I

Random Events and Events Algebra

<u>D. 1. 1.</u> (*Random Trial*)
A trial whose outcome cannot be predicted in advance is called a *random trial*.

<u>D. 1. 2.</u> (*Random Event*)
The outcome of a random trial is called a *random event*.

<u>Ex. 1. 1.</u>
Random trial: Rolling an unbiased die

Random events: $j = 1, 2, ..., 6$: Number appearing above

<u>Ex. 1. 2.</u>
Random trial: Quality control

Random events: $k = 0, 1, ...$: Number of defective products

<u>Ex. 1. 3.</u>
Random trial: Examining the lifetime of a certain kind of tyre

Random events: $t \in [0, \quad +\infty[$: lifetime of a certain kind of tyre.

<u>R. 1. 1.</u>
The algebra of events is the application of the set theory to random events.

<u>D. 1. 3.</u> (*Impossible and Certain Events*)
An event that in all repetitions of a certain random trial will never happen is called an *impossible event*. It will be denoted by $\emptyset$.
An event that in all repetitions of a certain event will always occur is called a *certain event*. It will be denoted by Ω.

<u>Ex. 1. 4.</u>
Random trial: Rolling an unbiased die.
The random event “Throw the number 8” or $\{8\}$ is an impossible event.
The random event “Throw at most the number 6“ or $\{1, 2, 3, 4, 5, 6\}$ is a certain event.

<u>D. 1. 4.</u> (*Subevent*)
The event E_1 is called a *subevent of the event* E_2 if it always accompanies the event E_2.

<u>Ex. 1. 5.</u>
Random trial: Rolling an unbiased die.

Let

E_1**:** "throw an odd number",
E_2**:** "throw at most 5".

We have:

$$E_1 = \{1,\ 3,\ 5\} \subseteq \{1, 2, 3, 4, 5\} = E_2 .$$

<u>D. 1. 5.</u> (*Equivalent Events*)

$$E_1 \coloneqq E_2 \quad \Leftrightarrow \quad \left(E_1 \subseteq E_2 \ \wedge \ E_2 \subseteq E_1\right)$$

<u>D. 1. 6.</u> (*Sum of Events*)
E is said to be the *sum* of the events $E_i, i = 1, 2, ..., n$, if <u>at least</u> one of the events E_i occurs:

$$E \coloneqq \bigcup_{i=1}^{n} E_i$$

<u>Ex. 1. 6.</u>
Random trial: Rolling an unbiased die.

Let

E_1**:** "throw either 2 or 4",
E_2**:** "throw either 2 or 6".

We have:

$$E_1 = \{2,\ 4\}, \qquad E_2 = \{2,\ 6\},$$

$$E = E_1 \cup E_2 = \{2,\ 4, 6\}$$

<u>D. 1. 7.</u> (*Product of Events*)
E is said to be the *product* of the events $E_i, i = 1, 2, ..., n$, if the events E_i occurs <u>at the same time:</u>

$$E \coloneqq \bigcap_{i=1}^{n} E_i$$

<u>Ex. 1. 7.</u>
Random trial: Rolling an unbiased die.

Let

E_1**:** “throw 1 or 4”,
E_2**:** “throw a prime number”.

We have:

$$E_1 = \{1,\ 4\}, \qquad E_2 = \{1, 2, 3, 5\},$$

$$E = E_1 \cap E_2 = \{1\}$$

<u>D. 1. 8.</u> (*Mutually exclusive events*)
The events E_1 and E_2 are said to be *mutually exclusive* if

$$E_1 \cap E_2 = \varnothing$$

<u>Ex. 1. 8.</u>
Random trial: Rolling an unbiased die.

Let

E_1**:** “throw an even number”,
E_2**:** “throw an odd number”.

We have

$$E_1 \cap E_2 = \varnothing$$

<u>D. 1. 9.</u> (*Complementary Events*)

The events E and $\bar{E}$ are said to be *complementary* if

$$E \cup \bar{E} = \Omega \quad \wedge \quad E \cap \bar{E} = \varnothing$$

<u>Ex. 1. 9.</u>
Random trial: Rolling an unbiased die.

Let

E_1**:** “throw an even number”,
E_2**:** “throw an odd number”.

We have:

$$E_1 = \{2, 4,\ 6\}, \qquad E_2 = \{1, 2, 3\},$$

$$E_1 \cup E_2 = \{1, 2, 3, 4, 5, 6\} = \Omega\ , \qquad E_1 \cap E_2 = \varnothing$$

D. 1. 10. (***Difference***)

The *difference* of the events E_1 and E_2, denoted by $E_1 \setminus E_2$ is defined as the case in which E_1 occurs while E_2 does not occur.

Ex. 1. 10.

Random trial: Rolling an unbiased die.

Let

E_1: "throw either 1 or 4",
E_2: "throw the number 4".

We have:

$E_1 = \{1, 4\}$, $E_2 = \{1\}$,

$E_1 \setminus E_2 = \{4\}$

T. 1. 1.

Let $E_i, i = 1, 2, ..., n$, be random events. Then we have:

The Commutative Laws:

$$E_1 \cup E_2 = E_2 \cup E_1$$
$$E_1 \cap E_2 = E_2 \cap E_1.$$

The Associative Laws:

$$E_1 \cup (E_2 \cup E_3) = (E_2 \cup E_1) \cup E_3 = E_1 \cup E_2 \cup E_3$$
$$E_1 \cap (E_2 \cap E_3) = (E_2 \cap E_1) \cap E_3 = E_1 \cap E_2 \cap E_3$$

The Distributive Laws:

$$E_1 \cup (E_2 \cap E_3) = (E_1 \cup E_2) \cap (E_1 \cup E_3)$$
$$E_1 \cap (E_2 \cup E_3) = (E_1 \cap E_2) \cup (E_1 \cap E_3).$$

Further we have:

$E \cup E = E$, $E \cap E = E$

$E \cup \bar{E} = \Omega$ $E \cap \bar{E} = \varnothing$

$E \cup \varnothing = E$ $E \cap \varnothing = \varnothing$

$E \cup \Omega = \Omega$ $E \cap \Omega = E$

De Morgan Laws.

$$\overline{\bigcap_{i=1}^{n} E} = \bigcup_{i=1}^{n} \bar{E}_i \ ,$$

$$\overline{\bigcup_{i=1}^{n} E_i} = \bigcap_{i=1}^{n} \bar{E}_i \ .$$

<u>Ex. 1. 11</u>.

A factory consists of three departments D_1, D_2, D_3. Let

$E_i,\ \ i = 1, 2, 3$: "There is <u>no</u> disturbance in D_i.

Describe the following events:

A: "There is no disturbance in the three departments:"

B: "There is disturbance in all departments."

C: "There is no disturbance in at least one department."

D: "There is disturbance in at least one department."

E: "There is disturbance in at most one department."

F: "There is disturbance only in department D_3 "

G: "There is disturbance in department D_3."

Solution:

$$A = E_1 \cap E_2 \cap E_3$$

$$B = \bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 = \overline{E_1 \cup E_2 \cup E_3}$$

$$\begin{aligned} C &= E_1 \cup E_2 \cup E_3 \\ &= (E_1 \cap E_2 \cap E_3) \\ &\quad \cup (E_1 \cap E_2 \cap \bar{E}_3) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3) \\ &\quad \cup (E_1 \cap \bar{E}_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap \bar{E}_3) \\ &= \Omega \setminus (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) \end{aligned}$$

$$D = \bar{E}_1 \cup \bar{E}_2 \cup \bar{E}_3$$

$$E = (E_1 \cap E_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (E_1 \cap E_2 \cap \bar{E}_3)$$

$$F = E_1 \cap E_2 \cap \bar{E}_3$$

$$G = (E_1 \cap E_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap E_2 \cap \bar{E}_3) \cup (E_1 \cap \bar{E}_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)$$

D. 1. 11. (***System of Mutually Exclusive and Exhaustive Events***)
The events E_i, $i = 1, 2, \ldots, n$, form a *mutually exclusive and exhaustive* system if the following conditions are fulfilled:

1. $E_i \neq \emptyset, \qquad i = 1, 2, \ldots, n$
2. $\bigcup_{i=1}^{n} E_i = \Omega$
3. $E_i \cap E_j = \emptyset, \quad i \neq j, \quad i, j = 1, 2, \ldots, n$

Ex. 1. 12.
Determine if the following events form a *mutually exclusive and exhaustive* system:

1. Random trial: Rolling an unbiased die

E_1 : : „throw 1 or 4“
E_2 : „throw an odd number > 2“
E_3 : “throw an even number $\neq 4$“.

$$E_1 = \{1, 4\}, \quad E_2 = \{3, 5\}, \quad E_3 = \{2, 6\}.$$

The events form a *mutually exclusive and exhaustive* system, since

$$E_i \neq \emptyset, \quad i = 1, 2, 3$$

$$E_1 \cup E_2 \cup E_3 = \Omega$$

$$E_1 \cap E_2 = \emptyset, \quad E_1 \cap E_3 = \emptyset, \qquad E_2 \cap E_3 = \emptyset$$

2. Random trial: Rolling three unbiased dice.

E_1 : : „throw at least 17“
E_2 : „throw at most 5“

$$E_1 = \{17, 18\}, \; E_2 = \{3, 4, 5\}.$$

The events do not form a *mutually exclusive and exhaustive* system, since among other things

$$E_1 \cup E_2 \neq \Omega$$

D. 1. 12. (*Field of Events*)
The *field of events* F is a set with the following properties:

1. $\varnothing$, $\Omega \in F$
2. $E_1, E_2 \in F \quad \Rightarrow \quad (E_1 \cup E_2 \in F) \;\wedge\; (E_1 \cap E_2 \in F)$
3. $E \in F \;\Rightarrow\; \bar{E} \in F$
4. $E_i \in F,\; i = 1, 2, \ldots \;\Rightarrow\; \bigcup_i E_i \in F \;\wedge\; \bigcap_i E_i \in F$

Ex. 1. 13.
Random trial: Rolling an unbiased die. Let

E_1 : : „throw an odd number“
E_2 : „throw an even number“

$$F = \{ E_1, E_2, \Omega, \varnothing \}$$

Ω : „throw either 1 or 2 or…6”
$\varnothing$: „throw neither of the numbers 1, 2, ..., 6”

D. 1. 13. (*Elementary or Atomic Event*)
An *elementary* or *atomic event* is an event which is not further genuinely decomposable.

Ex. 1. 14.
Random trial: Rolling an unbiased die. Let

E_1 : : „throw an odd number“
E_2 : „throw a 6“.

E_1 is decomposable: $E_1 = \{1, 3, 5\} = \{1\} \cup \{3\} \cup \{5\}$, E_2 is not.

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