Chapter 9

Input-Output Analysis

Solutions

9. 1.

	Manufacture	Non-Manufacture	Consumption	Government	Export	Total
Manufacture	18	18	40	14	30	120
Non-Manufacture	20	37	43	7	18	125
Households	56	52	16	9	27	160
Government	6	7	20	22	0	55
Import	20	11	41	3	5	80
Total	120	125	160	55	80	540

1.

$$y = (84 \quad 68)^T.$$

2.

$$A = \begin{pmatrix} \frac{18}{120} & \frac{18}{125} \\ \frac{20}{120} & \frac{37}{125} \end{pmatrix} = \begin{pmatrix} 0.150 & 0.144 \\ 0.167 & 0.296 \end{pmatrix}.$$

$a_{12} = 0.144$ is the proportion of the sector “manufacture” in one unit of the product of the sector “non-manufacture”.

3.

$$I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.150 & 0.144 \\ 0.167 & 0.296 \end{pmatrix} = \begin{pmatrix} 0.850 & -0.144 \\ -0.167 & 0.704 \end{pmatrix}.$$

4.

$$(I - A)^{-1} = \begin{pmatrix} 0.850 & -0.144 \\ -0.167 & 0.704 \end{pmatrix}^{-1} = \begin{pmatrix} 1.2257 & 0.2507 \\ 0.2908 & 1.4799 \end{pmatrix}.$$

If only sector “non-manufacture” increases its final demand by 1 unit, the two sectors should produce additionally 0.2507 and 1.4799.

5.

$$(I-A)^{-1}-(I+A)=\begin{pmatrix}1.2257 & 0.2507\\ 0.2908 & 1.4799\end{pmatrix}-\begin{pmatrix}1.150 & 0.144\\ 0.167 & 1.296\end{pmatrix}=\begin{pmatrix}0.0757 & 0.1067\\ 0.1238 & 0.1839\end{pmatrix}.$$

6.

$$y=(I-A)x=\begin{pmatrix}0.850 & -0.144\\ -0.167 & 0.704\end{pmatrix}\begin{pmatrix}130\\ 120\end{pmatrix}=\begin{pmatrix}93.22\\ 62.77\end{pmatrix}$$

7.

$$x=(I-A)^{-1}y=\begin{pmatrix}1.2257 & 0.2507\\ 0.2908 & 1.4799\end{pmatrix}\begin{pmatrix}90\\ 70\end{pmatrix}=\begin{pmatrix}127.862\\ 129.765\end{pmatrix},$$

8.

$$\begin{pmatrix}0.850 & -0.144\\ -0.167 & 0.704\end{pmatrix}\begin{pmatrix}x_1\\ 130\end{pmatrix}=\begin{pmatrix}80\\ y_2\end{pmatrix}\quad\Rightarrow\quad x_1=116.141,\quad y_2=72.124,$$

9.

$$\tilde{A}=\begin{pmatrix}\frac{56}{120} & \frac{52}{125}\\ \frac{6}{120} & \frac{7}{125}\\ \frac{20}{120} & \frac{11}{125}\end{pmatrix}=\begin{pmatrix}0.467 & 0.416\\ 0.050 & 0.056\\ 0.167 & 0.088\end{pmatrix}.$$

For each unit of the production of sector „manufacture“ 0.167 units should be imported.

10.

$$\tilde{B}=\tilde{A}\cdot(I-A)^{-1}=\begin{pmatrix}0.467 & 0.416\\ 0.050 & 0.056\\ 0.167 & 0.088\end{pmatrix}\cdot\begin{pmatrix}1.2257 & 0.2507\\ 0.2908 & 1.4799\end{pmatrix}=\begin{pmatrix}0.693 & 0.733\\ 0.078 & 0.095\\ 0.230 & 0.172\end{pmatrix}$$

The elements in the first column of this vector give the additional amount of primary inputs if only the final demand of sector “manufacture” increases by one unit.

11.

$$z=\tilde{B}\cdot y=\begin{pmatrix}0.693 & 0.733\\ 0.078 & 0.095\\ 0.230 & 0.172\end{pmatrix}\begin{pmatrix}90\\ 70\end{pmatrix}=\begin{pmatrix}113.68\\ 13.67\\ 32.74\end{pmatrix}.$$

9. 2

1.

$$x = \begin{pmatrix} 20 & 10 & 10 \end{pmatrix}^T$$

2.

$$y = \begin{pmatrix} 3 & 9 & 6 \end{pmatrix}^T$$

3.

$$A = \begin{pmatrix} \frac{8}{20} & \frac{5}{10} & \frac{4}{10} \\ \frac{0}{20} & \frac{1}{10} & \frac{0}{10} \\ \frac{2}{20} & \frac{0}{10} & \frac{2}{10} \end{pmatrix} = \begin{pmatrix} 0.4 & 0.5 & 0.4 \\ 0.0 & 0.1 & 0.0 \\ 0.1 & 0.0 & 0.2 \end{pmatrix}.$$

4.

$$I - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.4 & 0.5 & 0.4 \\ 0.0 & 0.1 & 0.0 \\ 0.1 & 0.0 & 0.2 \end{pmatrix} = \begin{pmatrix} 0.6 & -0.5 & -0.4 \\ 0.0 & 0.9 & 0.0 \\ -0.1 & 0.0 & 0.8 \end{pmatrix}$$

$$\begin{pmatrix} 0.6 & -0.5 & -0.4 \\ 0.0 & 0.9 & 0.0 \\ -0.1 & 0.0 & 0.8 \end{pmatrix} \begin{pmatrix} a & 1.01 & 0.91 \\ 0.00 & 1.11 & 0.00 \\ 0.23 & 0.13 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$0.6a - 0.4 \cdot 0.23 = 1 \quad \Rightarrow \quad a = 1.82$$

$$-0.1 \cdot 0.91 + 0.8b = 1 \quad \Rightarrow \quad b = 1.36$$

Incraesing only the final demand of the first sector will require an additional production of 1.82 units by the first sector, 0 units of the second and 0.23 units of the third sector.

5.

$$y = (I - A)x = \begin{pmatrix} 0.6 & -0.5 & -0.4 \\ 0.0 & 0.9 & 0.0 \\ -0.1 & 0.0 & 0.8 \end{pmatrix} \begin{pmatrix} 24 \\ 8 \\ 9 \end{pmatrix} = \begin{pmatrix} 6.8 \\ 7.2 \\ 4.8 \end{pmatrix}$$

6.

$$x = (I - A)^{-1} y = \begin{pmatrix} 1.82 & 1.01 & 0.91 \\ 0.00 & 1.11 & 0.00 \\ 0.23 & 0.13 & 1.36 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \\ 6 \end{pmatrix} = \begin{pmatrix} 20.82 \\ 8.88 \\ 10.12 \end{pmatrix}.$$

7.

$$\tilde{A} = \begin{pmatrix} \frac{2}{20} & \frac{2}{10} & \frac{2}{10} \\ \frac{3}{20} & \frac{1}{10} & \frac{1}{10} \\ \frac{5}{20} & \frac{1}{10} & \frac{1}{10} \end{pmatrix} = \begin{pmatrix} 0.10 & 0.20 & 0.20 \\ 0.15 & 0.10 & 0.10 \\ 0.25 & 0.10 & 0.10 \end{pmatrix}.$$

The elements of this matrix give the amount of primary inputs per production unit.

8.

$$\tilde{B} = \tilde{A} \cdot (I - A)^{-1} = \begin{pmatrix} 0.10 & 0.20 & 0.20 \\ 0.15 & 0.10 & 0.10 \\ 0.25 & 0.10 & 0.10 \end{pmatrix} \cdot \begin{pmatrix} 1.82 & 1.01 & 0.91 \\ 0.00 & 1.11 & 0.00 \\ 0.23 & 0.13 & 1.36 \end{pmatrix} = \begin{pmatrix} 0.228 & 0.149 & 0.363 \\ 0.296 & 0.176 & 0.273 \\ 0.478 & 0.277 & 0.364 \end{pmatrix}$$

Incraesing only the final demand of the second sector will require an additional production of 0.149 units by the first sector, 0.176 units of the second and 0.277 units of the third sector.

9.

$$z = \tilde{B} \cdot y = \begin{pmatrix} 0.228 & 0.149 & 0.363 \\ 0.296 & 0.176 & 0.273 \\ 0.478 & 0.277 & 0.364 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 3.556 \\ 3.684 \\ 5.584 \end{pmatrix}.$$

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