Kapitel 4
Lineare Optimierung

Die Simplexmethode

Lösungen

4. 1.
Sei

x_1: Anzahl der hellen Stühle,
x_2: Anzahl der dunklen Stühle.

Das Modell:

$$
\begin{aligned}
z = 100x_1 + 75x_2 &\to \max! \\
x_1 + x_2 &\le 20 \\
x_1 &\le 12 \\
2x_1 + x_2 &\le 30 \\
x_1, x_2 \ge 0&: \text{ ganz.}
\end{aligned}
$$

Die Normalform:

$$
\begin{aligned}
z = 100x_1 + 75x_2 &\to \max! \\
x_1 + x_2 + x_3 &= 20 \\
x_1 + x_4 &= 12 \\
2x_1 + x_2 + x_5 &= 30 \\
x_i \ge 0,\ i &= 1, 2, \ldots, 5 \\
x_1, x_2&: \text{ ganz.}
\end{aligned}
$$

Simplextableau

BV	x_1	x_2	x_3	x_4	x_5	x_0
x_3	1	1	1	0	0	20
x_4	**1**	0	0	1	0	12
x_5	2	1	0	0	1	30
z	**-100**	-75	0	0	0	0
x_3	0	1	1	-1	0	8
x_1	1	0	0	1	0	12
x_5	0	**1**	0	-2	1	6
z	0	**-75**	0	100	0	1200
x_3	0	0	1	**1**	-1	2
x_1	1	0	0	1	0	12
x_2	0	1	0	-2	1	6
z	0	0	0	**-50**	75	1650
x_4	0	0	1	1	-1	2
x_1	1	0	-1	0	1	10
x_2	0	1	2	0	-1	10
z	0	0	50	0	25	1750

$$x^* = \begin{pmatrix}10 & 10 & 0 & 2 & 0\end{pmatrix}^T, \qquad z^* = 1750 \text{ €}$$

4. 2.
Sei

x_1 : Produktionsmenge nach A,
x_2 : Produktionsmenge nach B,
x_3 : Produktionsmenge nach C.

<u>Das Modell</u>:

$$
\begin{aligned}
z = 20x_1 + 25x_2 + 15x_3 &\to \max!\\
10x_1 + 8x_2 + 5x_3 &\le 500\\
5x_1 + 10x_2 + 10x_3 &\le 400\\
5x_2 + 10x_3 &\le 600\\
x_1, x_2, x_3 &\ge 0.
\end{aligned}
$$

<u>Die Normalform</u>:

$$
\begin{aligned}
&z = 20x_1 + 25x_2 + 15x_3 \to \max!\\
&10x_1 + 8x_2 + 5x_3 + x_4 = 500\\
&5x_1 + 10x_2 + 10x_3 + x_5 = 400\\
&5x_2 + 10x_3 + x_6 = 600\\
&x_i \ge 0,\ i = 1, 2, \ldots, 6.
\end{aligned}
$$

Simplextableau

BV	x_1	x_2	x_3	x_4	x_5	x_6	x_0
x_4	10	8	5	1	0	0	500
x_5	5	**10**	10	0	1	0	400
x_6	0	5	10	0	0	1	600
z	-20	**-25**	-15	0	0	0	0
x_4	6	0	-3	1	$-\frac{4}{5}$	0	180
x_2	$\frac{1}{2}$	1	1	0	$\frac{1}{10}$	0	40
x_6	$-\frac{5}{2}$	0	5	0	$-\frac{1}{2}$	1	400
z	$-\frac{15}{2}$	0	10	0	$\frac{5}{2}$	0	1000
x_1	1	0	$-\frac{1}{2}$	$\frac{1}{6}$	$-\frac{2}{15}$	0	30
x_2	0	1	$\frac{5}{4}$	$-\frac{1}{12}$	$\frac{1}{6}$	0	25
x_6	0	0	$\frac{15}{4}$	$\frac{5}{12}$	$-\frac{5}{6}$	1	475
z	0	0	$\frac{25}{4}$	$\frac{15}{12}$	$\frac{3}{2}$	0	1225

$$x^* = \begin{pmatrix}30 & 25 & 0 & 0 & 0 & 475\end{pmatrix}^T, \qquad z^* = 1225 \text{ €}$$

4. 3.

1.

Sei

$x_i,\ \ i = 1,\ 2:$ Produktionsmenge P_i

Das Modell:

$$z = 3x_1 + 2x_2 \to Max!$$
$$x_1 + 2x_2 \le 10$$
$$x_1 + \ \ x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

2.
<u>Vorbereitung des Modells für die Simplexmethode</u>:

$$z = 3x_1 + 2x_2 \rightarrow Max!$$
$$x_1 + 2x_2 + x_3 \qquad = 10$$
$$x_1 + \ x_2 \qquad - x_4 + x_5 = 2$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$
$$\tilde{z} = -x_5 \rightarrow Max!$$
$$x_5 = 2 - x_1 - x_2 + x_4$$
$$\tilde{z} = -2 + x_1 + x_2 - x_4 \rightarrow Max!$$

Simplextableau

BV	x_1	x_2	x_3	x_4	x_5	x_0
x_3	1	2	1	0	0	10
x_5	**1**	1	0	-1	1	2
z	-3	-2	0	0	0	(0)
$\tilde{z}$	-1	-1	0	1	0	-2
x_3	0	1	1	**1**	-1	8
x_1	1	1	0	-1	1	2
z	0	1	0	-3	3	6
$\tilde{z}$	0	0	0	0	1	0
x_4	0	1	1	**1**		8
x_1	1	2	1	0		10
z	0	4	3	0		30

$x^* = (10 \quad 0 \quad 0 \quad 8)^T$ $\qquad\qquad$ $z^* = 30$ GE

4. 4.

1.

Sei

$x_i, \;\; i=1,\; 2:$ Artikelmenge A_i

Das Modell:

$$
\begin{aligned}
z = 500x_1 + 800x_2 &\to Max!\\
5x_1 + 2x_2 &\le 24\\
x_1 + 5\,x_2 &\le 24\\
6x_1 + 6x_2 &\le 36\\
x_1, x_2 &\ge 0\,.
\end{aligned}
$$

Die Normalform:

$$
\begin{aligned}
z = 500x_1 + 800x_2 &\to \max!\\
5x_1 + 2x_2 + x_3 \qquad\quad &= 24\\
x_1 + 5x_2 \qquad + x_4 \qquad &= 24\\
6x_1 + 6x_2 \qquad\qquad + x_5 &= 36\\
x_i \ge 0,\; i = 1,2,\ldots,5
\end{aligned}
$$

Simplextableau

BV	x_1	x_2	x_3	x_4	x_5	x_0
x_3	5	2	1	0	0	24
x_4	1	**5**	0	1	0	24
x_5	6	6	0	0	1	36
z	-500	**-800**	0	0	0	0
x_3	$\frac{23}{5}$	0	1	$-\frac{2}{5}$	0	$\frac{72}{5}$
x_2	$\frac{1}{5}$	1	0	$\frac{1}{5}$	0	$\frac{24}{5}$
x_5	$\frac{24}{5}$	0	0	$-\frac{6}{5}$	1	$\frac{36}{5}$
z	**-340**	0	0	160	0	3840
x_3	0	0	1	$\frac{3}{4}$	$-\frac{23}{4}$	$\frac{15}{2}$
x_2	0	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{9}{2}$
x_1	1	0	0	$-\frac{1}{4}$	$\frac{5}{24}$	$\frac{3}{2}$
z	0	0	0	$\frac{150}{2}$	$\frac{425}{6}$	4350

$$x^* = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} & \frac{15}{2} & 0 & 0 \end{pmatrix}^T, \qquad z^* = 4350 \text{ €}$$

4. 5.

1.

Sei

$x_i,\ i = 1,\ 2$: Produktionsmenge P_i

<u>Das Modell</u>:

$$z = 2x_1 + 4x_2 \rightarrow Max!$$
$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 7$$
$$x_1 \le 6$$
$$x_1, x_2 \ge 0$$

2.

<u>Normalform</u>:

$$z = 2x_1 + 4x_2 \rightarrow Max!$$

$$x_1 + 2x_2 + x_3 \qquad = 10$$

$$x_1 + x_2 \qquad + x_4 \quad = 7$$

$$x_1 \qquad\qquad\quad + x_5 = 6$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Simplextableau

BV	x_1	x_2	x_3	x_4	x_5	x_0
x_3	1	2	1	0	0	10
x_4	1	1	0	1	0	7
x_5	**1**	0	0	1	1	6
z	-2	-4	0	0	0	0
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	5
x_4	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	2
x_5	1	0	0	0	1	6
z	**0**	0	2	0	0	20

Die Optimallösung ist mehrdeutig:

BV	x_1	x_2	x_3	x_4	x_5	x_0
x_2	0	1	1	-1	0	4
x_1	1	0	-1	2	0	2
x_5	0	0	1	-2	1	2
z	0	0	2	0	0	20

$$x^* = \alpha \cdot x^{*,1} + (1-\alpha) \cdot x^{*,2}, \qquad 0 \leq \alpha \leq 1, \qquad z^* = 20$$

mit

$$x^{*,1} = (0,\ 5,\ 0,\ 2,\ 6)^T, \qquad x^{*,2} = (2,\ 4,\ 0,\ 0,\ 2)^T.$$

(Letzte Aktualisierung: 16.11.18)