Chapter I

Preliminaries

<u>R. 1. 1.</u> (Important Properties of Real Numbers)

1. Addition

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x + y \in \mathbb{R}$ (*closure*)

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, (x + y) + z = x + (y + z)$ (*Associativity*)

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x + y = y + x$ (*Commutativity*)

$\forall x \in \mathbb{R}, \quad x + 0 = 0 + x = x$ (*Neutral Element*)

$\forall x \in \mathbb{R}, \quad x + (-x) = (-x) + x = 0$ (*Opposite Element*)

2. Multiplication

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x \cdot y \in \mathbb{R}$ (C*losure*)

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, (x \cdot y) \cdot z = x \cdot (y \cdot z)$ (*Associativity*)

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x \cdot y = y \cdot x$ (*Commutativity*)

$\forall x \in \mathbb{R}, \quad x \cdot 1 = 1 \cdot x = x$ (*Neutral Element*)

$\forall x \in \mathbb{R} \setminus \mathbf{0}, \quad x \cdot \frac{1}{x} = \frac{1}{x} \cdot x = 1$ (*Opposite Element*)

3. Distributivity

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, x \cdot (y + z) = x \cdot y + x \cdot z$ (*Distributivity*)

$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, (x + x) \cdot z = x \cdot z + y \cdot z$ (*Distributivity*)

<u>R. 1. 2.</u> (Ordering)

1.

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \quad x \le y \Leftrightarrow y - x \in \mathbb{R}^+$$

2.

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}, x \le y \Leftrightarrow x + z \le y + z$$

3.

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \forall z \in \mathbb{R}^+, x \le y \Rightarrow x \cdot z \le y \cdot z$$

4.

$$\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \forall z \in \mathbb{R}^{-},\ x \le y \Rightarrow x \cdot z \ge y \cdot z$$

<u>D. 1. 1.</u> (Intervals)

$$]a,\ b[:= \{x \in \mathbb{R} \mid a < x < b\}$$

$$[a,\ b] := \{x \in \mathbb{R} \mid a \le x \le b\}$$

$$[a,\ b[:= \{x \in \mathbb{R} \mid a \le x < b\}$$

$$]a,\ b] := \{x \in \mathbb{R} \mid a < x \le b\}$$

$$]-\infty,\ b] := \{x \in \mathbb{R} \mid x \le b\}$$

$$]-\infty,\ b[:= \{x \in \mathbb{R} \mid x < b\}$$

$$[a,\ +\infty[:= \{x \in \mathbb{R} \mid a \le x\}$$

$$]a,\ +\infty[:= \{x \in \mathbb{R} \mid a < x\}$$

<u>D. 1. 2.</u> (Absolute Value)

Let x be a real number. The *absolute value* of x is the number denoted by $|x|$ and defined as

$$|x| := \begin{cases} x & for \quad x \ge 0 \\ -x & for \quad x \le 0 \end{cases}$$

<u>R. 1. 3.</u> (Properties of the Absolute Value)

1.

$$\forall x \in \mathbb{R},\quad |x| = 0 \iff x = 0$$

2.

$$\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\quad |x \cdot y| = |x| \cdot |y|$$

3.

$$\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\quad |x + y| \le |x| + |y| \qquad (\textit{Triangle Inequality})$$

4.

$$\forall x \in \mathbb{R},\ \forall y \in \mathbb{R}^{+},\quad -y \le x \le y \iff |x| \le y$$