# Chapter IV

# Random Variables

**<u>D. 4. 1.</u> (*Random Variable*)**
A *random variable* is a function defined on the sample space.

$$X = X(E), \quad E \in S$$

**<u>Ex. 4. 1.</u>** (see Ex. 3.9.)
A box contains 25 items, 10 of which are defective. A sample of two items will be taken, without replacement:
Let

$X$ : „number of defective items".

$E_1$: „both items are good" $\Rightarrow$ $X = 0$

$E_2$: „exactly one item is defective" $\Rightarrow$ $X = 1$

$E_3$: „both items are defective" $\Rightarrow$ $X = 2$

**<u>Ex. 4. 2.</u>**
Let

$X$ : „temperature on 01.01.2005"
$E$ : $[0, \quad 24]$ : time

**<u>D. 4. 2.</u> (*Discrete and Continuous Random Variables*)**

1. A random variable is called a *discrete random variable* if it takes only a finite or countably infinite number of values. (See ex. 4. 1.).
2. A random variable is called a *continuous random variable* if it takes every real value within an interval. (See Ex. 4. 2.).

**<u>Ex. 4. 3.</u>**
1. Examples of *discrete* random variables:

- The number of cars entering a carwash an hour.
- The number of home mortgages approved by a bank.

2. Examples of *continuous* random variables:

- The time it takes an executive to drive to work.
- The length of time of a particular phone call.

**D. 4. 3.** (***Distribution Function***)
A *distribution function* is defined as:

$$F(X) = P(X < x), \quad x \in R^1$$

**Ex. 4. 4.** (See Ex. 3. 9 and Ex. 4. 1.)
We had :

$$P(X = 0) = \frac{7}{20},$$

$$P(X = 1) = \frac{10}{20},$$

$$P(X = 2) = \frac{3}{20}.$$

Therefore, we have

$$F(0) = P(X < 0) = 0,$$
$$F(1) = P(X < 1) = P(X = 0) = \frac{7}{20},$$
$$F(2) = P(X < 2) = P(X = 0) + P(X = 1) = \frac{7}{20} + \frac{10}{20} = \frac{17}{20},$$
$$F(3) = P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{7}{20} + \frac{10}{20}\frac{3}{23} = 1.$$

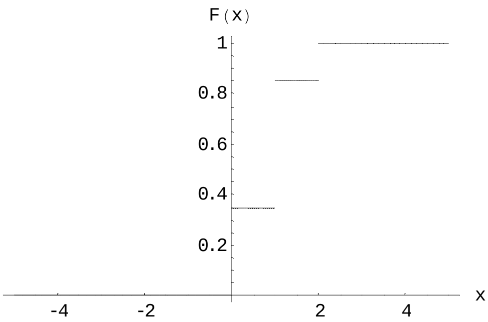
*Analytic form* of the distribution function:

$$F(x) = \begin{cases} 0 & \text{when} \quad -\infty < x \le 0 \\ \frac{7}{20} & \text{when} \quad 0 < x \le 1 \\ \frac{17}{20} & \text{when} \quad 1 < x \le 2 \\ 1 & \text{when} \quad 2 < x < +\infty \end{cases}$$

*Tabular form* of the distribution function:

| $x$ | $F(x)$ |
|---|---|
| $]-\infty, \quad 0]$ | $0$ |
| $]0, \quad 1]$ | $\frac{7}{20}$ |
| $]1, \quad 2]$ | $\frac{17}{20}$ |
| $]2, \quad +\infty[$ | $1$ |

*Graphical form* of the distribution function:




**<u>T. 4. 1.</u> (*Important Properties of the Distribution Function*)**

1.

$$0 \le F(x) \le 1, \quad \forall x \in R^1.$$

2.

$$\forall x_1, x_2 : x_1 < x_2 \quad \Rightarrow \quad F(x_1) \le F(x_2).$$

3.

$$\forall x_1, x_2 : x_1 < x_2 \quad \Rightarrow \quad P(x_1 \le X < x_2) = F(x_2) - F(x_1).$$

4.

$$x \to -\infty \quad \Rightarrow \quad F(x) \to 0$$

$$x \to +\infty \quad \Rightarrow \quad F(x) \to 1.$$

5.

$F(x)$ is at least left-sided continuous and has at most a finite number of jump discontinuities.

**<u>Ex. 4. 5.</u>**

Let $X$ be the time in hours elapsed between the arrival of two ships at a certain port with the following distribution function:

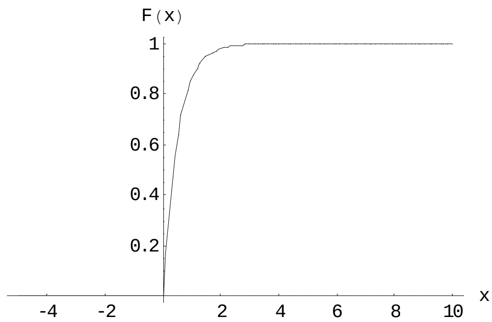
$$F(x) := \begin{cases} 0 & \text{when} \quad x \le 0 \\ 1 - e^{-2x} & \text{when} \quad x > 0 \end{cases}.$$

Find the probability that

1. the time elapsed between the arrival of two ships is less than 90 minutes.
2. 15 minutes elapse without any ship arriving.

3. the time elapsed between the arrival of two ships is at least 6 minutes but less than 30 minutes.

*Solution:*




1.

$$P\left( X < \frac{3}{2} \right) = F\left( \frac{3}{2} \right) = 1 - e^{-3} = 0.9502$$

2.

$$P\left( X \geq \frac{1}{4} \right) = 1 - P\left( X < \frac{1}{4} \right) = 1 - F\left( \frac{1}{4} \right) = 1 - 1 + e^{-0.5} = 0.6065$$

3.

$$P\left( \frac{1}{10} \leq X < \frac{1}{2} \right) = F\left( \frac{1}{2} \right) - F\left( \frac{1}{10} \right) = e^{-0.5} - e^{-1} = 0.8187 - 0.3679 = 0.4508$$

*(Last revised: 15.04.09)*