

# Functions

## Exercises (Solutions)

1.

1)

$$f(1) = 1^2 + 6 \cdot 1 - 4 = 3$$

2)

$$f(3) = 3^2 + 6 \cdot 3 - 4 = 23.$$

2.

$$f(x) = x^2 + 1$$

1)

$$f(4) = 4^2 + 1 = 17$$

2)

$$f(\sqrt{2}) = (\sqrt{2})^2 + 1 = 3$$

3)

$$f(a+1) = (a+1)^2 + 1 = a^2 + 2a + 2$$

4)

$$f(a) + 1 = a^2 + 1 + 1 = a^2 + 2$$

5)

$$f(a^2) = a^2 + 1$$

6)

$$[f(a)]^2 = (a^2 + 1)^2 = a^4 + 2a^2 + 1$$

7)

$$f(2a) = (2a)^2 + 1 = 4a^2 + 1.$$

3.

1.

$$\varphi\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{3 \cdot \frac{1}{x} + 5} = \frac{\frac{1-x}{x}}{\frac{3+5x}{x}} = \frac{1-x}{3+5x}, \quad x \neq 0, -\frac{5}{3}$$

2.

$$\frac{1}{\varphi(x)} = \frac{1}{\frac{x-1}{3x+5}} = \frac{3x+5}{x-1}$$

4.

1.

$$\psi(2x) = \sqrt{(2x)^2 + 4} = \sqrt{4x^2 + 4} = 2\sqrt{x^2 + 4}.$$

2.

$$\psi(0) = 2.$$

5.

$$f(2\theta) = \tan(2\theta) = \frac{2 \cdot \tan \theta}{1 - \tan^2 \theta} = \frac{2f(\theta)}{1 - [f(\theta)]^2}.$$

6.

$$\varphi(a) = \log \frac{1-a}{1+a}; \quad \varphi(b) = \log \frac{1-b}{1+b}$$

$$\varphi(a) + \varphi(b) = \log \frac{1-a}{1+a} + \log \frac{1-b}{1+b} = \log \left( \frac{1-a}{1+a} \cdot \frac{1-b}{1+b} \right) = \log \frac{1+ab-a-b}{1+ab+a+b}$$

$$\varphi \left( \frac{a+b}{1+ab} \right) = \log \frac{1 - \left( \frac{a+b}{1+ab} \right)}{1 + \left( \frac{a+b}{1+ab} \right)} = \log \frac{\frac{1+ab-a-b}{1+ab}}{\frac{1+ab+a+b}{1+ab}} = \log \frac{1+ab-a-b}{1+ab+a+b}$$

7.

$$a_1 = a_2 = a, \quad a \geq 3$$

*(To be continued.)*