Chapter 11

Decision Theory

Solutions

11. 1.
1.

Payoff Table

	State of Nature			
Action	s_1 : demand 10 $p_1 = 0.2$	s_1 : demand 30 $p_2 = 0.4$	s_1 : demand 50 $p_3 = 0.3$	s_1 : demand 70 $p_4 = 0.1$
a_1 : buy 20	50	550	450	350
a_2 : buy 40	-330	770	1270	1170
a_3 : buy 60	-650	450	1550	2050
a_4 : buy 80	-970	130	1230	2330

(*Explanation:*

1. Demand is 50, buy 60:
   The store buys 60 at $65 each for $3900. That is -$3900 since that is the money it spends. Now, it sells 50 bicycles at $100 each for $5000. The store has 10 bicycles left over at the end of month, and it sells those at $45 each of $450. That makes

   5000 + 450 -3900 = $1550

2. Demand is 70, buy 40:
   The store buys 40 at $67 each for $2680. That is -$2680 since that is money it spends. Now it sells 40 bicycles at $100 each for $4000. The other customers that want a bicycle und cannot get one cost the store $5 in goodwill for each of them. That is 30 customers at -$5 each or -$150. That makes:

   4000 – 2680 -150 =$1170.)

i. Maximax

v_{ij}		s_1 10	s_2 30	s_3 50	s_4 70	$\max_j v_{ij}$
a_1	20	50	550	450	350	550
a_2	40	-330	770	1270	1170	1270
a_3	60	-650	450	1550	2050	2050
a_4	**80**	-970	130	1230	2330	**2330**

∴ The store buys 80.

ii. Maximin

v_{ij}		s_1 10	s_2 30	s_3 50	s_4 70	$\min_j v_{ij}$
a_1	**20**	50	550	450	350	**50**
a_2	40	-330	770	1270	1170	-330
a_3	60	-650	450	1550	2050	-650
a_4	80	-970	130	1230	2330	-970

∴ The store buys 20.

iii. Minimax

w_{ij}		s_1 10	s_2 30	s_3 50	s_4 70	$\max_j w_{ij}$
a_1	**20**	2280	1680	1880	1980	**2280**
a_2	40	2660	1560	1060	1160	2660
a_3	6**0**	2980	1880	780	280	2980
a_4	80	3300	2200	1100	0	3300

∴ The store buys 20.

iv. Minimax regret

r_{ij}		s_1 10	s_2 30	s_3 50	s_4 70	$\max_j r_{ij}$
a_1	20	0	220	1100	1980	1980
a_2	40	380	0	280	1160	1160
a_3	**60**	700	320	0	280	**700**
a_4	80	1020	640	320	2330	2330

$\therefore$ The store buys 60.

v. Expected value

		s_1	s_2	s_3	s_4	
		0.2	0.4	0.3	0.1	
		10	30	50	70	μ_i
a_1	20	50	550	450	350	400
a_2	**40**	-330	770	1270	1170	**740**
a_3	60	-650	450	1550	2050	720
a_4	80	-970	130	1230	2330	460

$\therefore$ The store buys 40.

11. 2.
1.

Utility Matrix

	$p_1 = 0.1$	$p_2 = 0.5$	$p_3 = 0.4$	
Invest:	Strong market	Fair market	Poor market	μ_i
\$8000	1593.6	399.6	-801.6	38.52
\$4000	798.4	199.9	-400.4	19.63
\$2000	399.6	99.98	-200.1	9.91
\$1000	199.9	49.99	-100.03	4.98

$$Max\{38.74,\ 19.63,\ 9.91,\ 4.98\}=38.74$$

Therefore, Mrs. Greedy should invest \$8000.

2.

$$u'(v) = 2 - \frac{2v}{100000}, \qquad u''(v) = -\frac{1}{50000} < 0.$$

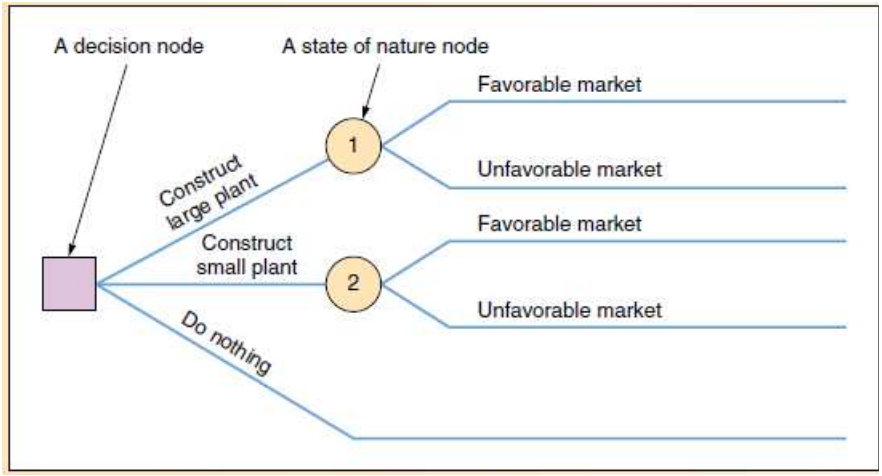
Therefore, Mrs. Greedy is risk averse.

11. 3.

Probabilities	0.40	0.35	0.25			
	s_1	s_2	s_3	$\mu(a_i)$	$\sigma(a_i)$	$\Phi_i(\mu_i, \sigma_i)$
a_1	-50	0	80	0.00	50.99	-2.5495
a_2	-10	30	35	15.25	20.70	29.4650
a_3	60	45	-30	32.25	36.52	62.6740
a_4	80	40	-45	34.75	49.18	**67.0410**

$\therefore$ The best decision would be to order a lot.

11. 4.



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