Inductive Statistics

Testing Hypotheses

- A statistical hypothesis is an assumption about a population parameter, based upon the investigation of a sample. This assumption may or may not be true.

- This chapter discusses how to make such tests of hypotheses about the population mean, μ, and the population proportion, P.

- We distinguish two hypotheses:

Null hypothesis:

A *null hypothesis* is a statement about a population parameter that is assumed to be true until it is declared false.
The null hypothesis will be denoted by H_0.

Alternative hypothesis

An *alternative hypothesis* is a statement about the population parameter that will be true if the null hypothesis is false.
The alternative hypothesis will be denoted by H_1.

Types of Error:

- **Type I Error**:

  A *type I error* occurs when a true null hypothesis is rejected. The value of α represents the probability of committing this type of error, that is,

  $$\alpha = P\left(H_0 \text{ is rejected/ } H_0 \text{ is true}\right)$$

  The value of α represents *significance level* of the test.

- **Type II Error**

  A *type II error* occurs when a false null hypothesis is not rejected. The value of β represents the probability of committing this type of error, that is,

  $$\beta = P\left(H_0 \text{ is not rejected/ } H_0 \text{ is false}\right)$$

  The value of $1-\beta$ is called *power of the test*. It represents the probability of not making a type II error.

Summary of Error Types

		Actual Situation	
		H_0 is true	H_0 is false
Decision	Do not reject H_0	Correct decision	Type II or β error
	Reject H_0	Type I or α error	Correct decision

Summary of relation between the signs in H_0 and H_1:

	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in the null hypothesis H_0	$=$	$=$ or $\geq$	$=$ or $\leq$
Sign in the alternative hypothesis H_1	$\neq$	$<$	$>$
Rejection region	In both tails	In the left tail	In the right tail

Test Procedures

We have the following two procedures to make tests of hypothesis:

1. ***The $p-$value approach***
   Under this procedure, we calculate what is called the $p-$value for the observed value of the sample statistic. If we have a predetermined significance level, then we compare the $p-$value with this significance level and make a decision (p stands for probability.)

2. ***The critical-value approach***
   In this approach, we find the critical value(s) from a table (such as the normal distribution or the t distribution table) and find the value of the test statistic for the observed value of the sample statistic. Then we compare these two values and make a decision.

Steps to Perform a Test of Hypothesis with the p-Value Approach:

1. Select the null and alternative hypothesis.

2. Select

   i. *The normal distribution if* σ *is known*

   with $z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}$ where $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

   ii. *t distribution if* σ *is unknown*

   with $t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}}$ where $s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$.

3. Calculate the *p-value*.

4. Reject the null hypothesis if $p - value < \alpha$. Do not reject the null hypothesis if $p - value \geq \alpha$

Steps to Perform a Test of Hypothesis with the Critical-Value Approach:

1. State the null and alternative hypothesis.
2. Select
   i. *The normal distribution if* σ *is known*
   ii. *t distribution if* σ *is unknown*

3. Determine the rejection and non-rejection regions by calculating by calculating

   z_{crit} if σ is known and t_{crit} if σ is unknown

4. Calculate the value of the test statistic.

$$z_{stat} = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{where} \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}, \quad \text{if } \sigma \text{ is known}$$

$$t_{stat} = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{where} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}, \quad \text{if } \sigma \text{ is unknown}$$

5. Compare z_{stat} with z_{crit} or t_{stat} with t_{crit} and make a decision.

A consumer advocacy group suspects that a local supermarket's 10-ounce packet of cheddar cheese actually weighs less than 10 ounces. The group took a random sample of 30 such packages and found the mean weight for the sample was 9.965 ounces. The population follows a normal distribution with the population standard deviation of 0.15 ounces.
Test the consumer advocacy group's suspicion at the significance level 0.01.

Solution:

$$n = 30, \quad \bar{x} = 9.965, \quad \sigma = 0.15, \quad \alpha = 0.01.$$

1.

$$H_0: \ \mu = 10; \quad H_1: \ \mu < 10. \quad .$$

2. Use the normal distribution.

3.

$$z_{critical} = -2.326$$

4.

$$z_{stat} = \frac{9.965 - 10}{\frac{0.15}{\sqrt{30}}} \approx -1.28$$

5.

$$-1.28 > -2.326.$$

$\therefore$ We do not reject H_0.

The manufacturer of a certain brand of car batteries claims that mean life of these batteries is 45 months. A consumer protection agency that wants to check this claim took a random sample of 24 such batteries and found that the mean life for this sample is 43.05 months. The lives of all such batteries have a normal distribution with the population standard deviation of 4.5 months.
Test the manufacturer's claim at the 0.025 degree of significance.

Solution:

$$n = 24, \quad \bar{x} = 43.05, \quad \sigma = 4.5, \quad \alpha = 0.025.$$

1.

$$H_0: \ \mu = 45; \quad H_1: \ \mu < 45. \quad .$$

2. Use the normal distribution.

3.

$$z_{critical} = -1.96$$

4.

$$z_{stat} = \frac{43.05 - 45}{\frac{4.5}{\sqrt{24}}} \approx -2.12$$

5. $-2.12 < -1.96$

$\therefore$ We reject H_0.

In the past, the mean running time for a certain type of flashlight battery has been 8.5 hours. The manufacturer has introduced a change in the production method which he hopes has increased the mean running time. The mean running time for a random sample of 40 light bulbs was 8.7 hours.
Do the data provide sufficient evidence to conclude that the mean running time of all light bulbs has increased from the previous mean of 8.5 hours?
Perform the appropriate hypothesis test using a significance level of 0.05. Assume that the standard deviation of the running time of all light bulbs is 0.5 hours.

Solution:

$$n = 40, \quad \bar{x} = 8.7, \quad \sigma = 0.5, \quad \alpha = 0.05.$$

1.

$$H_0: \ \mu = 8.5; \quad H_1: \ \mu > 8.5.$$

2. Use the normal distribution.

3.

$$z_{critical} = 1.645$$

4.

$$z_{stat} = \frac{8.7 - 8.5}{\frac{0.5}{\sqrt{40}}} \approx 2.53.$$

5.

$$2.53 > 1.645$$

The manager of a restaurant in a large city claims that waiters working in all restaurants in this city earn an average of $150 or more in tips per week. A random sample of 25 waiters selected from restaurants of this city yielded a mean of $139 in tips per week with a standard deviation of $28. Assume that the weekly tips for all waiters in this city have a normal distribution.
Using the 1% significance level, can you conclude that the manager's claim is true?

Solution:

$$n = 25, \quad \bar{x} = 139, \quad s = 28, \quad \alpha = 0.01.$$

1.

$$H_0: \ \mu \geq 150; \qquad H_1: \ \mu < 150.$$

2.
Use *t* distribution.

3.

$$t_{critical} = -2.492$$

4.

$$t_{stat} = \frac{139 - 150}{\frac{28}{\sqrt{25}}} = -1.964.$$

5.

$$-1.964 > -2.492.$$

$\because$ Do not reject H_0.

A company produces car batteries. The company claims that its top-of-the-line batteries are good, on average, for at least 65 months. A consumer protection agency tested 45 such batteries to check this claim. It found that the mean life of these 45 batteries is 63.4 months and the standard deviation is 3 months.
Test the claim of the company with a significance level of 2.5%

Solution:

$$n = 45, \quad \bar{x} = 63.4, \quad s = 3, \quad \alpha = 0.025$$

1.

$$H_0: \ \mu \geq 65; \qquad H_1: \ \mu < 65.$$

2.
Use *t* distribution.

3.

$$t_{critical} = -2.015$$

4.

$$t_{stat} = \frac{63.4 - 65}{\frac{3}{\sqrt{45}}} = -3.578$$

5.

$$-3.578 < -2.015.$$

$\because$ Reject H_0.