(Diese Datei wird zur Zeit überarbeitet.)

Chapter X

Functions of Several Variables
(Solutions)

1.
The expression $2x - y$ is meaningful for all value of x and y Therefore, the entire xy – Plane is the natural domain of the function.

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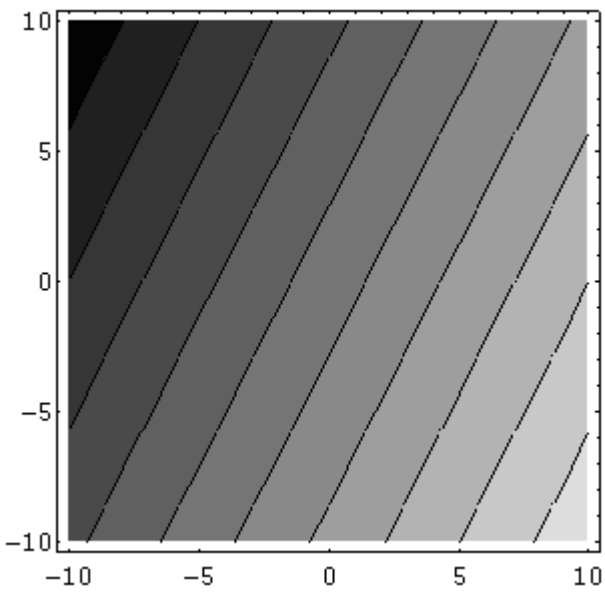
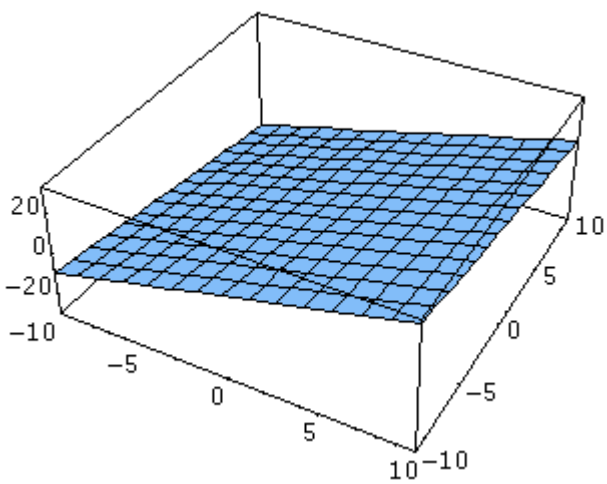
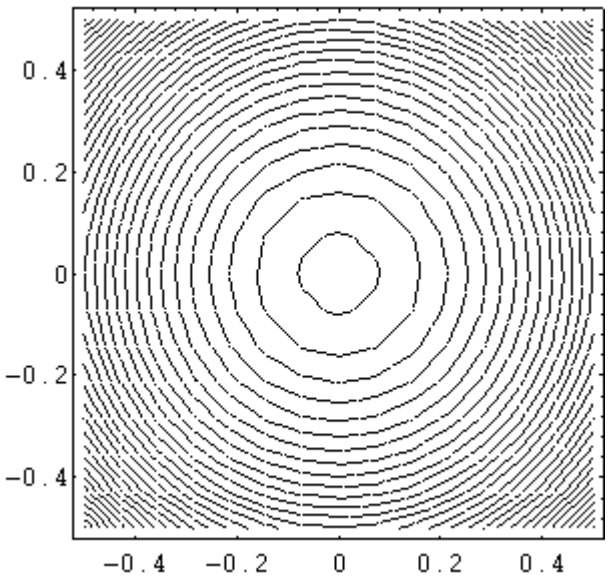


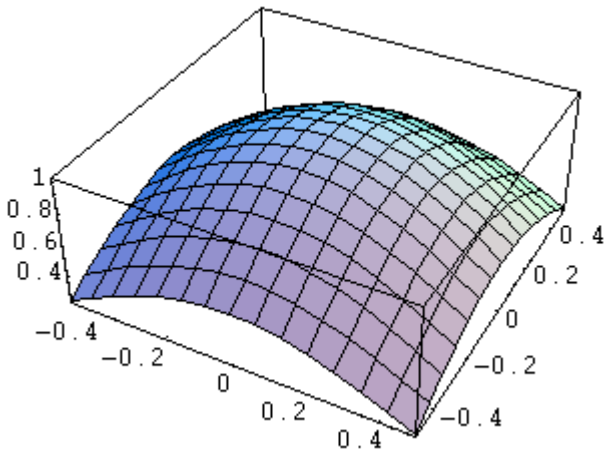


2.

$$1 - x^2 - y^2 \geq 0 \qquad \text{or} \qquad x^2 + y^2 \leq 1.$$

All the points $P(x, y)$ whose co-ordinates satisfy the given inequality lie in a circle of radius 1 with centre at the origin and on the boundary of this circle.





3.

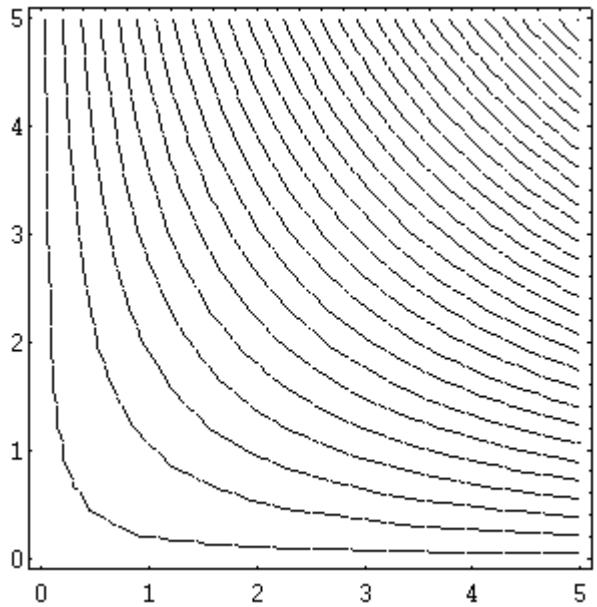
Since logarithms are defined only for positive numbers, the following inequality must be satisfied.

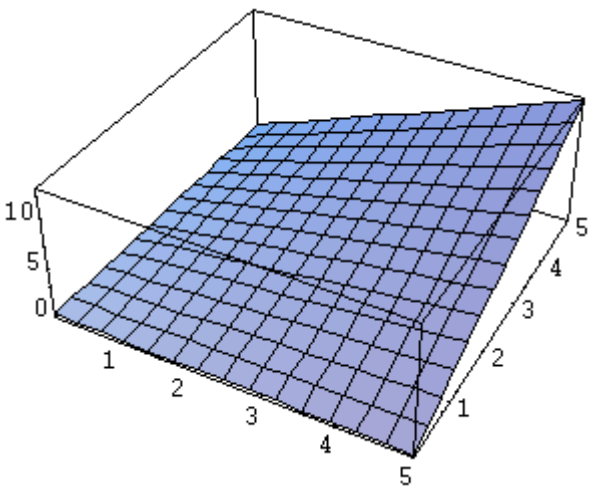
$$x + y > 0 \qquad \text{or} \qquad y > -x\,.$$

This means that the natural domain of definition of the function z is the half-plane above the straight line $y = -x$, the line itself not included.

4.

$$S = \frac{x \cdot y}{2}, \qquad x, y > 0$$





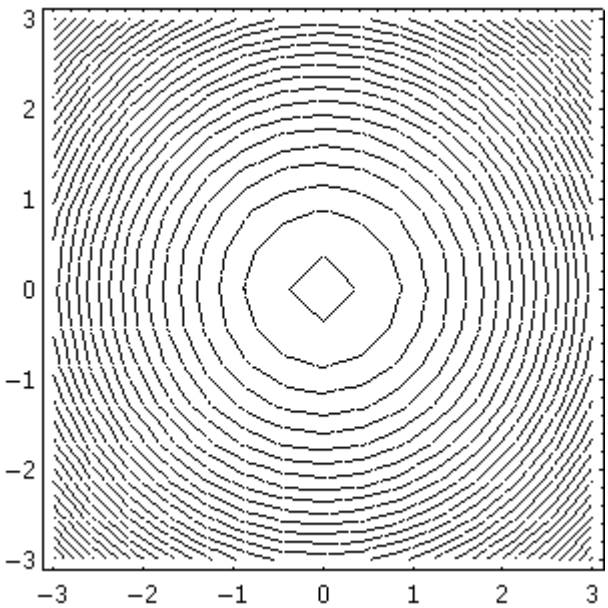
5.

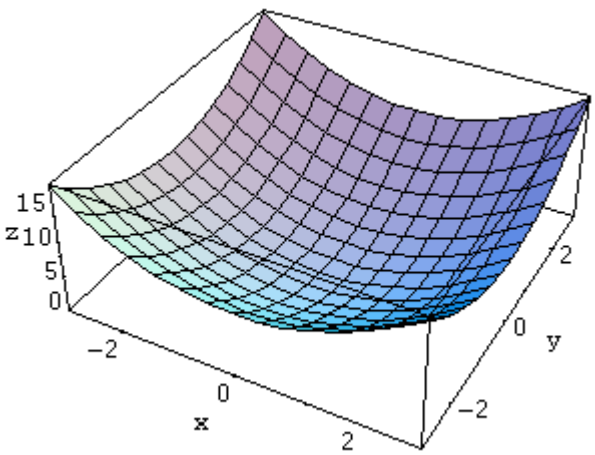
Indeed no matter what the numbers x and y, Δx and Δy, we have

$$\Delta z = \left[(x+\Delta x)^2 + (y+\Delta y)^2\right] - \left[x^2 + y^2\right] = 2x\cdot\Delta x + 2y\cdot\Delta y + \Delta x^2 + \Delta y^2 .$$

Consequently,

$$\lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}} \Delta z = 0$$





(To be continued.)