

Chapter I

Random Events, Events Algebra

Solutions

1. 1.

$$D = A \cap (B_1 \cup B_2 \cup B_3 \cup B_4) \cap (C_1 \cup C_2)$$

$$\bar{D} = \bar{A} \cup \left(\bar{B}_1 \cap \bar{B}_2 \cap \bar{B}_3 \cap \bar{B}_4 \right) \cup \left(\bar{C}_1 \cap \bar{C}_2 \right)$$

1. 2.

1.

$$A = (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap E_4) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3 \cap \bar{E}_4) \cup (\bar{E}_1 \cap E_2 \cap \bar{E}_3 \cap \bar{E}_4) \\ \cup (E_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4)$$

$$B = E_1 \cup E_2 \cup E_3 \cup E_4$$

$$C = \Omega \setminus (A \cup (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4))$$

$$D = (E_1 \cap E_2 \cap \bar{E}_3 \cap \bar{E}_4) \cup (E_1 \cap \bar{E}_2 \cap E_3 \cap \bar{E}_4) \cup \\ \cup (\bar{E}_1 \cap E_2 \cap E_3 \cap \bar{E}_4) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3 \cap E_4)$$

$$E = (\bar{E}_1 \cap E_2 \cap E_3 \cap E_4) \cup (E_1 \cap \bar{E}_2 \cap E_3 \cap E_4) \cup \\ (E_1 \cap E_2 \cap \bar{E}_3 \cap E_4) \cup (E_1 \cap E_2 \cap E_3 \cap \bar{E}_4)$$

$$F = E_1 \cap E_2 \cap E_3 \cap E_4$$

2.

a) $A \cup B = B$

b) $A \cap B = A$

c) $B \cup C = B$

d) $B \cap C = C$

e) $D \cup E \cup F = C$

f) $B \cap F = F$

1. 3.

$$A = \{6\}, \quad B = \{1, 3, 5\}, \quad C = \{4, 5, 6\}, \quad D = \{1, 2, 3\}, \quad E = \{2, 4\}$$

1.

D .

2.

A, E

3.

Fehler! Textmarke nicht definiert., since:

$$A, B, E \neq \emptyset$$

$$A \cup B \cup E = \Omega$$

$$A \cap B = \emptyset, \quad A \cap E = \emptyset, \quad B \cap E = \emptyset$$

1. 4.

The following table lists all possible cases:

Cases	Events		
1	E_1	E_2	E_3
2	E_1	E_2	\bar{E}_3
3	E_1	\bar{E}_2	E_3
4	\bar{E}_1	E_2	E_3
5	E_1	\bar{E}_2	\bar{E}_3
6	\bar{E}_1	E_2	\bar{E}_3
7	\bar{E}_1	\bar{E}_2	E_3
8	\bar{E}_1	\bar{E}_2	\bar{E}_3

1.

Cases to be considered: 1, 2, 3, 4:

$$A := (E_1 \cap E_2 \cap E_3) \cup (E_1 \cap E_2 \cap \bar{E}_3) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3)$$

2.

Cases to be considered: 2, 3, 4, 5, 6, 7, 8

$$B := \Omega \setminus (E_1 \cap E_2 \cap E_3)$$

3.

Case to be considered: 8

$$C := (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)$$

1. 5.

(See the exercise 1. 4. for a listing of all possible cases)

1.

Cases to be considered: 1, 2, 3, 4:

$$A := (E_1 \cap E_2 \cap E_3) \cup (E_1 \cap E_2 \cap \bar{E}_3) \cup (E_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap E_2 \cap E_3)$$

2.

Case to be considered: 1

$$B := (E_1 \cap E_2 \cap E_3)$$

3.

Cases to be considered: 3, 5, 7, 8

$$C := (E_1 \cap \bar{E}_2 \cap E_3) \cup (E_1 \cap \bar{E}_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3) \cup (\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)$$

1. 6.

Let H_i ($i = 1, 2$) and T_i ($i = 1, 2$) denote the events “head“ and “tail” respectively on the i^{th} coin.

Then the sample space will consist of (H_1, H_2) , (H_1, T_2) , (T_1, H_2) , (T_1, T_2) .

1. 7.

Denote by

E_i , $i = 1, 2$: "Student i suffers from math anxiety."

1.

$$\{(E_1, E_2), (E_1, \bar{E}_2), (\bar{E}_1, E_2), (\bar{E}_1, \bar{E}_2)\}$$

2.

a)

$$\{E_1, E_2\}: \quad \text{simple.}$$

b)

$$\{(E_1, \bar{E}_2), (\bar{E}_1, E_2)\}: \quad \text{compound.}$$

c)

$$\{\bar{E}_1, E_2\}: \quad \text{simple.}$$

d)

$$\{\bar{E}_1, \bar{E}_2\}: \quad \text{simple.}$$

(Last revised: 15.05.09)