## Chapter III

# Probability Algebra

## Solutions

**3. 1.**

1.

$$P(2)=\frac{100}{500}=0.20\,.$$

2.

$$P(1\text{ or }2)=\frac{60}{500}+\frac{100}{500}=0.32\,.$$

3.

$$P(1\text{ or }5)=\frac{60}{500}+\frac{40}{500}=0.20\,.$$

**3. 2.**

Denote by

$I$: "A computer owner shops on the Internet",
$D$: "A computer owner downloads software",

1.

$$P(\bar{I})=1-0.17=0.83\,.$$

2.

$$P(I\cup D)=P(I)+P(D)-P(I\cap D)=0.17+0.33-0.14=0.36.$$

3.

$$P(\bar{I}\cap\bar{D})=1-(I\cup D)=1-0.36=0.64.$$

**3. 4.**

$$P=\frac{1}{3}\cdot\frac{1}{3}\approx 0.11$$

**3. 5.**

Denote by

$V$: „A senior citizen has been victimised“

1.

$$P\left(V\cup B\right)=P(V)+P(B)-P(V\cap B)$$

$$P\left(V\cup B\right)=\frac{106+145+61}{1800}+\frac{145+447}{1800}-\frac{145}{1800}=\frac{759}{1800}=0.421666666\approx 0.42\,.$$

2.

$$P(\bar{V}\cup C)=P(\bar{V})+P(C)-P(\bar{V}\cap C)$$

$$P(\bar{V}\cup C)=\frac{698+447+343}{1800}+\frac{61+343}{1800}-\frac{343}{1800}=\frac{1549}{1800}=0.860555555\approx 0.86..$$

**3. 6.**

$$P=(1-0.78)^2=0.22^2=0.0484.$$

**3. 7.**

$$P=0.528^3\approx 0.15$$

**3. 8.**

Denote by

$J$: "A teenager has a part time job."
$C$: "A teenager plans to attend college."

$$P(J\cap C)=P(C)\cdot P(J)=0.47\cdot 0.78=0.3666$$

(Note: It will be assumed that two events are independent.)

**3. 9.**

a)

$$P(\bar{A})=1-P(A)=1-0.3=0.7$$

b)

$$P(\bar{B})=1-P(B)=1-0.5=0.5$$

c)

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.3+0.5-0.2=0.6$$

d)

$$P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-0.6=0.4$$

**3. 10.**

Let

$A$: „The article is defective."

$B_i$: "the article has been produced on machine $M_i$, $i=1,2,3$."

We have:

$P(B_1)=0.20$, $P(B_2)=0.45$, $P(B_3)=0.35$,

$$P(A/B_1)=0.02,\qquad P(A/B_2)=0.05,\qquad P(A/B_3)=0.03.$$

1.

$$P(\bar{B}_2/A)=1-P(B_2/A)$$

$$=1-\frac{0.45\cdot 0.05}{0.20\cdot 0.02+0.45\cdot 0.05+0.35\cdot 0.03}$$

$$=1-\frac{0.0225}{0.037}\approx 0.39$$

2.

$$P(\bar{B}_1/\bar{A})=1-P(B_1/\bar{A})$$

$$=1-\frac{0.20\cdot 0.98}{0.20\cdot 0.98+0.45\cdot 0.95+0.35\cdot 0.93}$$

$$=1-\frac{0.1960}{0.949}\approx 0.79.$$

**3. 11.**

Let

$A$: „The article is of the highest quality."

$B_i$: "The article has been produced on machine $M_i$, $i=1,2,3$."

We have:

$$P(B_1)=0.30,\qquad P(B_2)=0.45,\qquad P(B_3)=0.25,$$

$$P(A/B_1)=0.95,\qquad P(A/B_2)=0.92,\qquad P(A/B_3)=0.98.$$

1.

$$P(\bar{B}_1/\bar{A})=1-P(B_1/\bar{A})$$

$$=1-\frac{0.30\cdot 0.05}{0.30\cdot 0.05+0.45\cdot 0.08+0.25\cdot 0.02}$$

$$=1-\frac{0.015}{0.056}\approx 0.73$$

2.

$$P(\bar{B}_2/A)=1-P(B_2/A)$$

$$= 1 - \frac{0.45 \cdot 0.92}{0.30 \cdot 0.95 + 0.45 \cdot 0.92 + 0.25 \cdot 0.98}$$

$$= 1 - \frac{0.414}{0.944} \approx 0.56$$

**3. 12.**
Definition of events:

$D$: „An individual favours death penalty",
$M$: „An individual is a man. "

$$P(D/M) = \frac{P(D \cap M)}{P(M)}$$
$$= \frac{0.459}{0.459 + 0.441} = 0.51$$

**3. 13.**
Definition of events:

$A$: „ The defect will not be present in any particular product"
$B$: „The quality control will yield a positive result".

Therefore, we have:

$P(A) = 0.005$, , $P(B/A) = 0.99$, $P(B/\bar{A}) = 0.05$.

1.

$$P(\bar{A}) = 1 - P(A) = 1 - 0.005 = 0995.$$

2.

$$P(\bar{B}/A) = 1 - (B/A) = 1 - 0.99 = 0.01.$$

3.

$$P(\bar{B}/\bar{A}) = 1 - P(B/\bar{A}) = 1 - 0.05 = 0.95.$$

4.

$$P(B) = P(B/A) \cdot P(A) + P(B/\bar{A}) \cdot P(\bar{A})$$

$$= 0.99 \cdot 0.005 + 0.05 \cdot 0.995 = 0.0547.$$

5.

$$P(\bar{B}) = P(\bar{B}/A) \cdot P(A) + P(\bar{B}/\bar{A}) \cdot P(\bar{A})$$

$$= 0.01 \cdot 0.005 + 0.95 \cdot 0.995 = 0.9453$$

$(=1-P(B))$.

6.

$$P(A/B)=\frac{P(B/A)\cdot P(A)}{P(B)}=\frac{0.99\cdot 0.005}{0.0547}=0.0905\,.$$

7.

$$P(\bar{A}/B)=\frac{P(B/\bar{A})\cdot P(\bar{A})}{P(B)}=\frac{0.05\cdot 0.995}{0.0547}=0.9095\quad (=1-P(A/B))$$

8.

$$P(\bar{A}/\bar{B})=\frac{P(\bar{B}/\bar{A})\cdot P(\bar{A})}{P(\bar{B})}=\frac{0.95\cdot 0.995}{0.9453}=0.99995$$

9.

$$P(A/\bar{B})=\frac{P(\bar{B}/A)\cdot P(A)}{P(\bar{B})}=\frac{0.01\cdot 0.005}{0.9453}=0.00005\ (=1-P(\bar{A}/\bar{B})).$$

**3. 14.**

Denote by

$I$: "Increase of capital investment."
$R$: "Rise of structural steel prices."

1.

$$P(\bar{R}/I)=0.10.$$

2.

$$P(R)=P\left(I\cap R\right)+P(\bar{I}\cap R)$$

$$=P(I)\cdot P(R/I)+P(\bar{I}).P(R(R/\bar{I})$$

$$=0.60\cdot 0.90+0.40\cdot 0.40=0.70.$$

3.

$$P(R/I)=\frac{P(I\cap R)}{P(R)}=\frac{P(I)\cdot P(R/I)}{P(I)\cdot P(R/I)+P(\bar{I}).P(R(R/\bar{I})}$$

$$=\frac{0.60\cdot 0.90}{0.60\cdot 0.90+0.40\cdot 0.40}=\frac{0.54}{0.70}\approx 0.77.$$

**3. 15.**
Denote by

$A$: „An item is of high quality“,

$B_i \ (i=1,2,3)$: „An item is produced on machine $i$“.

We have:

$P(B_1)=0.30$, $P(B_2)=0.50$, $P(B_3)=0.20$,

$P(A/B_1)=0.80$, $P(A/B_2)=0.70$, $P(A/B_3)=0.90$.

1.

$$P(A)=\sum_{i=1}^{3} P(B_i)\cdot P(A/B_i)=0.30\cdot 0.80+0.50\cdot 0.70+0.20\cdot 0.90=0.77 .$$

2.

$$P(\bar{A})=1-P(A)=1-0.77=0.23.$$

3.

$$P(B_1/A)=\frac{P(B_1)\cdot P(A/B_1)}{P(A)}=\frac{0.30\cdot 0.8}{0.77}=0.311688311\approx 0.31.$$

4.

$$P(B_3/A)=\frac{P(B_3)\cdot P(A/B_3)}{P(A)}=\frac{0.20\cdot 0.9}{0.77}=0.233766233\approx 0.23.$$

5.

$$P(\bar{B}_3/\bar{A})=1-P(B_3/\bar{A})=1-\frac{P(\bar{B}_3)\cdot P(\bar{A}/B_3)}{P(\bar{A})}=1-\frac{0.20\cdot 0.10}{0.23}=0.913043478\approx 0.91$$

6.

$$P(\bar{B}_2/\bar{A})=1-P(B_2/\bar{A})=1-\frac{P(\bar{B}_2)\cdot P(\bar{A}/B_2)}{P(\bar{A})}=1-\frac{0.50\cdot 0.30}{0.23}=0.347826087\approx 0.35.$$

*(Last revised: 24.11.14)*