Analysis in Economics (I)

Solutions

1.

$$x = \sqrt[3]{4r^2}, \quad r \geq 0 \quad \Rightarrow \quad r = \frac{1}{2}\sqrt{x^3}$$

$$C(x) = 40 + 10\sqrt{x^3} \ .$$

2.

1.

$x-$ *intercept*:

$$\left(-x^2 + 20x + 312 = 0 \ \wedge \ x \geq 0\right) \ \Rightarrow \ x = 30.29778313 \approx 30.30 \ ,$$

i. e. for an output of 20.30 units the company's profit will be equal to zero.

$y-$ *intercept*:

$$x = 0 \ \Rightarrow \ P(x) = 312,$$

i. e. for an output of zero units the company's profit will be equal to 312.

2.

$$P'(x) = -2x + 20 \ , \quad P'(x) = 0 \ \Rightarrow \ -2x + 20 = 0 \ \Rightarrow \ x = 10 \ ,$$

$$P''(x) = -2 < 0 \ .$$

Hence, $x = 10$ gives the maximum profit. This will be equal to $P(10) = 412$.

3.

(Do it yourselves.)

4.

Changing the value of the constant simply moves the profit curve up or down and, in particular, it does not change the $x-$ coordinate of the maximum. In this case, changing the original constant 312 to 156 means that the profit function is now

$$\tilde{P}(x) = -x^2 + 20x + 156 \ ,$$

and so the curve will move down by 312-156 = 156 which means that the maximum value will now be 412 – 156 = 256 and this will still occur when $x = 10$.
The effect of changing the constant in this way is illustrated in the following figure:

5.

$$x = 4 \;\Rightarrow\; P(4) = 620, \qquad x = 8 \;\Rightarrow\; P(8) = 700,$$

$$C(x) - 620 = \frac{700-620}{8-4}\left(x-4\right),$$

$$C(x) = 20x + 540$$

3.

$$C(x) = x \cdot c(x)$$
$$C(x) = 1.5x^2 + 4x + 46$$
$$C'(x) = 3x + 4$$

4.
1.

$$x(r) = -20 + 0.1r \qquad \Rightarrow \qquad r(x) = 10x + 200$$

$$C(x) = C_{\text{var}} + C_{fix}$$
$$= \left(10x + 200\right) \cdot 2 + 300 = 20x + 700$$

$$C'(x) = 20.$$

2.
The cost function has because of $C'(x) = 20 \neq 0$ no minimum.

3.

$$R(x) = p(x) \cdot x$$
$$= 220x - 4x^2,$$

$$R'(x) = 220 - 8x.$$

4.

$$P(x) = R(x) - C(x), \qquad P'(x) = R'(x) - C'(x) = 220 - 8x - 20.$$

$$P'(x) = 0 \qquad \Rightarrow \qquad x = 25,$$

$P^{''}(x) = -8 < 0$, $P(25) = 1800$ €.

The firm has, therefore, to produce 25 units in order to gain a maximum profit of 1800 €.

5.

$p(25) = 120$ €

5.

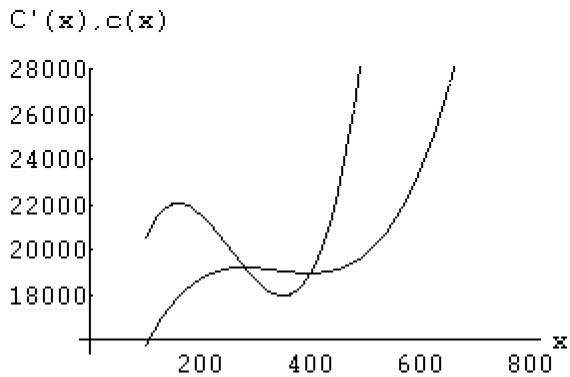
$$P(x) = R(x) - C(x)$$
$$= 4000x - 33x^2 - \left(2x^3 - 3x^2 + 400x + 5000\right)$$
$$P\left(x\right) = -2x^3 - 30x^2 + 3600x - 5000$$
$$\left(P'\left(x\right) = -6x^2 - 60x + 3600 \ \wedge \ x > 0\right) \Rightarrow x = 20$$
$$P''\left(x\right) = -12x - 60, \quad P''\left(20\right) = -300 < 0.$$

$\therefore$ Profit is maximised at $x = 20$ where $P(20) = 39000.$

6.

$$C'(x) = 8400 + 10^{-4}(2016000x - 9180x^2 + 12x^3)$$
$$c(x) = 8400 + 10^{-4}(1008000x - 3060x^2 + 3x^3)$$

$$C'(x) < c(x)$$
$$\Leftrightarrow$$
$$8400 + 10^{-4}(2016000x - 9180x^2 + 12x^3) < 8400 + 10^{-4}(1008000x - 3060x^2 + 3x^3)$$
$$\Leftrightarrow$$
$$x^3 - 680x^2 + 112000\text{x} < 0 \ \Leftrightarrow \ 280 < x < 400$$



7.

1.

$$R(p) = x(p) \cdot p$$
$$= 100pe^{-0.04p}, \quad 0 \le p \le 100$$

$$P(p) = R(p) - C(p)$$
$$= 100pe^{-0.04p} - 2000 - 3000e^{-0.04p}, \qquad 0 \le p \le 100$$

2.

$$P'(p) = e^{-0.04p}(220 - 4p), \qquad 0 < p < 100$$

$$P''(p) := 0 \qquad \Rightarrow \qquad p = 55$$

$$P''(p) = -0.04e^{-0.04p}(220 - 4p) - 4e^{-0.04p} \qquad \Rightarrow \qquad P''(p) < 0.$$

The profit function $P(p)$ has because of $P(100) < P(55)$ its absolute maximum at $p = 55$

8.

1.

$$C'(x) = 30 + \frac{1160000x}{(400 + x^2)^2}, \qquad 10 < x < 50$$

$$C'(20) = 66.25\,€$$

An increase of the output by one unit leads to an approximate increase of the total costs by 66.25 €.

$$c(x) := \frac{C(x)}{x}$$
$$= 30 + \frac{1450x}{400 + x^2}, \qquad 10 \le x \le 50$$
$$c'(x) = \frac{-1450x^2 + 580000}{(400 + x^2)^2}, \qquad 10 < x < 50$$

$$c'(x) := 0 \qquad \Rightarrow \qquad x = 20$$
$$c''(x) = \frac{-2900x \cdot (400 + x^2) - 4x \cdot (580000 - 1450x^2)}{(400 + x^2)^3}, \qquad 10 < x < 50$$
$$c''(20) < 0$$

Consequently, at $x = 20$ the function $c(x)$ has over the interval $]10,\ 50[$ a relative maximum. Because of $c(10) = 59,\ c(50) = 55$ the function $c(x)$ has in $x = 50$ its absolute minimum.

3.
From 2. it follows that $c(x)$ increases on $[10,\ 20]$ and decreases on $[20,\ 50]$.

9.

1.

$$\varepsilon_{p,x}(x) = \frac{-0.5x}{16-0.5x}$$

$$\varepsilon_{p,x}(8) = -\frac{1}{3}.$$

2.

$$p(x) = 16 - 0.5p \;\;\Rightarrow\;\; x(p) = 32 - 2p$$

$$\varepsilon_{x,p}(p) = \frac{-2p}{32-2p}$$

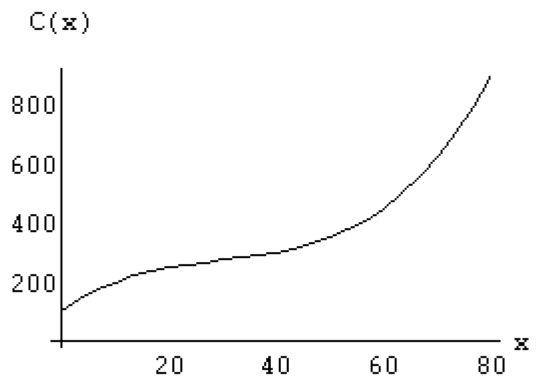
$$p(8) = 12$$
$$\varepsilon_{x,p}(12) = -3$$

10.

1.

$$C(40) = 300,\;\; C(50) = 350, C(60) = 450, C(70) = 625$$

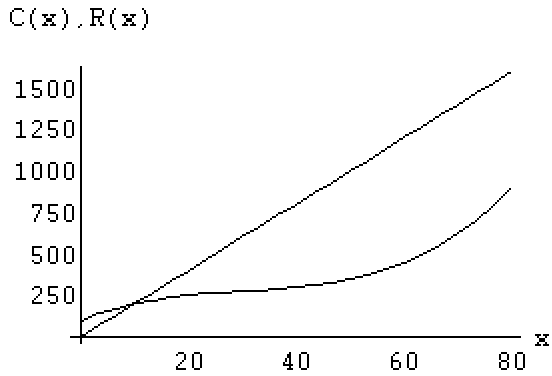
2.



3.

$$R(x) = 20x$$

4.



5.

$C(x) = R(x) = 10$ (See the curves above.)

6.

$$P(x) = R(x) - C(x) = -\frac{x^3}{240} + \frac{3x^2}{8} + \frac{20x}{3} - 100$$

$$P'(x) = -\frac{x^2}{80} + \frac{3x}{4} + \frac{20}{3} = 0$$

$x = 67.86$

$P''(x) = -\frac{x}{40} + \frac{3}{4}$, $\qquad P''(67.86) < 0$,

Maximum profit: $P(67.86) = 777.21$€

7..

$$c_v(x) = \frac{C_v(x)}{x} = \frac{x^2}{240} - \frac{3x}{8} + \frac{40}{3}$$

$c_v'(x) = \frac{x}{120} - \frac{3}{8} := 0 \Rightarrow x = 45$, $\quad c_v''(x) = \frac{1}{120} > 0$

8.

i)

$C'(70) = 22.0833$

ii)

$C(71) - C(70) = 647.5875 - 625 = 22.5875$

9.

i)

$$\varepsilon_{K,x}(x) = \frac{\left(\frac{x^2}{80} - \frac{3x}{4} + \frac{40}{3}\right)x}{\left(\frac{x^3}{240} - \frac{3x^2}{8} + \frac{40x}{3} + 100\right)}$$

$$\varepsilon_{K,x}(48) = 0.874$$

The cost function is inelastic at $x = 48$.

ii)

$$C(48) = 336.8$$

$$C(48 \cdot 1.01) = C(48.48) = 339.79$$

$$\frac{339.79}{336.8} \cdot 100 - 100 = 0.89$$

11.

1.

$$x'(p) = \frac{-500}{(p+5)^2}$$

2.

$$\varepsilon_{x,p}(p) = \frac{\frac{-500p}{(p+5)^2}}{\frac{500}{p+5} - 10} = -\frac{50p}{(5+p)(45-p)}$$

3.

$$10 = \frac{500}{p+5} - 10 \quad \Rightarrow \quad p = 20$$

$$40 = \frac{500}{p+5} - 10 \quad \Rightarrow \quad p = 5$$

$$\varepsilon_{x,p}(20) = -1.6, \qquad \varepsilon_{x,p}(5) = -0.625$$

4.

$$\frac{-50p}{(5+p)(45-p)} = -3 \qquad \Rightarrow \quad 3p^2 - 70p - 675 = 0 \quad \Rightarrow \quad p = 30.67\,€$$

$x(30.67) \approx 4.02$

5.

$$\frac{-50p}{(5+p)(45-p)} = -1 \quad \Rightarrow \quad p^2 + 10p - 225 = 0 \quad \Rightarrow \quad p \approx 10.81 \text{ €}$$

i. e. the demand function is elastic for $p > 10.81$ and inelastic for $p < 10.81$.

12.

1.

$$\begin{aligned} R(x) &= p(x) \cdot x \\ &= -0.5x^2 + 100x \end{aligned}$$

2.

$$\begin{aligned} P(x) &= R(x) - R(x) \\ &= -0.5x^2 + 90x - 1000 \end{aligned}$$

3.

$$P'(x) = -x + 90 \quad , \quad P'(x) := 0 \quad \Rightarrow \quad x = 90$$

$$P''(x) = -1 < 0.$$

There will, therefore, be a maximum profit of 3050 € for $x = 90$. The corresponding price will be $p(90) = 55$ €.

13.

1.

$$C'(x) = 20.$$

A change of output by one unit from any level x_0 will result in a change of total costs by 20 €.

2.

$$c(x) := \frac{C(x)}{x} = \frac{1000}{x} + 20, \qquad x > 0$$

3.

$$\lim_{x \to +\infty} C(x) = \lim_{x \to +\infty} (1000 + 20x) = +\infty$$

$$\lim_{x \to +\infty} c(x) = \lim_{x \to +\infty} \left(\frac{1000}{x} + 20\right) = 20 = C'(x)$$

14.

1.

$$C'(x) = -12 + 4x$$

$$C'(x) := 0 \quad \Rightarrow \quad x = 3$$

$$C''(x) = 4 > 0 .$$

Thus, the total costs function has its (absolute) minimum at $x = 3$ where $C(3) = 42$

2.

$$c(x) := \frac{C(x)}{x}$$
$$= \frac{60}{x} - 12 + 2x, \quad x > 0$$

$$c'(x) = -\frac{60}{x^2} + 2, \quad x > 0 ,$$

$$c'(x) := 0 \quad \Rightarrow \quad x = \sqrt{30} ,$$

$$c''(x) = \frac{120}{x^3} > 0$$

Thus, the average costs function $c(x)$ has its (absolute) minimum at $x = \sqrt{30}$.

3.

Since

$$C'(x) = -12 + 4x, \qquad C''(x) = 4 \neq 0$$

4.

$$-12 + 4x = \frac{60}{x} - 12 + 2x \quad \Leftrightarrow \quad x = \sqrt{30} .$$

5.

$C'(x_0)$ gives the approximate change of costs after a change of the output x_0 by one unit.

$c(x_0)$ gives the costs per unit for an output x_0 .

15.

1.

$$y' = \frac{\sqrt{10}}{2\sqrt{r}}, \qquad y'' = -\frac{\sqrt{10}}{4\sqrt{r^3}} < 0 ,$$

Thus, the marginal production function is decreasing.

2.

$$y = \sqrt{10r}\,, \qquad y^2 = 10r\,, \qquad r = 0.1y^2, \qquad K(r) = 2r$$

$$K(y) = 0.2y^2$$

16.
1.

$$\begin{aligned} x'(r) &= -r^2 + 4r \\ &= r(4-r) \end{aligned}$$

$$x'(r) := 0 \;\Rightarrow\; r = 4$$

Because of

$$x''(r) = -2r + 4, \quad x''(4) < 0$$

has $x(r)$ a relative (its absolute) maximum for $r = 4$.

2.

$$x(r) = -\frac{1}{3}r^3 + 2r^2 = r^2(2 - \frac{1}{3}r) < 0 \;\Leftrightarrow\; 2 - \frac{1}{3}r < 0 \;\Leftrightarrow\; r > 6$$

3.

$$\frac{x(r)}{r} = -\frac{1}{3}r^2 + 2r$$

4.
Because of

$$\left(\frac{x(r)}{r}\right)' = -\frac{2}{3}r + 2 := 0 \;\Rightarrow\; r = 3, \quad \left(\frac{x(r)}{r}\right)'' = -\frac{2}{3} < 0$$

the function $\frac{x(r)}{r}$ has a relative (its absolute) maximum at $r = 3$.

5.
$x'(r_0)$ means: the production will approximately change by $x'(r_0)$ units, if the input r_0 will be changed by one unit.

17.
1.

$$dC(x) = C'(x) \cdot dx$$
$$C'(x) = 0.18x^2 - 4x + 60$$

i) $dC(x=10, dx=2) = 76$ €

ii) $dC(x=10, dx=-1) = -38$ €

2.

i) $\Delta C = C(12) - C(10) = 75.68$ €

ii) $\Delta C = C(9) - C(10) = -38.26$ €

iii) $\Delta C = C(12) - C(10) = 75.70$ €

iv) $\Delta C = C(9) - C(10) = -38.30$ €

18.

1.

$$\lim_{x \to 0^+} C(x) = 16.$$

2.

$$C(0) = 0.$$

3.

No, since $\lim_{x \to 0} C(0) \neq C(0)$.

19.

1.

$$C'(x) = 5 - 0.02x + 0.03x^2$$

An increase of output from 5 to 6 units will lead to an approximate increase of costs by 5.65 units.

2.

$$P(x) = R(x) - C(x)$$

$$= p(x) \cdot x - C(x)$$

$$= (50 - 0.01x) \cdot x - \left(200 + 5x - 0.01x^2 + 0.01x^3\right)$$

$$= -200 + 45x - 0.01x^3$$

3.

$$P'(x) = 45 - 0.03x^2$$

$$P'(x) = 0 \ \wedge \ x \geq 0 \ \Rightarrow \ x = 38.73$$

$$P''(x) = -0.06x, \quad P''(38.73) < 0.$$

∴ The firm will maximise its profit for an output of 38.73 units. The maximum profit will amount to $P(38.73x) = 961.90$.

4.

i)

$$\varepsilon_{P,x}(x) = \frac{x}{P(x)} \cdot P'(x)$$

$$\varepsilon_{P,x}(x) = \frac{x}{-0.01x^3 + 45x - 200} \cdot \left(-0.03x^2 + 45x\right)$$

$$\varepsilon_{P,x}(40) = \frac{40}{-0.01 \cdot 40^3 + 45 \cdot 40 - 200} \cdot \left(-0.03 \cdot 40^2 + 45 \cdot 40\right) \approx -0.125\,\%.$$

ii)

$P(40 \cdot 1.01) = P(40.4) = 958.61$, $\qquad P(40) = 960.00$.

$$\frac{958.61}{960.00} \cdot 100 - 100 = -0.145\%\ \%$$

20.

$$C(x) = \frac{1}{12}x^3 - \frac{3}{4}x^2 + \frac{13}{4}x, \quad x \in [1,\ 8], \qquad (x\text{: output; } C\text{: costs}).$$

1.

$$C'(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}, \quad x \in [1,\ 8]$$

$$C'(2) = 2.$$

An increase of output from 5 to 6 units will lead to an approximate increase of costs by 2 units.

2.

$$P(x) = R(x) - C(x)$$

$$= \left(6 - \frac{x}{2}\right) \cdot x - \left(\frac{1}{12}x^3 - \frac{3}{4}x^2 + \frac{13}{4}x\right)$$

$$P(x) = -\frac{1}{12}x^3 + \frac{1}{4}x^2 + \frac{11}{4}x, \quad x \in [1,\ 8]$$

$$P'(x) = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4}, \quad x \in [1,\ 8]$$

$$P'(x) = 0, \quad -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4} = 0, \quad x \in [1,\ 8] \quad \Rightarrow \quad x \approx 4.46$$

$P''(4.46) < 0.$
$P(4.46) \approx 9.85, \quad P(1) \approx 2.92, \quad P(8) \approx -4.67$.

Hence, the firm's profit will be maximal for an output of $x \approx 4.46.$ It will be approximately equal to 9.85.

3.
i)

$$\varepsilon_{P,x}(x) = \frac{x}{P(x)} \cdot P'(x)$$

$$= \frac{x}{-\frac{1}{12}x^3 + \frac{1}{4}x^2 + \frac{11}{4}x} \cdot \left(-\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4}\right).$$

$$\varepsilon_{P,x}(3) = \frac{3}{8.25} \cdot 2 \approx 0.73$$

ii)

$P(3) = 8.25$, $\quad P(3.03) \approx 8.31$

$$\frac{8.31}{8.25} \cdot 100 - 100 \approx 0.73$$

21.
1.

2.
Strictly concave.

22.
1.

$$C(x) - 1500 = \frac{1800 - 1500}{100 - 0} \cdot (x - 0)$$

$C(x) = 3x + 1500$
$R(x) = 7x$

$P(x) = R(x) - C(x) = 4x - 1500$.

2.
The company will break even when $P(x) = 0$. Therefore,

$4x - 1500 = 0$, $\quad x = 375$.

So, by making and selling 375 mugs, the company will neither gain nor lose money.

23.

$$R(q) = 15q - q^2$$

$$P(q) = R(q) - C(q)$$

$$= -2q^2 + 12q + 2$$

$$P'(q) = -4q + 12, \qquad -4q + 12 = 0, \qquad q = 3.$$

Because of $P''(q) = -4 < 0$, $q = 3$ maximises the firm's profit. The corresponding price level is $p(3) = 12$.

24.

1.

$$p = 10 - 0.5q \quad \Rightarrow \quad q = 20 - 2p, \quad p \in [0,\ 10]$$

$$\varepsilon_{q,p}(p) = \frac{-2p}{20 - 2p}$$

2.

$$\frac{-2p}{20 - 2p} < -1, \qquad \frac{-p}{10 - p} + 1 < 0, \qquad \frac{-2p + 10}{10 - p} < 0.$$

Case 1:

$$\begin{cases} -2p + 10 > 0 \\ -p + 10 < 0 \end{cases} \Rightarrow \; p \in \varnothing$$

Case 2:

$$\begin{cases} -2p + 10 < 0 \\ -p + 10 > 0 \end{cases} \Rightarrow \; 5 < p < 10.$$

The demand is elastic for $p \in \,]5,\ 10[$ and inelastic for $p \in \,]0,\ 5[$.

25.

$$R(q) = p(q) \cdot q, \qquad R(q) = 15q - 6q^2,$$

$$P(q) = R(q) - C(q), \qquad P(q) = -2q^3 - 3q^2 + 12q - 2,$$

$$P'(q) = -6q^2 - 6q + 12$$

$$\left(P'(q)=0 \ \wedge \ q \geq 0\right) \quad \Rightarrow \quad q=1, \qquad P''(1)=-12<0,$$
$$p(1)=9, \qquad P(1)=5.$$

26.
1.
$$Q'(L)=6L-0.3L^2.$$

2.
$$Q''(L)=6-0.6L,$$

$$Q''(L)=0 \quad \Rightarrow \quad L=10, \qquad Q'''(L)=-0.6<0$$

$$q(L):=\frac{3L^2-0.1L^3}{L}=3L-0.1L^2,$$
$$q'(L)=3-0.2L, \qquad q'(L)=0 \ \Rightarrow L=15, \quad q''(L)=-0.2<0.$$

3.
$$Q'(L)=q(L) \quad \Rightarrow \quad 6L-0.3L^2=3L-0.1L^2 \quad \Rightarrow \quad L=15.$$

27.
1.
$$C(x)=12x-4x^2+x^3$$

$$C'(x)=12-8x+3x^2$$

2.
(Do it yourselves.)
3.
$$R(x)=16x$$

$$P(x)=R(x)-C(x)=16x-\left(12x-4x^2+x^3\right)$$
$$P(x)=4x+4x^2-x^3, \qquad P'(x)=4+8x-3x^2$$

$$\left(4+8x-3x^2=0 \ \wedge \ x \geq 0\right) \quad \Rightarrow \quad x \approx 3.10,$$

Because of

$$P''(x)=8x-6x, \quad P''(3.10)<0$$

the firm should sell 3.10 units in order to maximise profit. The maximum profit will be $P(3.10) \approx 21.05$ €.

28.

1.

$$R(x) = p(x)\cdot x = 4x - \frac{x^2}{25}$$

$$P(x) = R(x) - C(x) = 4x - \frac{x^2}{25} - \left(\frac{x}{2500}\cdot(x-100)^2 + x\right)$$

$$= 4x - \frac{x^2}{25} - \left(\frac{x^3}{2500} - \frac{2x^2}{25} + 5x\right)$$

$$= -\frac{x^3}{2500} + \frac{x^2}{25} - x,$$

$$P'(x) = -\frac{3x^2}{2500} + \frac{2x}{25} - 1 = 0 \quad \Rightarrow \quad x_1 = 50, \quad x_2 = \frac{50}{3} \approx 16.67,$$

$$P''(x) = -\frac{6x}{2500} + \frac{2}{25}, \qquad P''(50) = -0.04 < 0, \quad P''(16,67) = 0.039992 > 0.$$

Therefore, the monopolist's profit will be maximised at $x = 50$, with $P(50) = 0.$ The corresponding price will be $p(50) = 2$.

2.

(Do it yourselves.)

29.

1.

$$R(x) = p(x)\cdot x = 1200x - 10x^2$$

$$P(x) = R(x) - C(x) = 1200x - 10x^2 - 200x - 15x^2$$

$$P(x) = -25x^2 + 1000x$$
$$P'(x) = -50x + 1000$$

$$P'(x) = 0 \quad \Rightarrow \quad x = 20.$$

Because of $P''(x) = -50 < 0$ the output $x = 20$ maximises the monopolist's profit.

2.

$$p(20) = 1200 - 10\cdot 20 = 1000.$$

3.

$$P(20) = 10000.$$

30.

$$p+q^2+3q-20=0 \quad \Rightarrow \quad -q^2-3q+20=0$$

$$p-3q^2+10q=5 \quad . \Rightarrow \quad 3q^2-10q+5=0$$

$$-q^2-3q+20=q^2-10q+5$$

$$\left\langle 4q^2-7q-15=0 \quad \wedge \quad q\geq 0\right\rangle \Rightarrow \quad q=3 \quad \Rightarrow \quad p=2$$

31.

$$x(p)=\frac{1}{e^{p-2}}-2=e^{2-p}-2$$

$$t=2-p\,, \qquad \frac{dt}{dp}=-1$$

$$x(p)=e^t-2\,, \qquad \frac{dx(p)}{dt}=e^t$$

$$x'(p)=-e^t=-e^{2-p}<0\,.$$

Therefore, the demand function is strictly decreasing. This means that with an increase of prices the demand will decrease.

32.

1.

$$p(x)=10-0.5x \quad \Rightarrow \quad x(p)=-2p+20$$

$$\varepsilon_{x,p}(p)=\frac{-2p}{-2p+20}$$

2.

$$\frac{-2p}{-2p+20}=-1 \quad \Rightarrow \quad p=5 \qquad \Rightarrow \quad x(5)=10$$

The demand function $x(p)=-2p+20$

1. has a unitary elasticity in $p=5$ ($x=10$).

$$\varepsilon_{x,p}(4)=\frac{-8}{-8+20}=-0.4>-1 \qquad \Rightarrow \quad x(4)=12$$

2. is inelastic for $p\in\,]0,\ 5[$ ($x\in\,]0,\ 12[$)

$$\varepsilon_{x,p}(6) = \frac{-12}{-12+20} = -1.5 < -1 \qquad \Rightarrow \quad x(6) = 8$$

2. is elastic for $p \in \left]5,\ \infty\right[$ ($x \in \left]12,\ \infty\right[$).

33.

1.

$$P(x) = R(x) - C(x) = (180 - 2x) \cdot x - \left(128 + 60x + 8x^2\right)$$
$$P(x) = -10x^2 + 120x - 128$$
$$P'(x) = -20x + 120 := 0 \quad x = 6, \qquad P'(x) = -20 < 0.$$

Therefore, at the level of $x = 6$ the will be a maximum profit equal to $P(6) = 232$.

2.

$$c(x) = \frac{128}{x} + 60 + 8x$$
$$c(x) = \frac{128}{x} + 60 + 8x, \quad x > 0$$
$$\left\langle c'(x) = -\frac{128}{x^2} + 8 := 0 \ \wedge \ x > 0 \right\rangle \Rightarrow \quad x = 4$$

Because of $c''(x) = \frac{256}{x^3} > 0$ für $\forall x > 0$ the average cost function will assume its minimum at $x = 4$ with $c(4) = 124$.

3.

$$C'(x) = 60 + 16x > 0, \quad \forall x \geq 0$$

$$C''(x) = 16 > 0, \quad \forall x \geq 0$$

Therefore, the cost function does not increase degressively for any x.

(Last updated: 17.06.2022)