Chapter 10

Queuing Systems

Solutions

10. 1.

$$\lambda = 15/hr, \quad \mu = \frac{60}{3} = 20/hr$$

1.
a)

$$P(0) = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{20} = 0.25$$

b)

$$1 - P(0) = 1 - 0.25 = 0.75$$

c).

$$P(x > 1) = 1 - P(x \le 0) = 1 - (P(x = 0) + P(x = 1))$$

$$= 1 - \left(\left(\frac{\lambda}{\mu}\right)^0 \left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^1 \left(1 - \frac{\lambda}{\mu}\right) \right)$$

$$= 1 - \left(0.75^0 \cdot 0.25 + 0.75 \cdot 0.25\right) = 0.5625$$

d)

$$\bar{n}_q = \frac{\lambda}{\mu} \cdot \bar{n} = \frac{\lambda}{\mu} \cdot \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} = 0.75 \cdot \frac{0.75}{0.25} = 2.25$$

e)

$$W = \frac{1}{\mu - \lambda} = \frac{1}{20 - 15} = 0.2$$

$$W_q = \frac{\lambda}{\mu} \cdot W = 0.75 \cdot 0.20 = 0.15 \ hr / customer$$

$$Cost = 0.15 \cdot 60 \cdot 3 = 27 \text{ €} / customer$$

2.

$$\lambda = 15/hr, \quad \mu = \frac{60}{2} = 30/hr$$

Before	After
$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{15^2}{20(20-15)} = 2.25$ $Cost = 2.25 \cdot 60 \cdot 3 = 405$ €/*hr*	$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{15^2}{30(30-15)} = 0.5$ $Cost = 0.5 \cdot 60 \cdot 3 = 90$ €/*hr*
$Difference = 405 - 90 = 315$ € / *hr*	

2.

We have:

$$\mu = 3, \quad \lambda = 2$$

1.

$$\bar{n} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = 2 \text{ cars}$$

2.

$$W = \frac{1}{3-2} = 1 \text{ hour}$$

3.

$$\bar{n}_q = \frac{2}{3} \cdot 2 = \frac{4}{3} \approx 1.33 \text{ cars}$$

4.

$$W_q = \frac{2}{3} \cdot 1 = \frac{2}{3} \text{ hour (40 minutes)}$$

5.

$$\rho = \frac{2}{3} (\textit{i.e.} \text{ about66.6\%of the time the mechanic is busy})$$

6.

$$P(0) = 1 - \frac{2}{3} = \frac{1}{3} \approx 0.33$$

7.

$$P(1) = \frac{2}{3} \cdot \left(1 - \frac{2}{3}\right) = \frac{2}{9} \approx 0.22$$

(Last updated: 25.08.2014)