Chapter III

Measures of Location

(Solutions)

3. 1.
1.

$$\bar{x} = \frac{1}{5} \cdot (9 + 12 + 8 + 10 + 16) = 11 .$$

2.

$$\bar{x} = \frac{1}{25} \cdot (10 \cdot 7 + 15 \cdot 10 + 20 \cdot 5 + 25 \cdot 2 + 30 \cdot 1) = 16 .$$

3. 2.
1. Population.
2.

$$\mu = \frac{511 + 385 + ... + 23 + 13}{12} = \frac{2340}{12} = 195$$

3. 3.

$$\bar{x} = \frac{14 \cdot 16.50 + 10 \cdot 19.00 + 2 \cdot 26.00}{26} \approx 18.12 \text{ €}.$$

3. 4.
A perusal of the salaries reveals that the annual salary of $60000 appears most often (six times). The mode is, therefore, $60000.

3. 5.

i	C_i	F_i	f_i	$\sum_{j=1}^{i} f_j$	m_i	$m_i \cdot F_i$	$b_i \cdot F_i$	$B_i \cdot F_i$
1	[0, 2[	2	0.10	0.10	1	2	0	4
2	[2, 4[	5	0.25	0.35	3	15	10	20
3	[4, 6[	4	0.20	0.55	5	20	16	24
4	[6, 8[	8	0.40	0.95	7	56	48	64
5	[8,10[	1	0.05	1.00	9	9	8	10
Total		20	1.00			102	82	122

1.

$$\bar{x} \approx \frac{102}{20} = 5.10 \text{ km.}$$

$$\frac{82}{20} = 4.1 \le \bar{x} \le 6.1 = \frac{122}{20}$$

2.

$$f_1 = 0.10 < 0.5$$
$$f_1 + f_2 = 0.10 + 0.25 = 0.35 < 0.5$$
$$f_1 + f_2 + f_3 = 0.10 + 0.25 + 0.20 = 0.55 \ge 0.5 \quad \Rightarrow \quad Me \in C_3$$

$$Me \approx 4 + \frac{0.5 - 0.35}{0.20} \cdot 2 = 5.5 \text{ km.}$$

3.

$$Mo \in C_4$$

$$Mo \approx 6 + \frac{0.40 - 0.20}{2 \cdot 0.40 - 0.20 - 0.05} \cdot 2 \approx 6.73 \text{ km.}$$

3. 6.

$\bar{x}_g = \sqrt[4]{\frac{10368}{5000}} = 1.20$ or 120% (average ratio) or 20% (average annual rate of increase)

3. 7.

$$\bar{x}_g = \sqrt{1.05 \cdot 1.1} = 1.09886 .$$

Therefore, 9.886%.

3. 8.

$$\bar{x}_g = \sqrt[4]{1.3 \cdot 1.2 \cdot 0.6 \cdot 2.0} = 1.169705309 \approx 1.1697.$$

Therefore, 16.97%.

3. 9.
We organise the data from lowest to largest:

Position	1	2	3	4	5
Commission	1460	1471	1637	1721	1758
Position	6	7	8	9	10
Commission	1787	1940	2038	2047	2054
Position	11	12	13	14	15
Commission	2097	2205	2287	2311	2406

$$Me = x_{\left[\frac{15+1}{2}\right]} = x_{[8]} = 2038.$$

$$n \cdot 0.25 = 15 \cdot 0.25 = 3.75, \qquad \tilde{x}_{0.25} = x_{[4]} = 1721$$

$$n \cdot 0.75 = 15 \cdot 0.25 = 11.25, \qquad \tilde{x}_{0.75} = x_{[12]} = 2205$$