Chapter 8

Project Management

CPM/PERT

<u>R. 8. 1.</u>
The CPM and PERT are widely used techniques for the purpose of planning, scheduling and controlling the process and completion of large and complex projects.

<u>R. 8. 2</u> (*Critical Path Method - CPM*)
The *Critical Path Method* (*CPM*) is one of several related techniques for doing project planning. It is for projects that are made up of individual "*activities*". If some of the activities require other activities to finish before they can start, then the project becomes a complex web of activities.
CPM was developed by Du Pont and the emphasis was on the trade-off between the cost of the project and its overall completion time.
CPM and PERT (see later) were independently developed in the late 1950s. They have been among the most widely used OR techniques. These methods have been used for a variety of projects, including the following types:

1. Construction of a new plant
2. Research and development of a new product
3. NASA space exploration projects
4. Movie productions
5. Building a ship
6. Government-sponsored projects for developing a new weapon system
7. Relocation of a major facility
8. Maintenance of a nuclear reactor
9. Installation of a management of information system
10. Conducting an advertising campaign.

CPM is mainly used for the jobs of repetitive nature where the activity time estimates can be predicted with considerable certainty due to the existence of past experience.

CPM can help us answer following questions:

- How long a project will take to complete?
- Are the activities on schedule?
- Is the project within budget?
- Which activities are *critical*, meaning that they have to be done on time or else the whole project will take longer?
- How can the project be finished early at the least cost?

<u>ALG. 8. 1</u> (*CPM*)

Step 1 (List the activities):

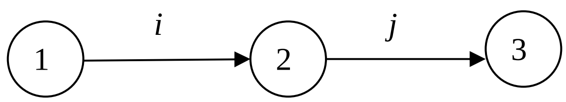
Set up a table giving following informations:

- The name of activities and their description
- The required predecessors
- The duration of each activity
-

Step 2 (Draw the Diagram):

There are two types of formats:

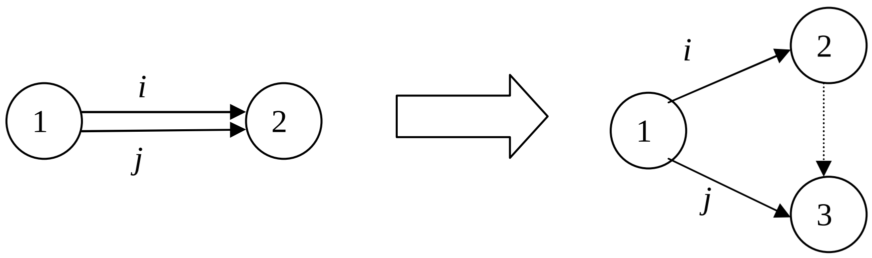
1. Activity-on-Arrow (AoA)
   In an AoA format, activities are displayed by means of arrows in the network. The nodes are events denoting the start and/or finish of a set of activities of the project. The technological links between activity i or activity j :



Since activities can be labeled with their corresponding start and end node event, it is said that activity (2, 3) is a *successor* of (1, 2) and activity (1, 2) is a *predecessor* of (2, 3).
The following rules have been suggested to construct AoA networks:

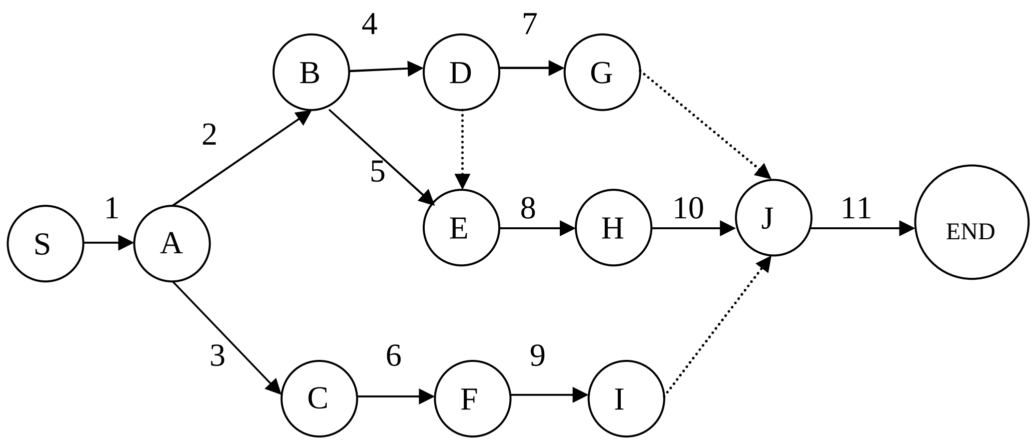
1. Before any activity may begin, all activities preceding it must be completed.
2. Arrows imply logical precedence only. Neither the length nor their "compass" direction has any difference.
3. Event numbers must not be duplicated in any network.
4. Any two events may be directly connected by no more than one activity.
5. Networks may have only one initial event (with no predecessor) and only one terminal event (with no successor)
6. The introduction of dummy activities is often necessary to model all precedence relations.

Dummies are introduced for the unique identification of activities and/or for displaying certain precedence relations. These activities are represented by dashed arrows in the network and do not consume time or resources.
The following figure displays an example project with a dummy arrow to identify all activities in a unique way. The network contains two activities that can be performed in parallel (i.e. there is no technological precedence relation between the two activities. Rule 4states that two events cannot be connected by more than one activity to ensure the unique identification of each activity (both i and j can be labeled as activity (1, 2)). Therefore, an extra dummy activity needs to be embedded in the project work, represented by the dashed arrows. In doing so, the network starts and ends with a single event node (rule 5) and each activity has been defined by a unique start/end event combination (rule 4):



Example:

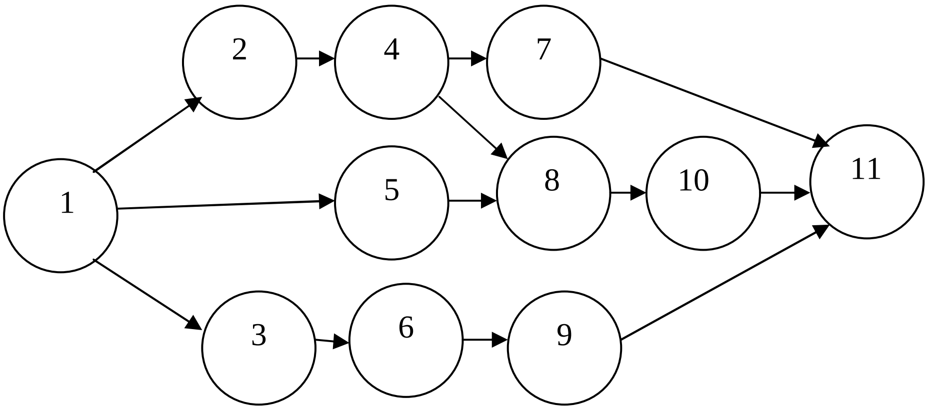
Activity	Predecessors
1	-
2	1
3	1
4	2
5	2
6	3
7	4
8	4, 5
9	6
10	8
11	7, 9, 10



2. Activity-on-Node (AoN)
   An AoN network displays the activities by nodes and precedence relations by arrows. Dummy activities are not necessary, apart from a single initial start and a single end activity, which makes an AoN network always unique.
   Following steps should be performed in order to construct an AoN network:

   1. Draw a node for each network activity.
   2. Draw an arrow for each immediate precedence relation between two activities.
   3. Possibly add a dummy start and dummy end node to force that the network begins with a single start activity and finish with a single end activity.

   Below, we have the AoN network of the above example:



Step 3 (Calculate the Critical Path):

The *critical path* determines the shortest time to complete the project. It is the longest duration path through a network of activities. *Critical activities* are activities on the critical path.

Denote by

$K := \{P_0, \ldots, P_n\}$

T_i^e : earliest possible point in time on which the activity $i \to j$ can start

T_j^e : earliest possible point in time on which the activity $i \to j$ can finish

T_i^l : latest possible point in time on which the activity $i \to j$ can start

T_j^l : latest possible point in time on which the activity $i \to j$ can finish

t_{ij} : duration of the activity $i \to j$.

Then we have:

$$T_0^e := 0$$

$$T_j^e = \max_i \left\{T_i^e + t_{ij}\right\}, \quad i < j;\ j = 1, 2, \ldots, n;\ (P_i, P_j) \in K$$

$$T_n^l := T_n^e$$

$$T_i^l = \min_j \left\{T_j^l - t_{ij}\right\}, \quad i < j;\ i = n-1, n-2, \ldots, 0,\ (P_i, P_j) \in K.$$

A path is *critical* if and only if all activities lying on it fulfil the following equation:

$$T_j^e - T_i^e - t_{ij} = 0.$$

Step 4 (Calculate the Float Times):

Slack time is the difference between the latest time and the earliest time of an event:

$$T_i^l - T_i^e$$

Positive slack is the amount of time an event can be delayed without delaying the project completion.

The most common types of floats are:

1. *Total float*
   It is the amount of time a single activity can be delayed without delaying project completion:

   $$\Delta^T t_{ij} = T_j^l - T_i^e - t_{ij}$$

2. *Free float*
   It is the amount of time a single activity can be delayed without delaying the early start of any successor activity:

   $$\Delta^F t_{ij} = T_j^e - T_i^e - t_{ij}$$

3. *Independent float*
   It is the amount of time that can be used without affecting either the predecessor or successor events. The independent float represents the amount of float time available for an activity when its preceding activities are completed at their latest and its succeeding activities are to begin at their earliest time:

   $$\Delta^I t_{ij} = \max\{0,\ T_j^e - T_i^l - t_{ij}\}$$

4. *Conditional slack*
   It is defined as:

   $$\begin{aligned}\Delta^C t_{ij} &:= \Delta^T t_{ij} - \Delta^F t_{ij} \\ &= T_j^l - T_j^e\end{aligned}$$

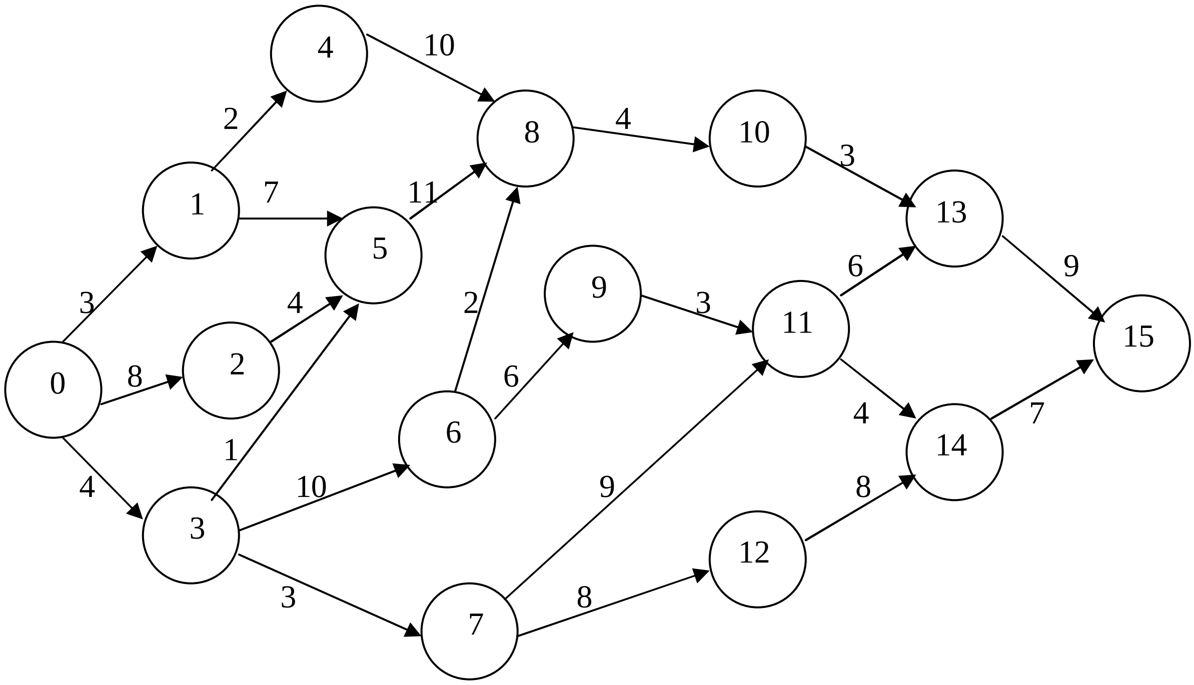
Slack time is the difference between the latest time and the earliest time of an event:

$$T_i^l - T_i^e$$

Positive slack is the amount of time an event can be delayed without delaying the project completion.

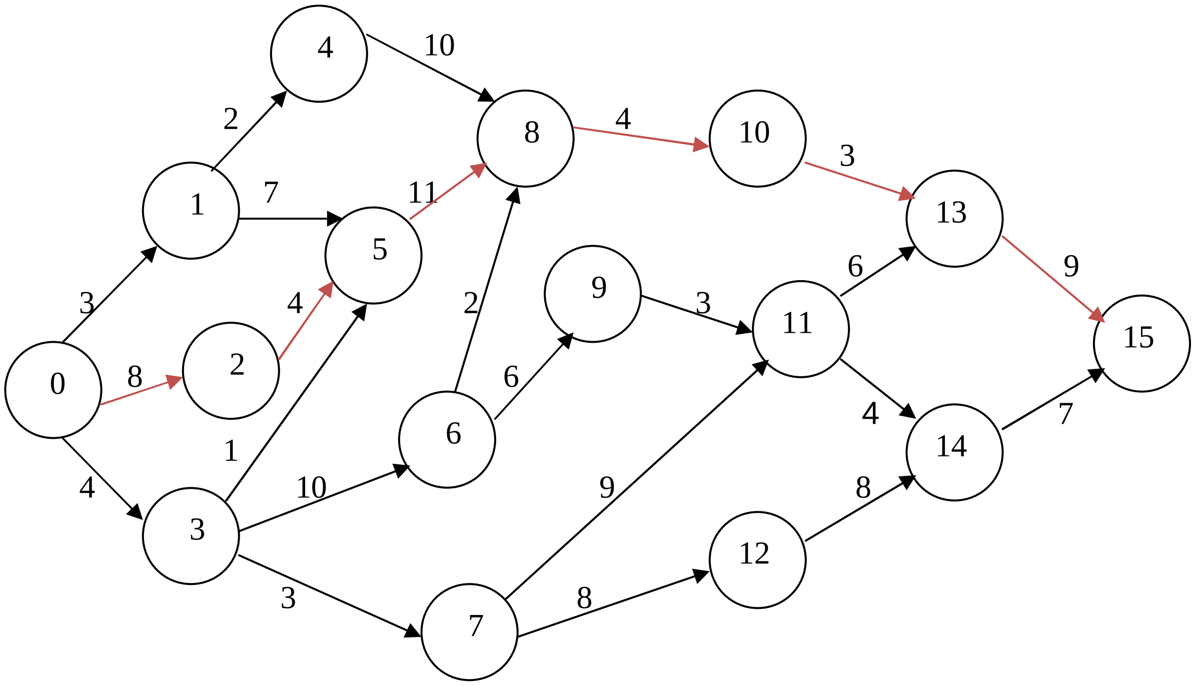
Ex. 8. 1.

Step 2:



Step 3:

T^e	Events	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0		3	8	4												
3	1					2	7										
8	2						4										
4	3						1	10	3								
5	4									10							
12	5									11							
14	6									2	6						
7	7												9	8			
23	8											4					
20	9												3				
27	10														3		
23	11														6	4	
15	12															8	
30	13																9
27	14																7
39	15																
T^l		0	5	8	5	13	12	15	15	23	21	27	24	24	30	32	39


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
3
8
4
2
7
4
1
10
3
10
11
2
6
4
9
8
3
3
6
4
8
9
7

Step 4:

i	j	t_{ij}	T_i^e	T_j^l	$T_i^e + t_{ij}$	$T_j^l - t_{ij}$	$\Delta^T t_{ij}$	$\Delta^F t_{ij}$	$\Delta^I t_{ij}$	$\Delta^C t_{ij}$
0	1	3	0	5	3	3	2	0	0	2
0	2	8	0	8	8	8	0	0	0	0
0	3	4	0	5	4	4	1	0	0	1
1	4	2	3	13	5	5	8	0	0	8
1	5	7	3	12	10	10	2	2	0	0
2	5	4	8	12	12	12	0	0	0	0
3	5	1	4	12	5	5	7	7	6	0
3	6	10	4	15	14	14	1	0	0	1
3	7	3	4	15	7	7	8	0	0	8
4	8	10	5	23	15	15	8	8	0	0
5	8	11	12	23	23	23	0	0	0	0
6	8	2	14	23	16	16	7	7	6	0
6	9	6	14	21	20	20	1	0	0	1
7	11	9	7	24	16	16	8	7	0	1
7	12	8	7	24	15	15	9	0	0	9
8	10	4	23	27	27	27	0	0	0	0
9	11	3	20	24	23	23	1	0	0	1
10	13	3	27	30	30	30	0	0	0	0
11	13	6	23	30	29	29	1	1	0	0
11	14	4	23	32	27	27	5	0	0	5
12	14	8	15	32	23	23	9	4	0	5
13	15	9	30	29	39	39	0	0	0	0
14	15	7	27	39	34	34	5	5	0	0

R. 8. 2 (*Program Evaluation Review Technique -PERT*)
In PERT activities are shown as a network of precedence relationships using AoA format

PERT was developed by the US Navy for the planning and control of the Polaris missile programme and the emphasis on completing the programme in the shortest possible time.

PERT is used mainly for non-repetitive jobs, where the time and cost estimates tend to be quite uncertain. This technique uses probabilistic time estimates.

ALG. 8. 2 (*PERT*)

Step 1 (List the activities):

Set up a table giving following informations:

- The name of activities and their description
- The required predecessors

- Three duration estimates:

    i) *Optimistic*
    This is the shortest possible time in which the activity can be completed, denoted by a_{ij}, $i, j = 1, 2, ..., n$.

    ii) *Most probable*
    This is the most likely time in which the activity can be completed under normal circumstances, denoted by m_{ij}, $i, j = 1, 2, ..., n$.

    iii) *Pessimistic*
    This is the longest time the activity might need, denoted by b_{ij}, $i, j = 1, 2, ..., n.$.

Step 2 (Draw the Diagram):

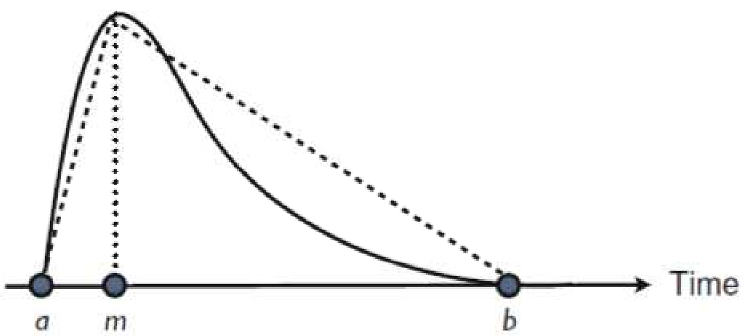
See ALG 8. 1.

Step 3 (Calculate the Expected Time $\bar{t}_{ij}$ and the Standard Deviation $\sigma_{t_{ij}}$):

$$\bar{t}_{ij} = \frac{a_{ij} + 4m_{ij} + b_{ij}}{6}, \quad i, j = 1, 2, ..., n; \ (P_i, P_j) \in K$$

$$\sigma_{t_{ij}} = \frac{b_{ij} - a_{ij}}{6}, \quad i, j = 1, 2, ..., n; \ (P_i, P_j) \in K$$

PERT assumes that each activity duration is a random variable between two extreme value (i.e. a_{ij} and b_{ij}) a follows a beta probability distribution. A typical beta distribution function and its triangular approximation looks as follows:



Step 4 (***Calculate the Critical Path***)

Denote by

$K := \{P_0, ..., P_n\}$

$\bar{T}_i^e$: earliest possible point in time on which the activity $i \to j$ can start

$\bar{T}_j^e$: earliest possible point in time on which the activity $i \to j$ can finish

$\sigma^2_{T_i^e}$: variance for the earliest possible point in time on which the activity $i \to j$ can start

$\sigma^2_{T_j^e}$: variance for the earliest possible point in time on which the activity $i \to j$ can finish

$\bar{T}_i^l$: latest possible point in time on which the activity $i \to j$ can start

$\bar{T}_j^l$: latest possible point in time on which the activity $i \to j$ can finish

$\sigma^2_{T_i^l}$: variance for the latest possible point on which the activity $i \to j$ can start

$\sigma^2_{T_j^l}$: variance for the latest possible point in time on which the activity $i \to j$ can finish

t_{ij} : duration of the activity $i \to j$

Then we have:

$$\bar{T}_0^e := 0,$$

$$\bar{T}_j^e = \max_i \{\bar{T}_i^e + \bar{t}_{ij}\},\ i < j;\ j = 1, 2, ..., n;\ (P_i, P_j) \in K,$$

$$\sigma^2_{T_0^e} := 0,$$

$$\sigma^2_{T_j^e} = \max_i \{\sigma^2_{T_i^e} + \sigma^2_{t_{ij}}\},\ i < j;\ j = 1, 2, ..., n;\ (P_i, P_j) \in K.$$

$$\bar{T}_n^l := \bar{T}_n^e,$$

$$\bar{T}_i^l = \min_j \{\bar{T}_j^l - \bar{t}_{ij}\},\ i < j;\ i = n-1,\ n-2, ...0;\ (P_i, P_j) \in K.$$

$$\sigma^2_{T^l_n} := 0,$$

$$\sigma^2_{T^l_i} = \max_j\{\sigma^2_{T^l_j} + \sigma^2_{t_{ij}}\},\ i < j;\ i = n-1,\ n-2,...0;\ (P_i, P_j) \in K.$$

Step 5 (***Calculation of probabilities***)

$$P\left[(T_i^l - T_i^e) \le x\right] = P\left[x \le -\frac{T_i^l - T_i^e}{\sqrt{\sigma^2_{T_i^l} + \sigma^2_{T_i^e}}}\right]$$

$$x = \frac{(T_i^l - T_i^e) - (\bar{T}_i^l - \bar{T}_i^e)}{\sqrt{\sigma^2_{T_i^l} + \sigma^2_{T_i^e}}}$$

$$P\left[\left(T_i^l - T_i^e\right) \le x\right] = P\left[x \le -\frac{T_i^l - T_i^e}{\sqrt{\sigma^2_{T_i^l} + \sigma^2_{T_i^e}}}\right]$$

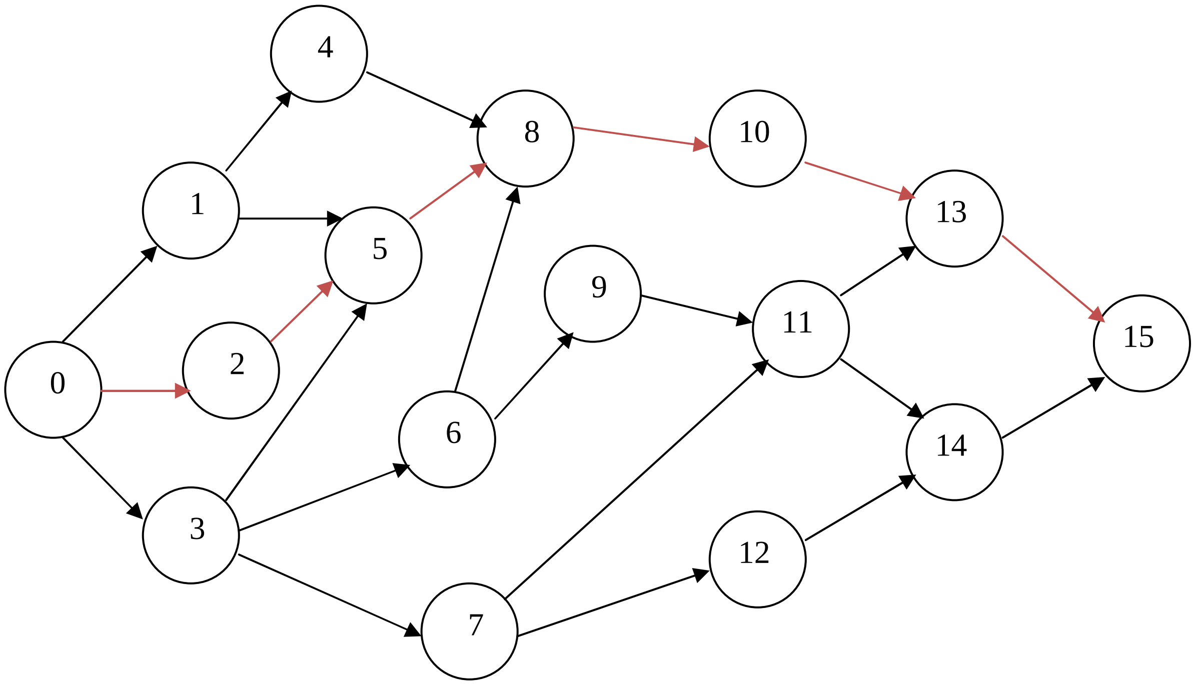
Ex. 8. 1. (*Continued*)

Steps 3:

i	j	a_{ij}	m_{ij}	b_{ij}	$\bar{t}_{ij}$	$\sigma_{t_{ij}}$	$\sigma^2_{t_{ij}}$
0	1	1	3	6	3.17	0.83	0.69
0	2	4	8	10	7.67	1.00	1.00
0	3	3	4	5	4.00	0.33	0.11
1	4	1	2	4	2.17	0.50	0.25
1	5	4	7	12	7.33	1.33	1.78
2	5	3	4	6	4.17	0.50	0.25
3	5	1	1	2	1.17	0.17	0.03
3	6	8	10	13	10.17	0.83	0.69
3	7	1	3	4	2.83	0.50	0.25
4	8	7	10	15	10.33	1.33	1.78
5	8	11	11	11	11.00	0.00	0.00
6	8	2	2	4	2.33	0.33	0.11
6	9	3	6	8	5.83	0.83	0.69
7	11	6	9	12	9.00	1.00	1.00
7	12	5	8	10	7.83	0.83	0.69
8	10	2	4	7	4.17	0.83	0.69
9	11	1	3	4	2.83	0.50	0.25
10	13	2	3	4	3.00	0.33	0.11
11	13	3	6	8	5.83	0.83	0.69
11	14	1	4	7	4.00	1.00	1.00
12	14	7	8	9	8.00	0.33	0.11
13	15	6	9	10	8.67	0.67	0.44
14	15	5	7	10	7.17	0.83	0.69

Steps 4:

$\sigma^2_{T_i^e}$	$\bar{T}_i^e$		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.00	0.00	0		3.17 0.69	7.67 1.00	4.00 0.11												
0.69	3.17	1					2.17 0.25	7.33 1.78										
1.00	7.67	2						4.17 0.25										
0.11	4.00	3						1.17 0.03	10.17 0.69	2.83 0.25								
0.94	5.34	4									10.33 1.78							
2.47	11.84	5									11.00 0.00							
0.80	14.17	6									2.33 0.11	5.88 0.69						
0.36	6.83	7												9.00 1.00	7.83 0.69			
2.72	22.84	8											4.17 0.69					
1.49	20.00	9												2.83 0.25				
3.41	27.01	10														3.00 0.11		
1.74	22.83	11														5.83 0.69	4.00 1.00	
1.05	14.66	12															8.00 0.11	
3.52	30.01	13																8.67 0.44
2.74	26.83	14																7.17 0.69
3.96	38.68	15																
		$\bar{T}_j^l$	0.00	4.51	7.67	5.35	12.51	11.84	15.52	15.18	22.84	21.35	27.01	24.18	23.51	30.01	31.51	38.68
		$\sigma^2_{T_j^l}$	3.96	3.27	1.49	3.32	3.02	1.24	2.63	2.69	1.24	1.94	0.55	1.69	0.80	0.44	0.69	0.00



Step 5

i	$\bar{T}_i^e$	$\sigma^2_{T_i^e}$	$\bar{T}_i^l$	$\sigma^2_{T_i^l}$	$\bar{T}_i^l - \bar{T}_i^e$	$P\left[(T_i^l - T_i^e)\right] \leq 0$
0	0.00	0.00	0.00	3.96	0.00	0.50
1	3.17	0.69	4.51	3.27	1.34	0.25
2	7.67	1.00	7.67	1.49	0.00	0.50
3	4.00	0.11	5.35	3.32	1.35	0.24
4	5.34	0.94	12.51	3.02	7.17	0.00
5	11.84	2.47	11.84	1.24	0.00	0.50
6	14.17	0.80	15.52	2.63	1.35	0.24
7	6.83	0.36	15.18	2.09	8.35	0.00
8	22.84	2.72	22.84	1.24	0.00	0.50
9	20.00	1.49	21.35	1.94	1.35	0.24
10	27.01	3.41	27.01	0.55	0.00	0.50
11	22.83	1.74	24.18	1.69	1.35	0.24
12	14.66	1.05	23.51	0.80	8.85	0.00
13	30.01	3.52	30.01	0.44	0.00	0.50
14	26.83	2.74	31.51	0.69	4.68	0.01
15	38.68	3.96	38.68	0.00	0.00	0.50

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