

# Kapitel 5

## Lineare Optimierung

### Dualität

### Lösungen

#### 5. 1.

1.  
Sei

$x_1$  : „Menge Obst“

$x_2$  : „Menge milch“

$$z = 3x_1 + 2.5x_2 \rightarrow \text{Min!}$$

$$2x_1 + 4x_2 \geq 40$$

$$4x_1 + 2x_2 \geq 50$$

$$x_1, x_2 \geq 0.$$

2.

$$Z = 40\lambda_1 + 50\lambda_2 \rightarrow \text{Max!}$$

$$2\lambda_1 + 4\lambda_2 \leq 3$$

$$4\lambda_1 + 2\lambda_2 \leq 2.5$$

$$\lambda_1, \lambda_2 \geq 0$$

3.

Normalform:

$$Z = 40\lambda_1 + 50\lambda_2 \rightarrow \text{Max!}$$

$$2\lambda_1 + 4\lambda_2 + \lambda_3 = 3$$

$$4\lambda_1 + 2\lambda_2 + \lambda_4 = 2.5$$

$$\lambda_i \geq 0, i = 1, 2, 3, 4$$

*Simplextableau*

BV	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_0$
$\lambda_3$	2	<b>4</b>	1	0	3
$\lambda_4$	4	2	0	1	$\frac{5}{2}$
Z	-40	<b>-50</b>	0	0	0
$\lambda_2$	$\frac{1}{2}$	1	$\frac{1}{4}$	0	$\frac{3}{4}$
$\lambda_4$	3	0	$-\frac{1}{2}$	1	1
Z	-15	0	$\frac{25}{2}$	0	$\frac{75}{2}$
$\lambda_2$	0	1	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{7}{12}$
$\lambda_1$	1	0	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
Z	0	0	10	5	$\frac{85}{2}$

4.

$$\lambda^* = \left( \frac{1}{3} \quad \frac{7}{12} \quad 0 \quad 0 \right)^T, \quad Z^* = \frac{82}{2} \text{ €}$$

$$x^* = (10 \quad 5 \quad 0 \quad 0)^T, \quad z^* = \frac{82}{2}$$

**5. 2.**

1.

Sei

$x_i, i = 1, 2$  . „Menge des Nahrungsmittels  $N_i$ “

Das Modell:

$$z = 2x_1 + 4x_2 \rightarrow \text{Min!}$$

$$3x_1 + x_2 \geq 15$$

$$x_1 + x_2 \geq 11$$

$$2x_1 + 8x_2 \geq 40$$

$$x_1, x_2 \geq 0$$

2.

Das Dual:

$$Z = 15\lambda_1 + 11\lambda_2 + 40\lambda_3 \rightarrow \text{Max!}$$

$$3\lambda_1 + \lambda_2 + 2\lambda_3 \leq 2$$

$$\lambda_1 + \lambda_2 + 8\lambda_3 \leq 4$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

Normalform:

$$Z = 15\lambda_1 + 11\lambda_2 + 40\lambda_3 \rightarrow \text{Max!}$$

$$3\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4 = 2$$

$$\lambda_1 + \lambda_2 + 8\lambda_3 + \lambda_5 = 4$$

$$\lambda_i \geq 0, i = 1, 2, \dots, 5$$

*Simplextableau*

BV	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\lambda_0$
$\lambda_4$	3	1	2	1	0	2
$\lambda_5$	1	1	8	0	1	4
Z	-15	-11	40	0	0	0
$\lambda_4$	$\frac{11}{4}$	$\frac{3}{4}$	0	1	$-\frac{1}{4}$	1
$\lambda_3$	$\frac{1}{8}$	$\frac{1}{8}$	1	0	$\frac{1}{8}$	$\frac{1}{2}$
Z	-10	-6	0	0	5	20
$\lambda_1$	1	$\frac{3}{11}$	0	$\frac{4}{11}$	$-\frac{1}{11}$	$\frac{4}{11}$
$\lambda_3$	0	$\frac{1}{11}$	1	$-\frac{1}{22}$	$\frac{3}{22}$	$\frac{5}{11}$
Z	0	$-\frac{36}{11}$	0	$\frac{40}{11}$	$\frac{45}{11}$	$\frac{260}{11}$
$\lambda_2$	$\frac{11}{3}$	1	0	$\frac{4}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$
$\lambda_3$	$-\frac{1}{3}$	0	1	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
z	12	0	0	8	3	28

4.

$$\lambda^* = (0 \quad 4/3 \quad 1/3 \quad 0 \quad 0)^T, \quad Z^* = 28 \text{ €}$$

$$x^* = (8 \quad 3 \quad 12 \quad 0 \quad 0)^T, \quad z^* = 28$$

(Letzte Aktualisierung: 16.10.13)