Chapter VIII

Some Special Continuous Distribution Functions

Solutions

8. 1.
Let X denote the weekly income of workers.

$$\mu = 500 \text{ €}, \quad \sigma = 100 \text{ €}$$

a)

$$P(X < 500) = F(500) = \Phi\left(\frac{500-500}{100}\right) = \Phi(0) = 0.5 .$$

The number of workers having a weekly income below 500 is: $10000 \cdot 0.5 = 5000$.

b)

$$P(500 < X < 600) \approx P(500 \le X < 600)$$

$$= \Phi\left(\frac{600-500}{100}\right) - \Phi\left(\frac{600-500}{100}\right)$$

$$= \Phi(1) - \Phi(0) = 0.84134 - 0.50000 = 0.34134 .$$

The number of workers having a weekly income above 500 € but below 600 € is:
$10000 \cdot 0.34134 = 3413$.

c)

$$P(X > 600) = 1 - P(X \le 600) \approx 1 - P(X < 600)$$

$$= 1 - P(X < 600) = 1 - F(600) = 1 - \Phi\left(\frac{600-500}{100}\right)$$
$$= 1 - \Phi(1) = 1 - 0.84134 = 0.15866 .$$

The number of workers having a weekly income above 600 € is : $10000 \cdot 0.15866 = 1587$.

8. 2.
1.

$$[65.5 - 2.4,\ 65.5 + 2.4] = [63.1,\ 67.9] .$$

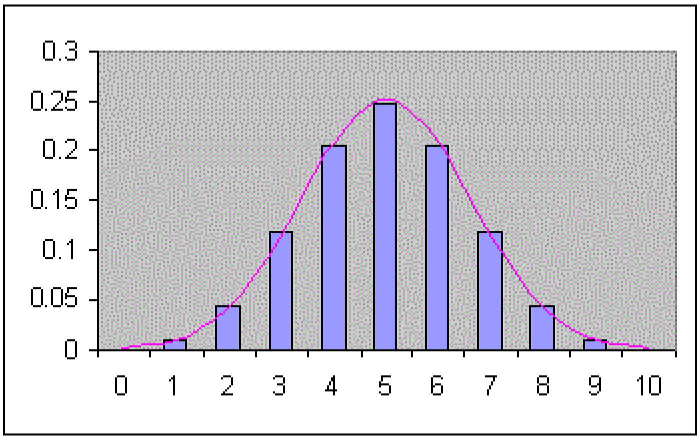
2.

$$[65.5 - 2 \cdot 2.4,\ 65.5 + 2 \cdot 2.4] = [60.7,\ 70.3] .$$

8. 3.

$$E(X) = \mu = n \cdot p = 10 \cdot 0.5 = 5$$
$$D(X) = \sigma = \sqrt{n \cdot p \cdot q} = \sqrt{10 \cdot 0.5 \cdot 0.5} \approx 1.58$$

x	Binomial Distribution	Normal Distribution
0	0.000977	0.001700
1	0.009766	0.010285
2	0.043945	0.041707
3	0.117188	0.113372
4	0.205078	0.206577
5	0.246094	0.252313
6	0.205078	0.206577
7	0.117188	0.113372
8	0.043945	0.041707
9	0.009766	0.010285
10	0.000977	0.001700



8. 4.
Applying the "empirical rule", we obtain a probability of approximately 68%.

8. 5.
Let X denote the thickness of a machined cylinder.

$$\mu = 8.1 \text{ cm}, \quad \sigma = 0.1 \text{ cm}$$

1.
$$P(7.9 \le X < 8.2) = F(8.2) - F(7.9)$$
$$= \Phi\left(\frac{8.2-8.1}{0.1}\right) - \Phi\left(\frac{7.9-8.1}{0.1}\right) = \Phi(1) - \Phi(-2)$$
$$= \Phi(1) - (-\Phi(2))$$

$$= 0.841345 - 1 + 0.977250 = 0.818595.$$

2.

$$P\left(|X - 8.1| < 0.5\right) = 2 \cdot \Phi\left(\frac{0.5}{0.1}\right) - 1 = 2 - 1 = 1.$$

3.

$$P\left(X \geq 8\right) = 1 - P(X < 8) = 1 - F(8)$$

$$= 1 - \Phi\left(\frac{8 - 8.1}{0.1}\right) = 1 - \Phi\left(-1\right)$$

$$= 1 - \left(1 - \Phi\left(1\right)\right) = 0.841345.$$

8. 6.

Let X denote the cash sale amounts.

1.

a)

$$P(55.00 \leq X < 72.50) = F(72.50) - F(55.00)$$

$$= \Phi\left(\frac{72.50 - 60.00}{10}\right) - \Phi\left(\frac{55.00 - 60.00}{10}\right)$$

$$= \Phi\left(1.25\right) - \Phi\left(-0.50\right) = \Phi\left(1.25\right) - \left(1 - \Phi\left(0.50\right)\right)$$

$$= 0.894350 - 1 + 0.691462 = 0.585812.$$

b)

$$P(X \geq 72.50) = 1 - P(X < 72.50)$$

$$= 1 - F(72.50) =$$

$$= 1 - \Phi\left(1.25\right) = 1 - 0.844350 = 0.105650.$$

c)

$$P(X < 55.00) = F(55.00)$$

$$= \Phi(-0.5) = 1 - \Phi(0.5) = 1 - 0.691462 = 0.308538.$$

2.

$$P(X > c) \approx P(X \geq c) = 0.20,$$

$$1 - P(X < c) = 0.2\ ; \qquad P(X < c) = 0.8,$$

$F(c) = 0.8;$ $\Phi\left(\frac{c-60}{10}\right) = 0.8 = \Phi(0.84)$,

$\frac{c-60}{10} = 0.84 \quad \Rightarrow \quad c = 68.4$.

8. 7.
Let X denote the exam mark.

$$\mu = 45, \quad \sigma = 15.$$

1.
$$P(X \geq 40) = 1 - P(X < 40) = 1 - F(40)$$
$$= 1 - \Phi\left(\frac{40-45}{15}\right) = 1 - \Phi(-0.33) = 1 - (1 - \Phi(0.33)) = 0.629300,$$
i.e. nearly 63%

2.

$$P(X \geq c) = 1 - P(X < c) = 0.75,$$
$$1 - F(c) = 1 - \Phi\left(\frac{c-45}{15}\right) = 0.75,$$
$$\Phi\left(\frac{c-45}{15}\right) = 0.25 = \Phi(-0.68)$$
$$\frac{c-45}{15} = -0.68 \quad \Rightarrow \quad c = 34.8 \approx 35.$$

3.
$$P(X \geq 6) = 1 - \sum_{x=6}^{10} \binom{10}{x} \cdot 0.63^x \cdot 0.37^{10-x}$$
$$= 1 - (0.246076 + 0.239425 + 0.152876 + 0.057845 + 0.009849)$$

$= 1 - 0.706071 = 0.293929$, i.e. nearly 29%.

8. 8.
Let

X : "The amount of time [hours] a household personal computer is used for entertainment

X is normally distributed with

$$\mu = 2, \quad \sigma = 0.5.$$

1.

$$P(1.80 \le X < 2.75) = F(2.75) - F(1.80)$$

$$= \Phi\left(\frac{2.75 - 2.00}{0.5}\right) - \Phi\left(\frac{1.80 - 2.00}{0.5}\right) = \Phi(1.5) - \Phi(-0.4)$$

$$= \Phi(1.5) - (1 - \Phi(0.4)) = \Phi(1.5) - 1 + \Phi(0.4)$$

$$= 0.9332 - 1 + 0.6554 = 0.5886.$$

2.

$$P(X \ge 1.4) = 1 - P(X < 1.4) = 1 - F(1.4)$$

$$= 1 - \Phi\left(\frac{1.4 - 2}{0.5}\right) = 1 - \Phi(-1.2)$$

$$= 1 - (1 - \Phi(1.2)) = 0.8849$$

3.

$$P(X < x) = 0.25$$

$$F(x) = 0.25$$

$$\Phi\left(\frac{x-2}{0.5}\right) = 0.25 = \Phi(-0.67)$$

$$\frac{x-2}{0.5} = -0.67$$

$$x = 1.67$$

8. 9.

Let

X : "the number of passengers with a ticket who show up for a given flight."

X is binomially distributed with

$$n = 330, \quad p = 0.95.$$

1.

$$\mu = n \cdot p = 330 \cdot 0.95 = 313.5$$

$$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{330 \cdot 0.95 \cdot 0.05} \approx 3.959.$$

2.

$$n \cdot p = 313.5 > 5 \quad \wedge \quad n \cdot q \approx 313.5 > 5.$$

3.

$$P(X \le 320) \approx P(X < 320) = F(320)$$

$$= \Phi\left(\frac{320 - 313.5}{3.959}\right) \approx \Phi(1.64) = 0.9495.$$

(Last revised: 16.08.2010)