Chapter II

Estimation Theory

Solutions

1.

The population consists of all public school teachers, $\mu = \$32000$, $\bar{x} = \$31895$. The parameter is μ. A point estimate of the parameter μ is \$31895.

2.

The population consists of all homes in the city. $P = 98\%$, $\bar{p} = 96.25\%$. The parameter is P. A point estimate of is 96.25%.

3.

1.

$$\mu = 24.5.$$

2.

$$n = 20, \quad \sigma = 3.1, \quad \bar{x} = 24.5, \quad \alpha = 0.01.$$

$$\mu \in \left[24.5 - 2.58 \cdot \frac{3.1}{\sqrt{20}},\ 24.5 + 2.58 \cdot \frac{3.1}{\sqrt{20}} \right] = [22.71,\ 26.29]$$

4.

$$n = 1500, \quad \bar{x} = 269720, \quad \sigma = 68650, \quad \alpha = 0.01.$$

$$\mu \in \left[269720 - 2.575 \cdot \frac{68650}{\sqrt{1500}},\ 269720 + 2.575 \cdot \frac{68650}{\sqrt{1500}} \right] \approx [265155.72,\ 274284.28].$$

5.

$$n = 100, \quad \bar{x} = 5.46, \quad \sigma = 2.47.$$

1.

$$\mu \in \left[5.46 - 1.96 \cdot \frac{2.47}{\sqrt{100}},\ 5.46 + 1.96 \cdot \frac{2.47}{\sqrt{100}} \right] \approx [4.98,\ 5.94].$$

$$\mu \in \left[5.46 - 2.575 \cdot \frac{2.47}{\sqrt{100}},\ 5.46 + 2.575 \cdot \frac{2.47}{\sqrt{100}} \right] \approx [4.82,\ 6.10].$$

2.
Yes, the entire interval is below 6.

3.
No, the interval contains 6.

4.
95% convinced, but not 99% convinced.

6.

$$\bar{x} = 5.46,\ s = 2.47,\quad s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.47}{\sqrt{100}} = 0.247,\quad df = 99,\quad t_{99,0.05} = 1.984$$

$$\mu \in \left[5.46 - 1.984 \cdot \frac{2.47}{\sqrt{100}},\ 5.46 + 1.984 \cdot \frac{2.47}{\sqrt{100}} \right] \approx [4.97,\ 5.95].$$

Yes.

7.

We have

$$\bar{x} = 24.57\text{ €},\ s = 6.60\text{ €},\quad s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{6.60}{\sqrt{100}} = 0.66.$$

1.

$$\mu \in [24.57 - 1.96 \cdot 0.66,\ 24.57 + 1.96 \cdot 0.66\ ] = [23.28,\ 25.86].$$

2.
The confidence interval for the Euro amount of purchases is simply the total number of customers in the population multiplied by the confidence limits fort he mean purchase amount per customer:

$$[23.28 \cdot 4000,\ 25.86 \cdot 4000] = [93120,\ 103440].$$

8.

$$n = \left(\frac{z \cdot \sigma}{E} \right)^2$$

$$= \left(\frac{2.58 \cdot 3.20}{1.00} \right)^2 \cong 69\,.$$

9.

$$n = \left(\frac{z \cdot \sigma}{E} \right)^2 = \left(\frac{2.58 \cdot 3.20}{2.00} \right)^2 \cong 18.$$

However, because the population is not assumed to be normally distributed, the minimum sample size is $n = 30$, so that the central limit theorem can be invoked as the basis for using the normal probability distribution for constructing the confidence interval.

10.

1.

$$p = \frac{75}{119} = 0.6302521 \approx 0.63$$

2.

$$z \cdot \sqrt{\frac{p(1-p)}{n}} = 1.96 \cdot \sqrt{\frac{0.63 \cdot (1-0.63)}{119}} = 0.086746852.$$

3.

$$P \in [0.63 - 0.086746852,\ 0.63 + 0.086746852\ ] = [0.543253147,\ 0.716746852].$$

11.

$$P \in \left[0.40 - 2.58 \cdot \sqrt{\frac{0.40 \cdot 0.60}{1600}},\ 0.40 + 2.58 \cdot \sqrt{\frac{0.40 \cdot 0.60}{1600}}\ \right] =$$

$$= [0.368401582,\ 0.431598417] \approx [0.37,\ 0.43].$$

12.

$$n = \frac{z^2 \cdot p \cdot q}{E^2} = \frac{1.96^2 \cdot 0.50 \cdot 0.50}{0.02^2} = 2401.$$

SPSS Problems:

1.

- ***Analyze -> Compare Means -> One- Sample T Test…***

- Move the variable *exp_total* to the ***Test Variable(s)*** box.

  Type 0 for ***Test Value***.
  Click on ***Options*** to confidence level. (The original setting is 95%).

  Continue

- ***OK.***

Output:

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Total expenditure (HUF/month)	222	99692,7162	57045,34814	3828,63308

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Total expenditure (HUF/month)	26,039	221	,000	99692,71622	92147,4136	107238,0189

The average total expenditure of households is equal to 96692.7162. It lies between 92 147.4136 and 107 238.0189 at 95% confidence level.

(Last updated: 03.12.19)