Chapter III

Testing Hypotheses

Solutions

Part I:

1.

1. Parameter: the average life span [in km] of this manufacturer's tires.

2. $H_0: \ \mu = 50000, \quad H_1: \mu < 50000.$
   (Note, you can also write:

   $H_0: \ \mu \geq 50000, \quad H_1: \mu < 50000.$

3. Type I error: rejecting the manufacturer's claim when in fact it is true:
   Type II error: not rejecting the manufacturer's claim when in fact it is false.

4. A ten percent significance level means that we are setting an upper limit of 10% for the probability of making type I error.

2.

$$n = 15, \quad \bar{x} = 8800, \quad \sigma = 500, \quad \alpha = 0.05$$

1.

$$H_0: \ \mu \geq 9000; \quad \mu < 9000$$

2.
Normal distribution.

3.

$$z_{crit} = -1.645$$

4.

$$z_{stat} = \frac{8800 - 9000}{\frac{500}{\sqrt{15}}} = -1.549193338 \approx -1.55.$$

5.

$-1.55 > -1.645.$

$\therefore H_0$ cannot be rejected.

3.

$$n = 40, \quad \bar{x} = 1950, \quad \sigma = 500, \quad \alpha = 0.05$$

1.
$H_0: \ \mu \le 1800; \quad \mu > 1800$

2.
Normal distribution.

3.
$z_{crit} = 1.645$

4.
$$z_{stat} = \frac{1950 - 1800}{\frac{500}{\sqrt{40}}} = 1.897366596 \approx 1.897.$$

5.
$1.897 > 1.645.$

$\therefore$ We reject H_0.

4.

$$n = 15, \quad \bar{x} = 44000, \quad \sigma = 2000, \quad \alpha = 0.05$$

1.
$H_0: \ \mu = 45000; \quad \mu \ne 45000$

2.
Normal distribution.

3.
$z_{crit} = 1.96$

4.

$$z_{stat} = \frac{44000 - 45000}{\frac{2000}{\sqrt{15}}} = -1.936491673 \approx -1.94.$$

5.

$-1.94 > -1.96.$

$\therefore$ The community representative's claim cannot be rejected.

5.

$n = 6, \quad \bar{x} = 493, \quad s = 4, \quad \alpha = 0.05$

1.

$H_0: \ \mu \geq 500; \quad \mu < 500$

2.
Use the t-distribution.

3.

$t_{crit} = -2.01$

4.

$$t_{stat} = \frac{493 - 500}{\frac{4}{\sqrt{6}}} = -4.28660705 \approx -4.29.$$

5.

$-4.29 > -2.01.$

$\therefore$ We reject H_0.

6.

$$\bar{x} = \frac{62+92+75+68+83+95}{6} = \frac{475}{6} \approx 79.17$$

$$s = \sqrt{\frac{(62-79.17)^2 + (92-79.17)^2 + \left(75-79.17\right)^2 + \left(68-79.17\right)^2 + \left(83-79.17\right)^2 + \left(95-79.17\right)^2}{5}}$$

≈ 13.17

$$n = 6, \quad \bar{x} = 79.17, \quad s = 13.17, \quad \alpha = 0.05.$$

1.

$H_0: \ \mu < 70; \quad H_1: \ \mu \geq 70.$

2.

Use the $t-$distribution.

3.

$t_{0.05,5} = 2.01$

4.

$$t = \frac{79.17-70}{\frac{13.17}{\sqrt{6}}} = 1.705529305 \approx 1.71$$

5.

$1.71 < 2.01.$

$\therefore$ We do not reject H_0.

7.

$\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = 0.04798$ **;** $\qquad s^2 = \frac{1}{9}\sum_{i=1}^{10}(x_i - 0.04798)^2 = 0.000233693$

$$n = 10, \quad \bar{x} = 0.047798, \quad s = 0.015287017 \approx 0.0153, \quad \alpha = 0.05.$$

1.

$H_0: \ \mu = 0.05; \quad H_1: \ \mu \neq 0.05.$

2.
Use the $t-$distribution.

3.

$$t_{crit} = \pm 2.26$$

4.

$$t_{stat} = \frac{0.0478 - 0.05}{\frac{0.0153}{\sqrt{10}}} = -0.4547065917 \approx -0.4547$$

5.

$\therefore$ Calculated value of t is not in the critical region. At the 5% level of significance, we do not reject H_0. There is no evidence to suggest that the machinery is not functioning properly.

8.

1.

$$H_0: \mu = 50; \quad H_1: \mu > 50.$$

2. Use the normal distribution.

3.

$$z = \frac{52 - 50}{\frac{8}{\sqrt{36}}} = 1.50,$$

$$p - value = 1 - 0.933198 = 0.066802.$$

4.

$$0.066802 > 0.05.$$

Therefore, the null hypothesis will be accepted. This means that at the 0.05 level of significance there does not seem to be sufficient evidence that the mean amount of cash people have on them is significantly higher than 50 €.

9.

1.

$H_0: \mu \le 15; \quad H_1: \mu > 15.$.

2. Use the normal distribution.

3.

$$z = \frac{17-15}{\frac{0.5}{\sqrt{10}}} \approx 12.65,$$

$$p-value \approx 1-1=0.$$

4.

$0 < 0.05.$

Therefore, the evidence is strongly against the null hypothesis (the average height is at most 15 cm.). There is sufficient evidence that the true average height for the population of the baker's loaves of bread is greater than 15 cm.

10.

$$\bar{x} = \frac{1}{12}\sum_{i=1}^{12} x_i = \frac{29.5}{12} = 2.4583;$$

$$s = \sqrt{\frac{\sum_{i=1}^{12} x_i^2 - \frac{\left(\sum_{i=1}^{12} x_i\right)^2}{12}}{11}} = \sqrt{\frac{122.611 - \frac{(29.5)^2}{12}}{11}} = 2.1339$$

$$n = 12, \quad \bar{x} = 2.4583, \quad s = 2.1339, \quad \alpha = 0.05.$$

1.

$H_0: \mu = 0; \quad H_1: \mu > 0.$.

2.

Use the $t-$distribution.

3.

$$t = \frac{2.4583 - 0}{\frac{2.1339}{\sqrt{12}}} = 3.990721684 \approx 3.99,$$

$$p - value = P(t \geq 3.99) = 0.00106.$$

4.

$$0.00106 < 0.05.$$

$\therefore$ We reject H_0.

Therefore, there exists enough evidence that there is an average weight loss under this diet programme.

11.

$$n = 1520, \quad p = \frac{58}{1520} = 0.038158, \quad \alpha = 0.01.$$

1.

$$H_0:\ P \geq 0.058; \quad H_1:\ P < 0.058.$$

2.

$$n \cdot P = 1520 \cdot 0.058 > 5; \quad n \cdot (1 - P) = 1520 \cdot (1 - 0.058) > 5$$

Therefore, we use the normal distribution.

3.

$$z_{-0.01} = -2.58$$

4.

$$z = \frac{p - P}{\sqrt{\frac{P \cdot (1 - P)}{n}}} = \frac{0.038158 - 0.058}{\sqrt{\frac{0.058 \cdot (1 - 0.058)}{1520}}} = -3.309541194 \approx -3.31$$

5.

$$-3.31 < -2.58.$$

$\therefore$ We reject H_0.

12.

$$n = 1000, \quad p = 0.056, \quad \alpha = 0.05.$$

1.

$$H_0: \ P = 0.06; \quad H_1: \ P < 0.06.$$

2.

$$n \cdot P = 1000 \cdot 0.06 > 5; \quad n \cdot (1-P) = 1000 \cdot 0.94 > 5$$

Therefore, we use the normal distribution.

3.

$$z_{-0.05} = -1.645$$

4.

$$z = \frac{p-P}{\sqrt{\frac{P \cdot (1-P)}{n}}} = \frac{0.056-0.06}{\sqrt{\frac{0.06 \cdot 0.94}{1000}}} = -0.532623641 \approx -0.53$$

5.

$$-0.53 > -1.645.$$

$\therefore$ We fail to reject H_0.

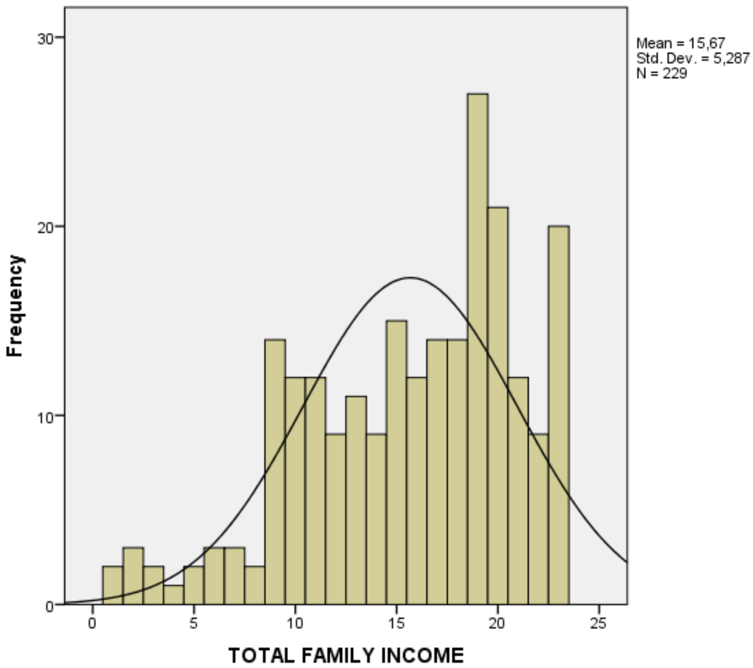
Part II: *SPSS*

1.

1**.**
Total family income [*income98*] is quantitative (ordinal treated as quantitative). It satisfies the level of measurement requirement for a one-sample t-test of a population mean.

2.

- ***Graphs -> Legacy Dialogs -> Histogram…***
- Move the variable *income98* to the ***Variable*** box.
  Check ***Display normal curve***
  OK.



The distribution does not seem to be normal.

The following should be tested:

H_0 : *income98* is normally distributed H_1 : *income98* is not normally distributed

- ***Analyze -> Descriptive Statistics -> Explore***
  Move the variable *data* to the ***Dependent List*** Box.
- Click ***Plots*** on the right. A new window pops out. Check ***None*** for ***Boxplot***, uncheck everything for ***Descriptive*** and make sure the box ***Normality plots with tests*** is checked.

  Continue
- ***OK.***

Output:

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
TOTAL FAMILY INCOME	229	84,8%	41	15,2%	270	100,0%

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
TOTAL FAMILY INCOME	,124	229	,000	,945	229	,000

a. Lilliefors Significance Correction

The test statistics are shown in the second table. Here two tests for normality are run. For datasets smaller than 2000 elements, we use the Shapiro-Wilk test, otherwise, the Kolmogorov-Smirnov test. In our case, since we have only 229 elements, the Shapiro-Wilk test is used. We have:

$$p-value = 0.000 < 0.05 = \alpha .$$

Therefore, we reject the null hypothesis. This means that the variable *data* is not normally distributed.

3.
To justify the use of probabilities based on a normal sampling distribution in testing hypotheses, either the distribution of the variable must satisfy the nearly normal condition or the size of the sample must be sufficiently large to generate a normal sampling distribution under the Central Limit Theorem.
A one-sample t-test of a population mean requires that the distribution of the variable satisfy the nearly normal condition, which we will operationally define as having skewness and kurtosis between -1.0 and +1.0, and having no outliers with standard scores equal to or smaller than -3.0 or equal to or larger than +3.0.
To evaluate the variables conformity to the nearly normal condition, we will use descriptive statistics and standard scores.

- ***Analyse -> Descriptive Statistics -> Descriptives***

- Move the variable for the analysis *income98* to the ***Variable(s)*** list box.

  Click on the ***Options*** button to select optional statistics.

  (The check boxes for *Mean* and *Std. Deviation* are already marked by default.)

  Mark the ***Kurtosis*** and ***Skewness*** check boxes. This will provide the statistics for assessing normality.

  Continue

- Mark the check box

  Save standardized values as variables.

  OK.

Output:

Descriptive Statistics

	N	Mean	Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
TOTAL FAMILY INCOME	229	15,67*	5,287	-,628	,161	-,248	,320
Valid N (listwise)	229						

*More exactly: 15.668122

The skewness of the distribution (-0.628) is between -1.0 and +1.0. The kurtosis of the distribution (-0.248) is between -1.0 and +1.0.

- Sort the column *Zincome98* in ascending order to show any negative outliers at the top of the column:

  There are no outliers that have a standard score less than or equal to -3.0.

- Sort the column *Zincome98* in descending order to show any positive outliers at the top of the column:

  There are no outliers that have a standard score greater than or equal to +3.0.

Therefore, the total family income [*income98*] satisfies the criteria for a nearly normal distribution.

4.
Though we have satisfied the nearly normal condition and do not need to utilize the Central Limit Theorem to justify the use of probabilities based on the normal distribution, we will still examine the sample size.
To apply the Central Limit Theorem for a one-sample t-test of a population mean requires that the sample have 30 or more cases.
The number of valid cases available for the test was 229, larger than requirement of 30 cases to apply the Central Limit Theorem.

5.

- ***Analyse -> Compare Means -> One-Sample T Test…***
- Move the variable *income98* to the ***Test Variable(s)*** list box.

  Type the value of the population mean that we are testing against, i.e. the mean based on previous research 16.5 as ***Test Value*** (See the ***Descriptives*** table above).

  OK

Output and Interpretation

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
TOTAL FAMILY INCOME	229	15,67	5,287	,349

One-Sample Test

	Test Value = 16.5					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
TOTAL FAMILY INCOME	-2,381	228	,018	-,832	-1,52	-,14

The value for the sample mean is 15.67. The standard error of the sampling distribution is identified as 0.349.

6.

$$H_0 : \mu = 16.5, \qquad H_1 : \mu \neq 16.5$$

7.

$$t_{stat} = \frac{15.668122 - 16.5}{\frac{5.287}{\sqrt{229}}} = -2.381047557$$

8.
The *p-value* is equal to 0.018 (See the last table above.). It is the probability that a sample with the mean of 15.7 can be drawn from a population with a mean of 16.5 and a standard error of 0.349

9.
The null hypothesis will be rejected, because:

$$p - value = 0.018 \leq 0.05 = \alpha$$

(And because: $t_{stat} = -2.381047557 < -1.9704 = t_{crit}$).

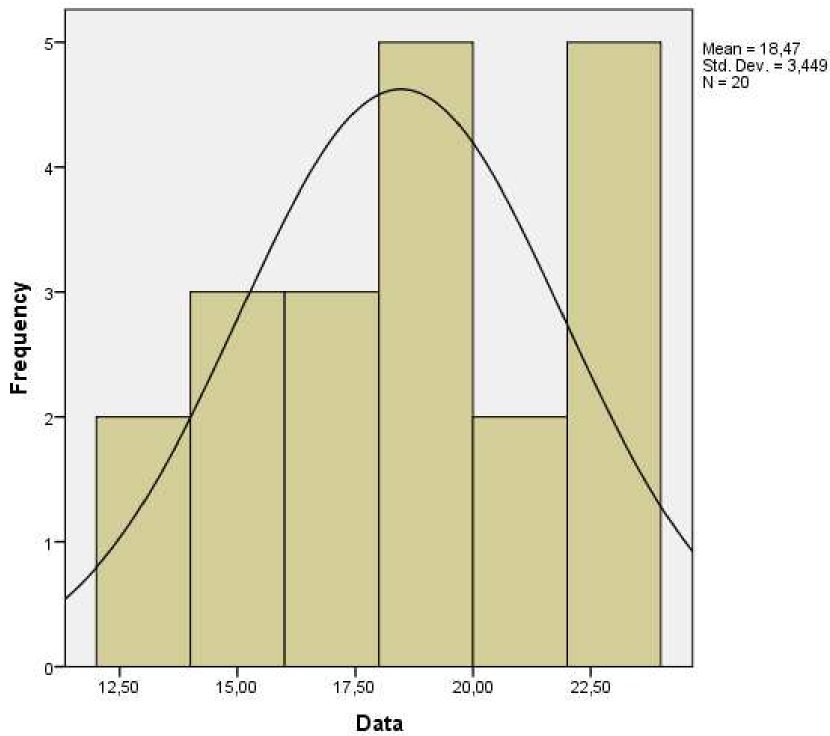
2.

Solution:

1.

- ***Graphs -> Legacy Dialogs -> Histogram…***
- Move the variable *date* to the ***Variable*** box.
  Check ***Display normal curve***
  OK.

Output and Interpretation:



2.
The following should be tested:

H_0 : *data* is normally distributed H_1 : *data* is not normally distributed

- ***Analyze -> Descriptive Statistics -> Explore***
  Move the variable *data* to the ***Dependent List*** Box.
- Click ***Plots*** on the right. A new window pops out. Check ***None*** for ***Boxplot***, uncheck everything for ***Descriptive*** and make sure the box ***Normality plots with tests*** is checked.

  Continue
- ***OK.***

Descriptives

				Statistic	Std. Error
Data	Mean			18,4719	,77126
	95% Confidence Interval for Mean	Lower Bound		16,8577	
		Upper Bound		20,0862	
	5% Trimmed Mean			18,4986	
	Median			18,3962	
	Variance			11,897	
	Std. Deviation			3,44918	
	Minimum			12,66	
	Maximum			23,80	
	Range			11,14	
	Interquartile Range			6,80	
	Skewness			,070	,512
	Kurtosis			-1,060	,992

- The average is equal to 18.4719

$$\mu \in [16.8577, \quad 20.0862]$$

- Skewness is rather small = 0.070.

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Data	,137	20	,200*	,946	20	,316

*. This is a lower bound of the true significance.

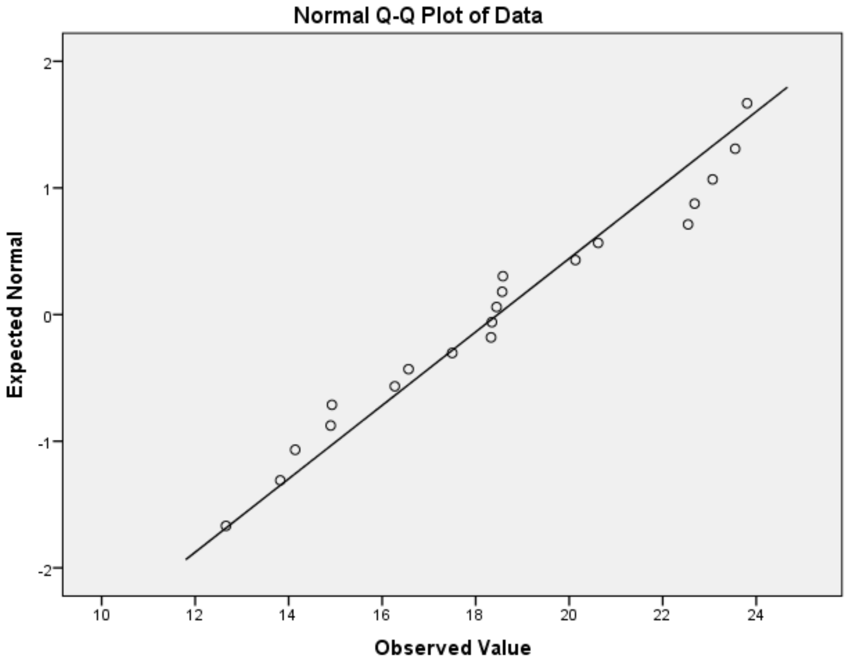
a. Lilliefors Significance Correction

The test statistics are shown in the third table. Here two tests for normality are run. For datasets smaller than 2000 elements, we use the Shapiro-Wilk test, otherwise, the Kolmogorov-Smirnov test. In our case, since we have only 20 elements, the Shapiro-Wilk test is used. We have:

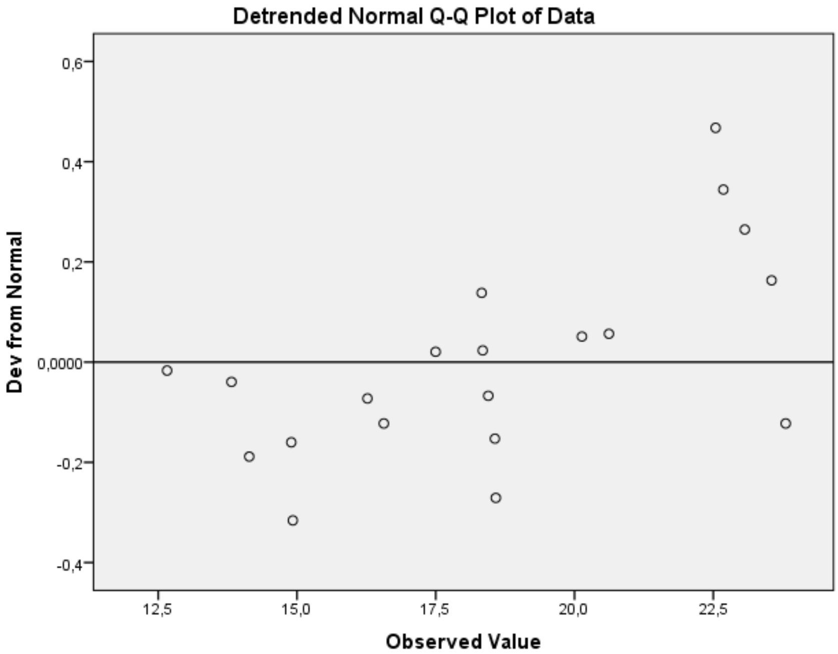
$$p-value = 0.316 > 0.05 = \alpha .$$

Therefore, do not reject the null hypothesis. This means that the variable *data* is normally distributed.

This will also be confirmed by the following Q-Q Plot and the Detrended Normal O-Q Plot:



In order to determine normality graphically, we can use the output of a normal Q-Q Plot. If the data are normally distributed, the data points will be close to the diagonal line. If the data points stray from the line in an obvious non-linear fashion, the data are not normally distributed. As we can see from the normal Q-Q plot above, the data is normally distributed.



The Detrended Normal Q-Q Plot shows the deviations from the normal distribution in detail

(Last revised: 18.01.2020)