) + (-1)^n - \left[2n + (-1)^n\right]$$

$$= 2\left[1 - (-1)^n\right] \geq 0.$$

4.

$$a_n = \frac{n+2}{n+3} = \frac{1+\frac{2}{n}}{1+\frac{3}{n}} \to 1 \quad \text{as} \quad n \to 0.$$

4.

The sequence $\{a_n\} = \{n^3\}$, $n \in \mathbb{N}$, is bounded below because the set $\{1, 8, 27, 64, \ldots\}$ is bounded below. Any number less than or equal to 1 is a lower bound of the sequence.

5.

The sequence $\{a_n\} = \left\{\dfrac{1}{n}\right\}$, $n \in \mathbb{N}$, is bounded , because the set $\{a_n\} = \left\{1, \dfrac{1}{2}, \dfrac{1}{3}, \ldots\right\}$ is bounded below and above. We have

$$0 < \frac{1}{n} \leq 1, \qquad \forall n \in \mathbb{N}$$

6.

$$a_n = \frac{4-7n^2}{n^2+3n} = \frac{\frac{4}{n^2}-7}{1+\frac{3}{n}} \to -7 \text{ as } n \to 0.$$

7.

$$a_n = a_n = \sqrt{n+1}-\sqrt{n} = (\sqrt{n+1}-\sqrt{n}) \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{(n+1)-1}{(\sqrt{n+1}+\sqrt{n})}$$

$$= \frac{1}{(\sqrt{n+1}+\sqrt{n})} \to 0 \quad \text{as} \quad n \to \infty.$$

(To be continued.)