

Chapter V
Discrete and Continuous Random Variables
Solutions

5. 1.

1.

$$p_3 = 1 - (0.70 + 0.10 + 0.05 + 0.05 + 0.03) = 1 - 0.93 = 0.07$$

2.

$$P(X \leq 3) = 0.70 + 0.10 + 0.05 + 0.05 = 0.90$$

3.

$$F(x) = \begin{cases} 0.00 & \text{when } -\infty < x \leq 0 \\ 0.70 & \text{when } 0 < x \leq 1 \\ 0.80 & \text{when } 1 < x \leq 2 \\ 0.85 & \text{when } 2 < x \leq 3 \\ 0.90 & \text{when } 3 < x \leq 4 \\ 0.93 & \text{when } 4 < x \leq 5 \\ 1.00 & \text{when } 5 < x < +\infty \end{cases}$$

4.

i)

$$F(3.8) = P(X < 3.8) = 0.9$$

The probability that the number of broken bricks is less than 3.8 is equal to 90%.

ii)

$$F(4.7) - F(1.8) = 0.93 - 0.80 = 0.13 = P(1.8 \leq X < 4.7)$$

The probability that the number of broken bricks is at least 1.8 and less than 4.7 is equal to 13%.

5. 2.

1.

$$p_2 = 1 - (0.70 + 0.10 + 0.05 + 0.05) = 1 - 0.90 = 0.10$$

2.

$$P(X \geq 2) = 0.10 + 0.05 + 0.05 = 0.20$$

3.

$$F(x) = \begin{cases} 0.00 & \text{when } -\infty < x \leq 0 \\ 0.70 & \text{when } 0 < x \leq 1 \\ 0.80 & \text{when } 1 < x \leq 2 \\ 0.90 & \text{when } 2 < x \leq 3 \\ 0.95 & \text{when } 3 < x \leq 4 \\ 1.00 & \text{when } 4 < x < +\infty \end{cases}$$

4.

i)

$$F(2.06) = 0.9$$

The probability that the number of passengers is less than 2.06 is equal to 90%.

ii)

$$F(3.9) - F(0.05) = 0.95 - 0.70 = 0.25 = P(0.05 \leq X < 3.9)$$

The probability that the number of passengers is at least 0.05 and less than 3.9 is equal to 25 %.

5. 3.

1.

$$F(t) = P(T < t) \approx P(T \leq t) = 1 - P(T > t) = 1 - e^{-\frac{t}{10}}$$

$$P(5 \leq T < 10) = (1 - e^{-1}) - (1 - e^{-0.5}) = 0.23865122$$

2.

1.

$$P(X > 2) = 0.05 + 0.05 = 0.10$$

2.

$$P(X \geq 2) = 0.10 + 0.05 + 0.05 = 0.20 .$$

3.

$$P(X = 0) = 0.7$$

4.

$$k = 3.$$

5.

0, 0.7, 0.7, 0.9, 1.

5.5.

$$P\left(0 < X < \frac{1}{3}\right) = F\left(\frac{1}{3}\right) - F(0) = \left(\frac{3}{4} \cdot \frac{1}{3} + \frac{3}{4}\right) - \frac{3}{4} = \frac{1}{4}.$$

5.6.

a)

$$\int_{-\infty}^{+\infty} f(x) dx = 1;$$
$$\int_{-3}^0 a \cdot (3+x) dx + \int_0^3 a \cdot (3-x) dx = \left[3ax + a \cdot \frac{x^2}{2}\right]_{-3}^0 + \left[3ax - a \cdot \frac{x^2}{2}\right] = 1;$$
$$-\left(3a \cdot (-3) + a \cdot \frac{9}{2}\right) + \left(9a - 9 \cdot \frac{a}{2}\right) = 1;$$

$$a = \frac{1}{9}.$$

b)

$$F(x) = \int_{-\infty}^x f(t) dt$$

$$F(x) = \begin{cases} 0 & -\infty < x \leq -3 \\ \frac{1}{18}x^2 + \frac{1}{3}x + \frac{1}{2} & -3 < x \leq 0 \\ -\frac{1}{18}x^2 + \frac{1}{3}x + \frac{1}{2} & 0 < x \leq 3 \\ 1 & 3 < x < +\infty \end{cases};$$

c)

$$P\left(\frac{1}{2} \leq X \leq 1\right) = F(1) - F(0.5) = \frac{1}{9} \cdot \int_{0.5}^1 (3-x) dx = \frac{1}{8}.$$

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