Chapter VI

Parameters of a Random Variable

Solutions

6. 1.

a)

$$F(x)=\begin{cases} 0 & -\infty < x \le -3 \\ 0.10 & -3 < x \le 0 \\ 0.25 & 0 < x \le 1 \\ 0.35 & 1 < x \le 2 \\ 0.60 & 2 < x \le 3 \\ 1 & 3 < x < +\infty \end{cases}$$

b)

$$P(X>0)=P(X=1)+P(X=2)+P(X=3)$$

$$=0.10+0.25+0.40=0.75$$

c)

$$E(X)=-3\cdot 0.1+0\cdot 0.15+1\cdot 0.1+2\cdot 0.25+3\cdot 0.4=1.5$$

$$D^2(X)=(-3)^2\cdot 0.1+0^2\cdot 0.15+1^2\cdot 0.1+2^2\cdot 0.25+3^2\cdot 0.4-(1.5)^2=3.35.$$

6. 2.

Let

X **: „**Number of elements that will fall out“.

$$E(X)=\sum_{i=1}^{3} p_i\cdot x_i = 0\cdot(1-p_1)\cdot(1-p_2)\cdot(1-p_3)$$

$$+1\cdot[p_1\cdot(1-p_2)\cdot(1-p_3)+p_2\cdot(1-p_1)\cdot(1-p_3)+p_3\cdot(1-p_1)\cdot(1-p_2)]$$

$$+2\cdot[p_1\cdot p_2\cdot(1-p_3)+p_1\cdot p_3\cdot(1-p_2)+p_2\cdot p_3\cdot(1-p_1)]+3\cdot p_1\cdot p_2\cdot p_3$$

$$=p_1+p_2+p_3.$$

6. 3.

a)

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{0}^{1} \alpha \cdot x^2 \cdot (1-x)\,dx = 1,$$

$$\alpha \cdot \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1 = 1, \qquad \alpha \cdot \left(\frac{1}{3} - \frac{1}{4} \right) = 1, \qquad \alpha = 12.$$

b)

$$F(x) = \begin{cases} 0 & x \le 0 \\ 12\left(\frac{x^3}{3} - \frac{x^4}{4}\right) & 0 < x \le 1, \\ 1 & 1 < x \end{cases}$$

$$E(X) = \int_0^1 x \cdot f(x)dx = \int_0^1 12 \cdot \left(x^3 - x^4\right)dx = 12 \cdot \left[\frac{x^4}{4} - \frac{x^5}{5} \right]_0^1 = 12 \cdot \left(\frac{1}{4} - \frac{1}{5} \right) = \frac{3}{5}.$$

$$D^2(X) = \int_0^1 x^2 \cdot f(x)dx - \left(E\left(X\right)\right)^2 = 12 \cdot \int_0^1 \left(x^4 - x^5\right)dx - \frac{9}{25} = 12 \cdot \left[\frac{x^5}{5} - \frac{x^6}{6} \right]_0^1 - \frac{9}{25} = \frac{1}{25}.$$

c)

$$P\left(X < \frac{1}{2} \right) = F\left(\frac{1}{2} \right) = 12 \cdot \left[\frac{\frac{1}{8}}{3} - \frac{\frac{1}{16}}{4} \right] = 0.3125.$$

$$P\left(X < E(X)\right) = P\left(X < \frac{3}{5} \right) = F\left(\frac{3}{5} \right) = 12 \cdot \left[\frac{\left(\frac{3}{5}\right)^3}{3} - \frac{\left(\frac{3}{5}\right)^4}{4} \right] = 0.4752.$$

6. 5.

1.

$E\left(X\right) = 4 \cdot 0.01 + 5 \cdot 0.08 + 6 \cdot 0.29 + 7 \cdot 0.42 + 8 \cdot 0.14 + 9 \cdot 0.06 = 6.78$ periods.

2.

$D^2\left(X\right) = 4^2 \cdot 0.01 + 5^2 \cdot 0.08 + 6^2 \cdot 0.29 + 7^2 \cdot 0.42 + 8^2 \cdot 0.14 + 9^2 \cdot 0.06 - (6.78)^2 = 1.0316.$

3.

$D\left(X\right) = \sqrt{1.0316} \approx 1.02$ periods.

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