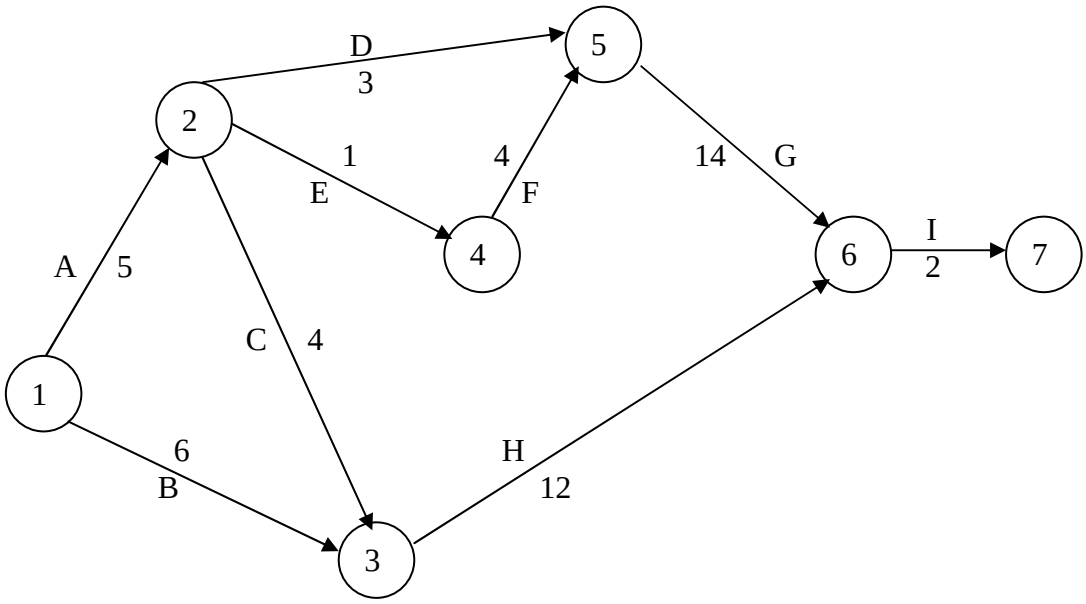
Chapter 8

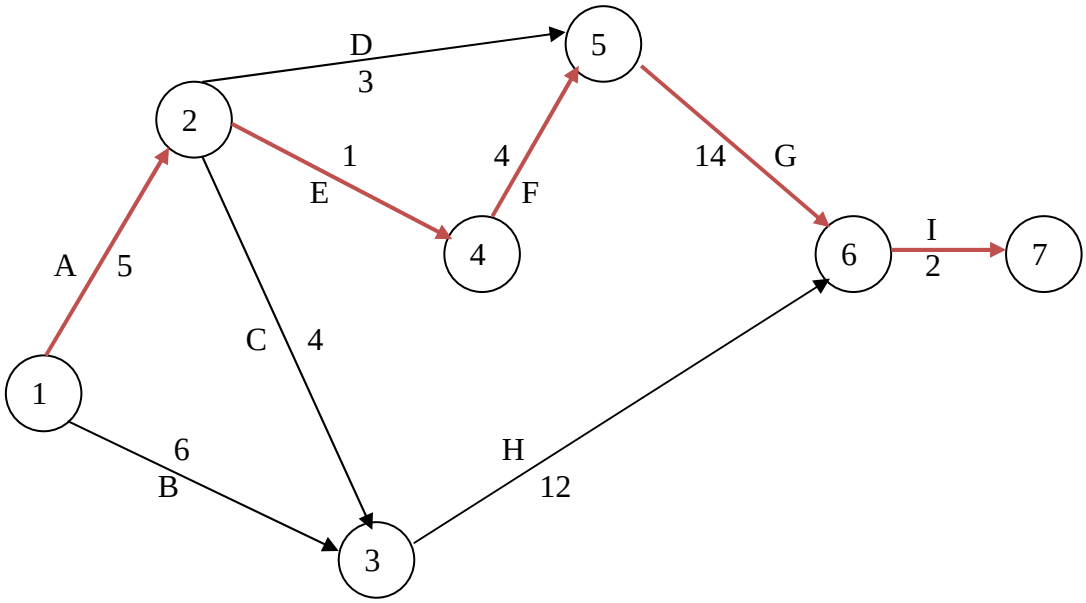
Project Management

Solutions

8. 1.
1.



T^e	Events	1	2	3	4	5	6	7
0	1		5	6				
5	2			4	1	3		
9	3						12	
6	4					4		
10	5						14	
24	6							2
26	7							
T^l		0	5	12	6	10	24	26



The project can be completed in 26 weeks if the individual activities are completed on schedule.

2.
See the following table:

i	j	Activity	t_{ij}	$\Delta^T t_{ij} = T_j^l - T_i^e - t_{ij}$	Critical Path?
1	2	A	5	0	Yes
1	3	B	6	6	No
2	3	C	4	3	No
2	4	E	1	0	Yes
2	5	D	3	2	No
3	6	H	12	3	No
4	5	F	4	0	Yes
5	6	G	14	0	Yes
6	7	I	2	0	Yes

3.

A, E, F, G, I

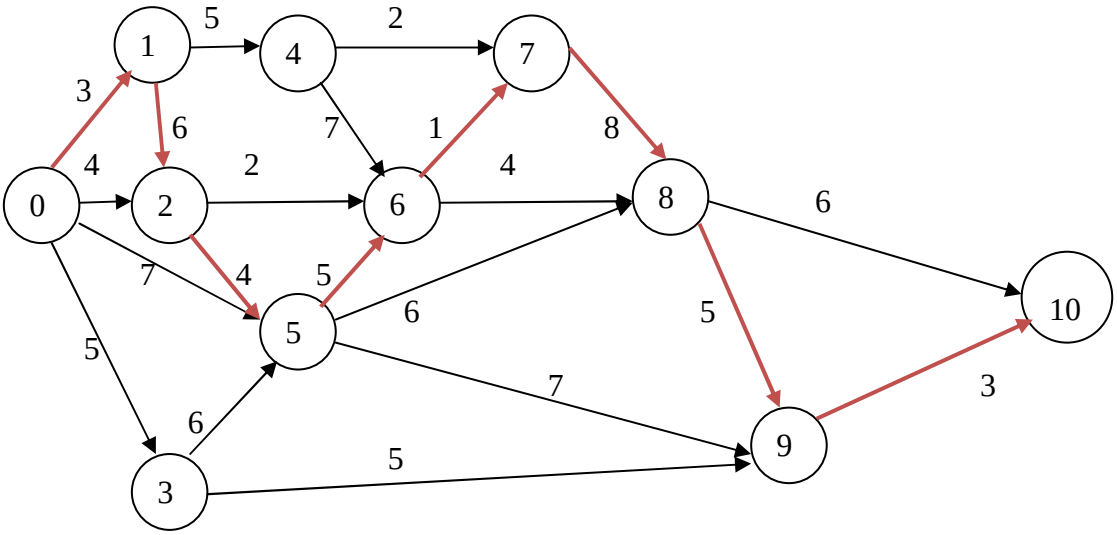
4.
It is the critical path that determines the project completion time. Changing time of the non-critical activities within the permissible range will not affect the project completion time. But changing time of the critical activities may cause the project completion time to change.

8. 2.

1.

0	t_{ij}	T^e	T^l
0	$t_{01}=3\ t_{02}=4\ t_{03}=5\ t_{05}=7$	0	$\min\{13-7,\ 7-5,\ 9-4,\ 3-3\}=0$
1	$t_{12}=6\ t_{14}=5$	3	$\min\{11-5,\ 9-6\}=3$
2	$t_{25}=4\ t_{26}=2$	$\max\{0+4,\ 3+6\}=9$	$\min\{18-2,\ 13-4\}=9$
3	$t_{35}=6\ t_{39}=5$	5	$\min\{32-5,\ 13-6\}=7$
4	$t_{46}=7\ t_{47}=2$	8	$\min\{19-2,\ 18-7\}=11$
5	$t_{56}=5\ t_{58}=6\ t_{59}=7$	$\max\{7,\ 9+4,\ 5+6\}=13$	$\min\{32-7,\ 27-6,\ 18-5\}=13$
6	$t_{67}=1\ t_{68}=4$	$\max\{9+2,\ 8+7,\ 13+5\}=18$	$\min\{27-4,\ 19-1\}=18$
7	$t_{78}=8$	$\max\{8+2,\ 18+1\}=19$	$\min\{27-8\}=19$
8	$t_{89}=5\ t_{8,10}=6$	$\max\{13+6,\ 18+4,\ 19+8\}=27$	$\min\{35-6,\ 32-5\}=27$
9	$t_{9,10}=3$	$\max\{5+5,\ 13+7,\ 27+5\}=32$	$\min\{35-3\}=32$
10		$\max\{27+6,\ 32+3\}=35$	35

T^e	Events	0	1	2	3	4	5	6	7	8	9	10
0	0		3	4	5		7					
3	1			6		5						
9	2						4	2				
5	3						6				5	
8	4							7	2			
13	5							5		6	7	
18	6								1	4		
19	7									8		
27	8										5	6
32	9											3
35	10											
T^l		0	3	9	7	11	13	18	19	27	32	35


0
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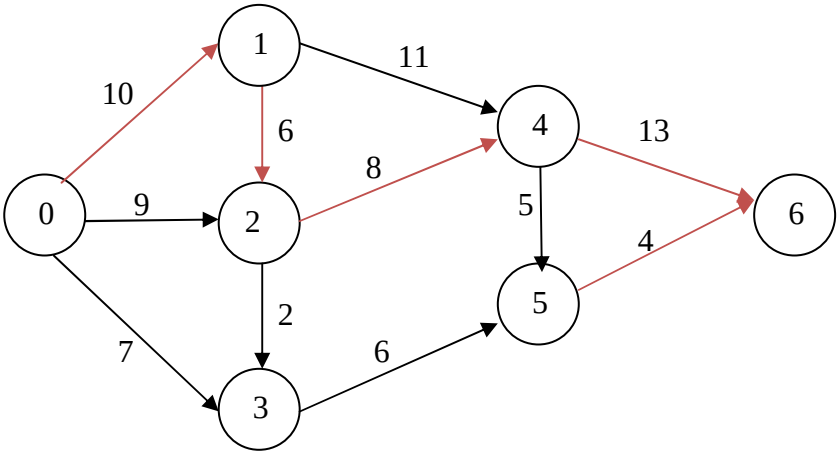
2.

i	j	t_{ij}	T_i^e	T_j^l	$T_i^e + t_{ij}$	$\Delta^T t_{ij}$	$\Delta^F t_{ij}$	$\Delta^C t_{ij}$	$\Delta^I t_{ij}$
1	2	3	4	5	6	7	8	9	10
0	1	3	0	3	3	0	0	0	0
0	2	4	0	9	4	5	5	0	5
0	3	5	0	7	5	2	0	2	0
0	5	7	0	13	7	6	6	0	6
1	2	6	3	19	9	0	0	0	0
1	4	5	3	11	8	3	0	3	0
2	5	4	9	13	13	0	0	0	0
2	6	2	9	18	11	7	7	0	7
3	5	6	5	13	11	2	2	0	0
3	9	5	5	32	10	22	22	0	20
4	6	7	8	18	15	3	3	0	0
4	7	2	8	19	10	9	9	0	6
5	6	5	13	18	18	0	0	0	0
5	8	6	13	27	19	8	8	0	8
5	9	7	13	32	20	12	12	0	12
6	7	1	18	19	19	0	0	0	0
6	8	4	18	27	22	5	5	0	5
7	8	8	19	27	27	0	0	0	0
8	9	5	27	32	32	0	0	0	0
8	10	6	27	35	33	2	2	0	2
9	10	3	32	35	35	0	0	0	0

8. 3.

T^e	i \ j	0	1	2	3	4	5	6
0	0		10	9	7			
10	1			6		11		
16	2				2	8		
18	3						6	
24	4						5	13
29	5							4
37	6							
T^l		0	10	16	27	24	33	37



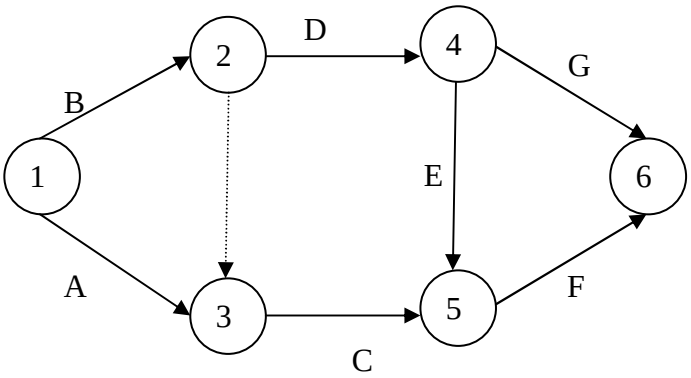
The project will be completed in 37 weeks.

2.

i	j	t_{ij}	T_i^e	T_j^l	$T_i^e + t_{ij}$	$\Delta^T t_{ij}$	$\Delta^F t_{ij}$	$\Delta^C t_{ij}$	$\Delta^I t_{ij}$
0	1	10	0	10	10	0	0	0	0
0	2	9	0	16	9	7	7	0	7
0	3	7	0	27	7	20	11	9	11
1	2	6	10	16	16	0	0	0	0
1	4	11	10	24	21	3	3	0	3
2	3	2	16	27	18	9	0	9	0
2	4	8	16	24	24	0	0	0	0
3	5	6	18	33	24	9	5	4	0
4	5	5	24	33	29	4	0	4	0
4	6	13	24	37	37	0	0	0	0
5	6	4	29	37	33	4	4	0	0

8. 4

1.

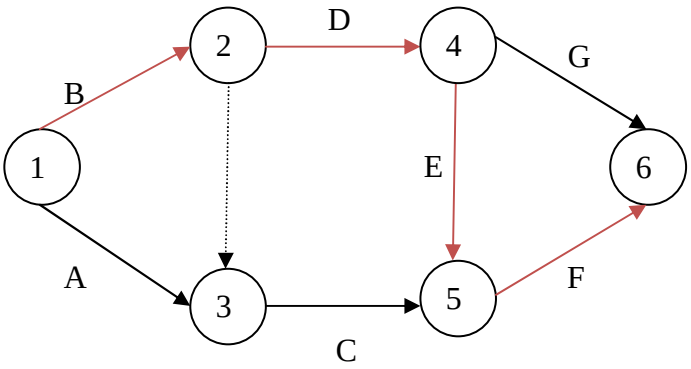


2.

T^e	Events	1	2	3	4	5	6
0	1		4	3			
4	2			0	6		
3	3					5	
10	4					7	9
17	5						8
25	6						
T^l		0	4	12	10	17	25

a)

B → D → E → F



b)

4 + 6 + 7 + 8 = 25 days.

3.

i	j	t_{ij}	$\Delta^T t_{ij} = T_j^l - T_i^e - t_{ij}$	Critical	$\Delta^F t_{ij} = T_j^e - T_i^e - t_{ij}$
1	2	4	0	Yes	0
1	3	3	9	No	0
2	4	6	0	Yes	0
3	5	5	9	No	9
4	5	7	0	Yes	0
4	6	9	0	Yes	6
5	6	8	0	Yes	0

A slack signifies the time which an activity can be delayed without delaying the project. A zero slack for an activity indicates that it cannot be delayed without delaying the project and hence it is called a critical activity. On the other hand, a positive slack for an activity means that it can be delayed by the length of the slack without delaying the project and hence it is called a non-critical activity.
It is the amount of time a single activity can be delayed without delaying the early start of any successor activity.

4.
a)

i	j	a_{ij}	m_{ij}	b_{ij}	$\bar{t}_{ij}$	$\sigma^2_{t_{ij}}$
1	2	3	4	5	4	0.1111
1	3	1	3	5	3	0.4444
2	4	3	5	7	5	0.4444
3	5	4	5	6	5	0.1111
4	5	5	6	13	7	1.7778
4	6	6	8	10	8	0.4444
5	6	4	7	10	7	1.0000

$\bar{T}^e$	$\sigma^2_{T^e}$		1	2	3	4	5	6
0	0.0000	1		4 0.1111	3 0.4444			
4	0.1111	2			0 0	5 0.4444		
4	0.4444	3					5 0.1111	
9	0.5555	4					7 1.7778	8 0.4444
16	2.3333	5						7 1.0000
23	3.3333	6						
	$\bar{T}^l$	$\bar{T}^l$	0	4	11	9	16	23
		$\sigma^2_{T^l}$	3.3333	3.2222	1.1111	2.7778	1.0000	0

Critical path: $\mathbf{B} \rightarrow \mathbf{D} \rightarrow \mathbf{E} \rightarrow \mathbf{F}$

Expected completion time $= \bar{t}_{12} + \bar{t}_{24} + \bar{t}_{45} + \bar{t}_{56} = 4 + 5 + 7 + 7 = 23$

Project variance $= \sigma^2_{T^e_{12}} + \sigma^2_{T^e_{24}} + \sigma^2_{T^e_{45}} + \sigma^2_{T^e_{56}}$

$= 0.1111 + 0.4444 + 1.7778 + 1.0000 = 3.3333$

b)
Let T be the completion time:

i) $$P(T \le 20) \approx P(T < 20) = \Theta\left(\frac{20-23}{1.83}\right) = \Theta(-1.64) = 1 - \Theta(1.64)$$
$$= 1 - 0.948449 = 0.051551$$

ii) $$P(T \le 25) \approx P(T < 25) = \Theta\left(\frac{25-23}{1.83}\right) = \Theta(1.09) = 0.862143.$$

8. 5.
1.

i	j	a_{ij}	m_{ij}	b_{ij}	$\bar{t}_{ij}$	$\sigma_{t_{ij}}$	$\sigma^2_{t_{ij}}$
0	1	3	5	8	5.17	0.83	0.69
1	2	12	13	16	13.33	0.67	0.44
1	3	8	11	15	11.17	1.17	1.37
2	5	13	15	21	15.67	1.33	1.78
3	4	6	8	10	8.00	0.67	0.44
3	5	7	8	10	8.17	0.50	0.25
4	8	6	7	9	7.17	0.50	0.25
5	6	13	14	16	14.17	0.50	0.25
5	7	3	4	5	4.00	0.33	0.11
5	8	10	14	19	14.17	1.50	2.25
6	7	7	9	12	9.17	0.83	0.69
7	9	4	5	6	5.00	0.33	0.11
8	9	15	16	18	16.17	0.50	0.25

$\sigma^2_{T_i^e}$	$\bar{T}_i^e$		0	1	2	3	4	5	6	7	8	9
0.00	0.00	0		5.17 0.69								
0.69	5.17	1			13.33 0.44	11.17 1.37						
1.13	18.50	2						15.67 1.78				
2.06	16.34	3					8.00 0.44	8.17 0.25				
2.50	24.34	4									7.17 0.25	
2.91	34.17	5							14.17 0.25	4.00 0.11	14.17 2.25	
3.16	48.34	6								9.17 0.69		
3.85	57.51	7										5.00 0.11
5.16	48.34	8										16.17 0.25
5.41	64.51	9										
		$\bar{T}_j^l$	0.00	5.17	18.50	26.00	41.17	34.17	50.34	59.51	48.34	64.51
		$\sigma^2_{T_j^l}$	5,41	4.72	4.28	2.75	0.50	2.50	0.80	0.11	0.25	0.00

Critical path: $\mathbf{0 \to 1 \to 2 \to 5 \to 8 \to 9}$

2

Expected completion time: $= \bar{t}_{01} + \bar{t}_{12} + \bar{t}_{25} + \bar{t}_{58} + \bar{t}_{89}$

$$= 5.17 + 13.33 + 15.67 + 14.17 + 16.17 = 64.51$$

Project variance $= \sigma^2_{T^e_{01}} + \sigma^2_{T^e_{12}} + \sigma^2_{T^e_{25}} + \sigma^2_{T^e_{58}} + \sigma^2_{T^e_{89}}$

$$= 0.69 + 0.44 + 1.78 + 2.25 + 0.25 = 5.41$$

$$P(T < 65) = \Phi\left(\frac{65 - 64.51}{\sqrt{5.41}}\right) = \Phi(0.21) = 0.5832$$

3.

i	$\bar{T}_i^e$	$\bar{T}_i^l$	$\sigma^2_{T_i^e}$	$\sigma^2_{T_i^l}$	$\sqrt{\sigma^2_{T_i^e}+\sigma^2_{T_i^l}}$	$\bar{T}_i^l-\bar{T}_i^e$	$\frac{\bar{T}_i^l-\bar{T}_i^e}{\sqrt{\sigma^2_{T_i^e}+\sigma^2_{T_i^l}}}$	$P\left[(\bar{T}_i^l-\bar{T}_i^e)\le 0\right]$
0	0.00	0.00	0.00	5.41	2.33	0.00	0.00	50%
1	5.17	5.17	0.69	4.72	2.33	0.00	0.00	50%
2	18.50	18.50	1.13	4.28	2.33	0.00	0.00	50%
3	16.34	26.00	2.06	2.75	2.19	9.66	4.41	0.0%
4	24.34	41.17	2.50	0.50	1.73	16.83	9.72	0.0%
5	34.17	34.17	2.91	2.50	2.33	0.00	0.00	50%
6	48.34	50.34	3.16	0.80	1.99	2.00	1.01	15.6%
7	57.51	59.51	3.85	0.11	1.99	2.00	1.01	15.6%
8	48.34	48.34	5.16	0.25	2.33	0.00	0.00	50%
9	64.51	64.51	5.41	0.00	2.33	0.00	0.00	50%

(Last updated: 16.10.2015)