Chapter X

Functions of Several Variables

<u>D. 10. 1.</u> (Function of two Variables)
If to each pair (x, y) of values of two independent quantities x and y (from some range D) there corresponds a definite value z, we say that z is a *function of the two independent variables* x *and* y defined in D.
A function of two variables is symbolically given as

$$z = f(x, y).$$

<u>D. 10. 2.</u> (Domain of Definition of a Function of two Variables)
The collection of pairs (x, y) of values of x and y, for which the function $z = f(x, y)$ is defined, is called the *domain of definition* of this function.

<u>D. 10. 3.</u> (Function of Several Variables)
If to every collection of values of the variables $x_1, x_2, ..., x_n$ there corresponds a definite value of the variable y, we shall then call y *the function of the independent variables* $x_1, x_2, ..., x_n$ and write

$$y = f(x_1, x_2, ..., x_n).$$

<u>D. 10. 4.</u> (Domain of Definition of a Function of Several Variables)
The collection of the variables $x_1, x_2, ..., x_n$ for which the function $y = f(x_1, x_2, ..., x_n)$ is defined, is called the *domain of definition* of this function.

<u>D. 10. 5.</u> (Limit of a Function of two Variables)
The number l is called the *limit of the function* $z = f(x, y)$ as $P(x, y)$ approaches $P_0(x_0, y_0)$
If for every $\varepsilon > 0$ there is an $r > 0$ such that for all points $P(x, y)$ for which the inequality $\overline{PP_0} < r$ is fulfilled, we have the inequality

$$|f(x, y) - l| < \varepsilon .$$

If l is the limit of $f(x, y)$ as $P(x, y) \to P_0(x_0, y_0)$, then we write

$$\lim_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = l$$

<u>D. 10. 6.</u> (Continuity at a Point)
Let the point $P_0(x_0, y_0)$ belong to the domain of definition of the function $f(x, y)$. The function $f(x, y)$ is called *continuous at the point* $P_0(x_0, y_0)$ if we have

$$\lim_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = f(x_0, y_0)$$

and $P(x, y)$ approaches $P_0(x_0, y_0)$ in arbitrary fashion all the while remaining in the domain of the function.