Analysis in Economics (I)

Exercises

1.
The production function of a firm is given by

$$x = \sqrt[3]{4r^2}, \quad r \geq 0,$$

Here are:

r: input,
x: output.

The firm has to pay 20 € for each unit of input and has 40 € fixed costs. Determine the firm's total cost function and plot it.

2.
A company has a profit function given by

$$P(x) = -x^2 + 20x + 312$$

where x denotes the quantity produced.

1. Find and interpret $x-$intercepts and $y-$intercepts of the curve $y = P(x)$.
2. What quantity produced gives the maximum profit? What is the maximum profit?
3. Use the above information to sketch the curve $y = P(x)$.
4. If the constant term in our expression for $P(x)$ is changed from 312 to 156, how does the answer to 2. change?
5. Given that the company has a linear cost function and that it costs 620 € to produce four units and 700 € to produce eight units, determine the cost function.

3.
Find the marginal cost function for the following average cost function:

$$c(x) = 1.5x + 4 + \frac{46}{x}, \ x > 0.$$

4.
The production function of a firm is given by

$$x(r) = -20 + 0.1r.$$

Here are:

r: input,
x: output.

The firm has to pay 2.00 € for each unit of input and has fixed costs amounting to 300.00 €.

A marketing research has estimated the following price function:

$$p(x) = 220 - 4x.$$

1. Determine the total and the marginal cost functions of the firm.
2. At what level of production will the total costs be minimal?
3. Find the revenue and the marginal revenue functions.
4. How much should be produced so that the firm's profit will be maximised? Find the maximal profit.
5. Determine the price for which the profit will be maximal.

5.
Maximise profits P for a firm, given total revenue

$$R(x) = 4000x - 33x^2 \quad (x: \text{ sales})$$

and total cost

$$C(x) = 2x^3 - 3x^2 + 400x + 5000, \ (x: \text{ output}),$$

assuming $x > 0$.

6.
The total cost function of a firm is given by

$$C(x) = 8400x + \left(1008000x^2 - 3060x^3 + 3x^4\right) \cdot 10^{-4}.$$

Find the intervals in which the marginal costs are less than the average costs.

7.
Consider the demand function

$$x(p) = 100e^{-0.04p}, \quad 0 \le p \le 100, \quad (x: \text{ demand}; \ p: \text{ price}).$$

Given the cost function

$$C(x) = 2000 + 30x$$

1. describe the revenue and the profit as functions of price.
2. determine the price for which the firm's profit will be maximal.

8.
The total costs of a firm are given by the function

$$C(x) = 30x + \frac{1450x^2}{400 + x^2}, \qquad 10 \le x \le 50.$$

1. Determine the marginal cost function. Calculate and interpret its value at $x = 20$.
2. Find the absolute minimum of the average cost function...
3. Investigate the monotonocity of the average costs function.

9.
Given the demand function

$$p(x) = 16 - 0.5x,$$

find and interpret

1. the elasticity of price with regards to a demand of $x_0 = 8$ units,
2. the elasticity of demand with regards to the corresponding price.

10.
The total cost function of a firm for a certain commodity is given by

$$C(x) = \frac{1}{240}x^3 - \frac{3}{8}x^2 + \frac{40}{3}x + 100$$

x: output,
$C(x)$: cost of producing x output units.

1. Determine the total cost of producing 40, 50, 60 and 70 units of output.
2. Sketch the total cost function.
3. Write down the revenue function, assuming that the firm charges 20 € for each unit of the commodity.
4. Draw the revenue function in the same sketch as the total cost function.
5. Which amount should be at least sold if no losses are to be incurred?
6. Find the firm's maximizing output as well as its maximum profit.
7. How much must be produced, if the firm's *variable average costs* are to be minimal?
8. The firm intends to increase its output from 70 to 71 units. Determine the increase in costs

   i) approximately,
   ii) exactly

9. Find the percentage increase of costs

   i) approximately,
   ii) exactly,

   if there is to be a one percent increase of output at the level of 48 units.
   How would you comment the results?

11.
Consider the following demand function:

$$x(p) = \frac{500}{p+5} - 10, \quad 0 \le p < 45 \qquad (x\text{: demand; } p\text{: price})$$

1. Find the marginal demand function.
2. Give a function describing approximately the percentage change of demand with regards to a percentage change of the price.
3. Calculate the elasticity of demand for prices for $x = 10$ and $x = 40$.
4. Determine the quantity of demand for which the price elasticity of demand is equal to –3.
5. Find the domains in which the demand function is elastic or inelastic.

12.
The following function gives the relation between the price of a commodity and the demand for it.

$$p(x) = 100 - 0.5x.$$

The total cost of producing the commodity is given by the function

$$C(x) = 1000 + 10x$$

Find

1. the revenue function.
2. the profit function
3. the output guaranteeing maximal profit.

13.
The total cost function of a firm is given by

$$C(x) = 1000 + 20x$$

1. Determine the marginal cost function and interpret it at the point x_0.
2. Write the average cost function.
3. Investigate the behaviour of the total and the average cost functions for $x \to +\infty$.

14.
The total cost of a firm is given by the function

$$C(x) = 60 - 12x + 2x^2.$$

1. Determine the output which minimises the total costs.
2. Find the output for which the average costs will be minimised.
3. Argue why the marginal costs function can not have a minimum.
4. Find the point of intersection of the marginal and the average costs curves.
5. Interpret the values of the marginal and average costs function at an arbitrary point x_0.

15.
The production function of a firm is given by

$$y = \sqrt{10r}\,, \qquad (r : \text{input}; y : \text{output})$$

1. Is the marginal production function increasing or decreasing?
2. Find the total cost function, assuming that each unit of input costs 2 €.

16.
The production function of a firm is given by

$$x(r) = -\frac{1}{3}r^3 + 2r^2 .$$

1. Find the input for which the production will be at its maximum level.
2. In which interval(s) will the production be „negative“?
3. Give the average production function.
4. Determine the amount of input which maximises the production per unit of input.
5. Interpret the first derivative of the production function.

17.
The costs C of producing a certain product are given by the function

$$C(x) = 0.06x^3 - 2x^2 + 60x + 200 .$$

C : total costs ,
x : output .

1. At present, we have the production level $x = 10$. Calculate the *approximate* change of costs if the production will be

   i) increased 2 units,
   ii) decreased by 1 unit.

2. Find the *exact* change in costs.

18.
A firm that produces some amount of output x has the following cost function:

$$C(x) = \begin{cases} (x-4)^2 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \end{cases} .$$

1. What is the right-hand limit of this function as output approaches zero?
2. What is the function actually equal to when output is zero?
3. Is the function continuous? If not, explain why not.

19.
The total cost of a firm is given by the function

$$C(x) = 200 + 5x - 0.01x^2 + 0.01x^3, \quad (x\text{: output; } C\text{: costs}).$$

1. Find the marginal cost function and interpret it for an output of 5 units.
2. Given the price function

$$p(x) = 50 - 0.01x,$$

   determine the profit function of the firm.

3. Find the output for which the firm's profit will be maximised. How high will be this profit?
4. Producing at a level of 40 units, the firm increases its output by one percent. Find the resulting

   i) approximate
   ii) exact

   percentage change of profit.

20.
The total cost of a firm is given by the function

$$C(x) = \frac{1}{12}x^3 - \frac{3}{4}x^2 + \frac{13}{4}x, \quad x \in [1,\ 8], \qquad (x\text{: output; } C\text{: costs}).$$

1. Find the marginal cost function and interpret it for an output of 5 units.
2. Given the price function

$$p(x) = 6 - \frac{x}{2}, \quad x \in [1,\ 8] \qquad (p\text{: price}),$$

   determine the profit function of the firm.
   Find the output for which the firm's profit will be maximised. How high will be this profit?

3. Producing at a level of 3 units, the firm increases its output by one percent. Find the resulting

   i) approximate
   ii) exact

   percentage change of profit.

21.
The amount of output (Q) that a company can produce when it uses some amount of labour ($L \geq 0$) is described by the function

$$Q(L) = 3\sqrt{L}.$$

1. Sketch this function.
2. Based upon your sketch, is this function concave, strictly concave, convex or strictly convex?

22.
A company is manufacturing and selling insulated mugs. The company has monthly fixed costs of 1500 € and there is a total monthly cost of 1800 € when producing 100 mugs. Each mug sells for 7 €,

1. Find the cost, revenue, and profit functions for the mug manufacturer, assuming each is a linear function.
2. How many mugs must the company make and sell in order to break even?

23.
A monopolist's demand function is given as

$$p = 15 - q \quad (p: \text{ price}; q: \text{ demand}).$$

The firm's cost function is

$$C(q) = q^2 + 3q + 2\,.$$

Find the level of output and the price level that maximises profit.

24.
The demand function for a commodity is given as

$$p = 10 - 0.5q \;\;,\;\; q \in [0,\; 20] \quad (p: \text{ price}; q: \text{ demand}).$$

1. Express the price elasticity of demand as a function of price
2. Determine the intervals in which the demand as a function of price is elastic or inelastic.

25.
A monopoly's demand function is given as

$$p = 15 - 6q$$

where q is output, p is price, and its total cost function is given by

$$C(q) = 2q^3 - 3q^2 + 3q + 2\,.$$

Find the firm's maximising level of output, the price it will charge and the amount of profit it will make.

26.
A firm has the following production function:

$$Q(L) = 3L^2 - 0.1L^3 \quad (Q: \text{ Production}; L: \text{ Labour})$$

1. Find the firm's marginal product of labour.
2. Find the values of Q for which the marginal product of labour and the average product of labour are maximised.
3. Show that, when average product of labour is at a maximum, marginal product of labour

equals average product of labour.

27.

The average cost function of a firm is

$$c(x) = 12 - 4x + x^2 \qquad (x: \text{ level of output }).$$

1. Derive the total and marginal cost functions.
2. Sketch the average and marginal cost curves in the same diagram.
3. If the firm takes the prices as given and price is 16.00 €, what quantity will it sell to maximise profit? What are the firm's profits at this output?

28.

A monopolist's cost function is

$$C(x) = \frac{x}{2500} \cdot (x - 100)^2 + x \qquad (x: \text{ level of output }).$$

It faces the demand function

$$p(x) = 4 - \frac{x}{25}.$$

1. The monopolist would like to maximise his profit. Find its output, the associated price, and its profit.
2. Sketch the marginal cost and the marginal revenue functions in the same diagram.

29.

Find the equilibrium price and quantity, given the demand function

$$p + q^2 + 3q - 20 = 0$$

and the supply function

$$p - 3q^2 + 10q = 5 \qquad (p: \text{ price; } q: \text{ quantity }).$$

30.

Plot the demand function:

$$x(p) = \frac{1}{e^{p-2}} - 2$$

Investigate and interpret its monotonicity.

31.

The demand function for a commodity is given as

$$p(x) = 10 - 0.5x \quad (p: \text{ price; } x: \text{ demand })$$

1. Express the elasticity of demand with regards to price as a function of price.
2. Determine the values of p and x for which the elasticity of demand with regards to price is elastic, inelastic and unitary.

32.
A manufacturer estimates his total costs to be

$$C(x) = 128 + 60x + 8x^2.$$

His product can be sold at a price of

$$p(x) = 180 - 2x$$

1. Determine the level of production that results in the maximum profit. What is the maximum profit?
2. At what level of production are the average costs minimal? What are the minimal average costs?
3. Find the values of output for which the costs increase degressively.

(Last updated: 17.07.2022)