# Combinatorics

Permutations are arrangements of objects.

The number of permutations of $n$ objects, *without repetition*:

$$P_n = P_n^n = n!$$

Example 1:
How many nine-digit numbers can be formed with the numbers 1. - 9.

*Solution:*

$$P_9^9 = 9! = 362880$$

A permutation *with repetition* is an arrangement of objects, where some objects are repeated a prescribed number of times. The number of permutations with repetitions of $k_1$ copies of 1, $k_2$ copies of 2, . . . , copies $k_r$ of $r$ is:

$$P_{k_1,\ldots k_r} = \frac{(k_1 + \ldots + k_r)!}{\prod_{i=1}^{r} k_i!}$$

Example 2:
How many eight-digit numbers can be formed with the numbers 2, 2, 2, 3, 3, 3, 4, 4?

*Solution:*

$$P_{3,3,2} = \frac{(3+3+2)!}{3!3!2!} = 560$$

Combinations are selections of objects where the *order does not matter.*

The number of $k-element$ combination of $n$ objects *without repetition* is

$$C_n^k = \binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Example 3:
There are 10 friends in your group but you can invite only 5 to your birthday party.
How many different combinations of friends could you invite?

*Solution:*

$$C_{10}^{5}=\binom{10}{5}=\frac{10!}{(10-5)!5!}=252.$$

Example 4:
Calculate the number of possibilities in Lotto 6/49.

*Solution:*

$$C_{49}^{6}=\binom{49}{6}=\frac{49!}{(49-6)!6!}=13983816$$

Example 5:
Out of 7 consonants and 4 vowels, how many words of 3 different consonants and 2 different vowels can be formed?

*Solution:*

$$C_{7}^{3}\cdot C_{4}^{2}=35\cdot 6=210$$

Example 6:
The dean of science wants to select a committee consisting of mathematicians and physicist to discuss a curriculum. There are 15 mathematicians and 20 physicists at the faculty.
How many possible committees are there, if there must be more mathematicians than physicists (but at least one physicists) on the committee?

*Solution:*
If 5 mathematicians and 3 physicists are chosen for the committee, there are

$$\binom{15}{5}\cdot\binom{20}{3}=3423420$$

possible committees.

If 6 mathematicians and 2 physicists are chosen for the committee, there are

$$\binom{15}{6}\cdot\binom{20}{2}=950950$$

possible committees.

Finally, if 7 mathematicians and 1 physicist are chosen for the committee, there are

$$\binom{15}{7}\cdot\binom{20}{1}=128700$$

possible committees.

Summing the three, we find

$$3423420+950950+128700=4503070$$

possible ways for the dean to select a committee.

The number of $k-element$ combination of $n$ objects *with repetition* is

$$\bar{C}_n^k=\binom{n+k-1}{k}$$

Example 7:
A person is going to a candy shop where there are 8 types of flavors, if this person is only going to buy 3. Calculate the number of possible combinations

*Solution:*

$$\bar{C}_4^3=\binom{8+3-1}{3}=\binom{10}{3}=120.$$

<u>Variations</u> are selections of objects, where the *order matters.*

The number of $k-element$ variation of $n$ objects *without repetition* is

$$V_n^k=\binom{n}{k}\cdot k!$$

Example 8:
How many possibilities are there for choosing three digits from 1, 2, 3, 4?

*Solution:*

$$V_{10}^4=\binom{4}{3}\cdot 3!=24.$$

The number of $k-element$ variation of $n$ objects *with repetition* is

$$\bar{V}_n^k = n^k.$$

Example 9:
In how many ways can a 3digit password be set from the letter *a, b, c, d, e*?

*Solution:*

$$\bar{V}_5^3 = 5^3 = 125.$$

*(Last revised: 13.12.2020)*