Chapter VII

Some Special Discrete Distribution Functions

Solutions

7. 1.
First we confirm that this experiment possesses the characteristics of a binomial experiment. This experiment consists of 5 trials, one corresponding to each random selected well. Each trail results in an S (the well contains impurity A) or an F (the well does not contain impurity A). Since the total number of wells in the country is large, the probability of drawing a single well and finding that it contains impurity A is equal to 0.30 and this probability will remain the same for each of the 5 selected wells. Further, since the sampling is random, we assume that the outcome on any one well is unaffected by the outcome of any other and that the trials are independent. Finally we are interested in the number of wells in the sample of 5 that contain impurity A.
Therefore, the sampling process represents a binomial experiment with $n = 5$ and $p = 0.30$.
Let X denote the number of wells containing impurity A.

1.
$$P(X = 3) = \binom{5}{3} \cdot 0.30^3 \cdot 0.70^2 = 0.1323.$$

2.
$$P(X \geq 3) = \sum_{x=3}^{5} \binom{5}{x} \cdot 0.30^x \cdot 0.70^{5-x} = 0.13230 + 0.02835 + 0.00243 = 0.16308.$$

3.
$$P(X < 3) = 1 - P(X \geq 3) = 1 - 0.16380 = 0.83692.$$

7. 2.
We have
$$N = 50, \quad M = 4, \quad n = 5.$$

1.
$$P(X = 1) = p_1 = \frac{\binom{4}{1} \cdot \binom{50-4}{5-1}}{\binom{50}{5}} = 0.308076422 \cdot$$

2.

$$F(2) = P\left(X < 2\right) = \sum_{x=0}^{1} \frac{\binom{4}{x} \cdot \binom{50-4}{5-x}}{\binom{50}{5}} = 0.646960486 + 0.308076422 = 0955036908 \cdot$$

7. 3.
We have

$$n = 20, \quad p = 0.90 .$$

a)

$$P(X = 20) = \binom{20}{20} \cdot 0.90^{20} \cdot 0.10^{0} = 0.121576654 .$$

b)

$$P(X = 19) = \binom{20}{19} \cdot 0.90^{19} \cdot 0.10^{1} = 0.270170343 .$$

c)

$$P(X = 18) = \binom{20}{18} \cdot 0.90^{18} \cdot 0.10^{2} = 0.285179807 .$$

7. 4.
Let the random variable X be the number of female. We have

$$n = 4, \quad p = 0.4878 \quad (100 \cdot 100 : 205 \approx 48.78\%) .$$

a)

$$P(X = 2) = \binom{4}{2} \cdot 0.5122^{2} \cdot 0.4878^{2} \cdot = 0.374553612 .$$

b)

$$P(X = 4) = \binom{4}{0} \cdot 0.4878^{0} \cdot 0.5122^{4} = 0.068826913 .$$

c)

$$P(X = 4) + P(X = 0) = \binom{4}{4} \cdot 0.4878^{4} \cdot 0.5122^{0} + \binom{4}{0} \cdot 0.4878^{0} \cdot 0.5122^{4}$$

$$= 0.068826913 + 0.05661965 = 0.137653826 .$$

7. 5.

Let the random variable X be the number of misprints on a page. We have $\lambda = \frac{50}{250} = 0.2$.

$$P(X = 0) = p_0 = \frac{0.2^{0}}{0!} \cdot e^{-0.2} = 0.818730753 .$$

7. 6.

Let the random variable X be the number of defective biros in a packet. X is binomially distributed with

$$n = 10, \quad p = 0.05 .$$

1.

$$P(X < 3) = \sum_{x=0}^{2} \binom{10}{x} \cdot 0.05^x \cdot 0.95^{10-x} .$$

$$= 0.598736939 + 0.315124704 + 0.074634798 = 0.988496441 .$$

2.

$$P(X \geq 3) = 1 - P(X < 3)$$

$$= 1 - P(X < 3) = 1 - \sum_{x=0}^{2} \binom{10}{x} \cdot 0.05^x \cdot 0.95^{10-x} = 0.011503558 .$$

7. 7.

Discrete, Poisson distribution.

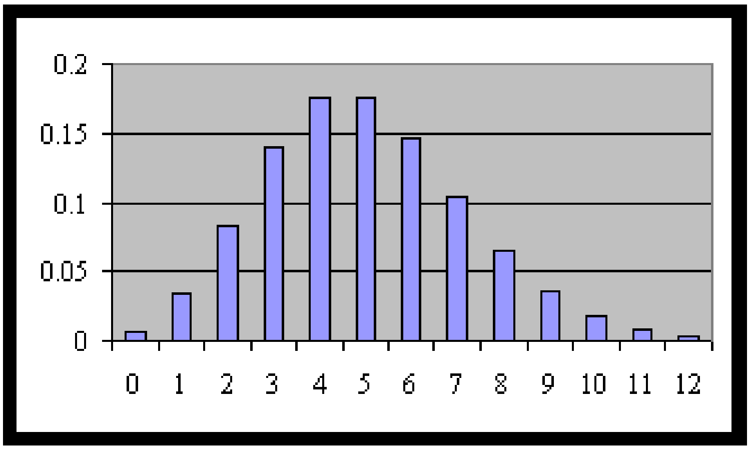
7. 8.

Let X denote the number of accidents per month. We have to calculate

$$P\left(X = x\right) = \frac{5^x}{x!} \cdot e^{-5}, \quad x = 0,\ 1,\ \ldots ,\ 12 .$$

The probability distribution of X is presented below:

Number of Accidents	Probability
0	0.006738
1	0.03369
2	0.084224
3	0.140374
4	0.175467
5	0.175467
6	0.146223
7	0.104445
8	0.065278
9	0.036266
10	0.018133
11	0.008242
12	0.003434
Poisson probability distribution of the number of accidents per month	



The Poisson probability distribution of the number of accidents

7. 9.
Let X denote the number employees belonging to a trade union. We have a hypergeometrically distributed random variable with

$$N = 50, \quad M = 40, \quad n = 5.$$

$$P(X = 4) = \frac{\binom{40}{4} \cdot \binom{50-40}{5-4}}{\binom{50}{5}} \approx 0.431.\cdot$$

7. 10.
Let X denote the number of late flights. We have a binomially distributed random variable with

$$n = 5, \quad p = 0.2.$$

1.

$$P(X = 0) = \binom{5}{0} \cdot 0.2^0 \cdot 0.8^5 = 0.32768..$$

2.

$$E(X) = 5 \cdot 0.2 = 1, \qquad D^2(X) = 0.8.$$

7. 11.
Let X denote the number of customers arriving within an interval of one minute. We have the Poisson distribution with $\lambda = 1$.

1. $P(X = 0) = 0.367879$.
2. $P(X = 1) = 0.367879$.
3. $P(X = 3) = 0.0.061313$.
4. $P(X > 3) = 1 - P(X \le 3) = 1 - (0.367879 + 0.367879 + 0.183840 + 0.061313) = 0.018989$.

7. 12.
Let X denote the number of ships arriving at the dock. We have the Poisson distribution with $\lambda = 0.5$.

$$P(X \ge 2) = 1 - P(X < 1) = 1 - (0.606531 + 0.303265) = 0.090204.$$

7. 13.
Denote by

X : "The number of corporations which incurred losses."

X is Hypergeometrically distributed with

$$N = 15, \quad M = 9, \quad n = 3.$$

1.

$$P(X = 2) = \frac{\binom{9}{2} \cdot \binom{15-9}{3-2}}{\binom{15}{3}} = \frac{36 \cdot 6}{455} \approx 0.47.47$$

2.

$$P(X = 0) = \frac{\binom{9}{0} \cdot \binom{15-9}{3-0}}{\binom{15}{3}} = \frac{1 \cdot 20}{455} \approx 0.0440.$$

3.

$$P(X = 0) + P(X = 1) = 0.0440 + \frac{\binom{9}{1} \cdot \binom{15-9}{3-1}}{\binom{15}{3}} = 0.0440 + \frac{9 \cdot 15}{455}$$

$$= 0.0440 + 0.0297 \approx 0.3407$$

7. 14.
Let

X : "the number of cellular phones in a U.S. household."

X is binomially distributed.

1.

$$n = 11, \quad p = 0.70$$

$$P(X = 11) = \binom{11}{11} 0.70^{11} \cdot 0.30^{11-11} \approx 0.01977326743.$$

2.

$$n = 11, \quad p = 0.70$$

$$P(X > 4) = 1 - P(X \le 4) = 1 - \sum_{x=0}^{4} P(X = x)$$

$$= 1 - \sum_{x=0}^{4} \binom{11}{x} \cdot 0.70^{x} \cdot 0.30^{11-x}$$

$$\approx 1 - \left(0.00000 + 0.00005 + 0.00053 + 0.00371 + 0.01733\right) = 0.97838.$$

3.

$$n = 11, \quad p = 0.70$$

$$P(X < 5) = P(X \le 4) = \sum_{x=0}^{4} P(X = x)$$

$$= \sum_{x=0}^{4} \binom{11}{x} \cdot 0.70^{x} \cdot 0.30^{11-x}$$

$$\approx 0.00000 + 0.00005 + 0.00053 + 0.00371 + 0.01733 = 0.02162$$

4.

$$n = 11, \quad p = 0.30$$

$$P(X > 7) = \sum_{x=8}^{11} P(X = x) = \sum_{x=8}^{11} \binom{11}{x} \cdot 0.30^{x} \cdot 0.70^{11-x}$$

$$\approx 0.00371 + 0.00053 + 0.00005 + 0.00000 = 0.00429.$$

(Last revised: 11.08.2010)