

Chapter 7

Linear Optimization (Transportation Problem)

Solution

7. 1.

We use the northwest corner method to find an initial solution:

To → From ↓	P_1	P_2	P_3	P_4	Supply
W_1	15	20	16	21	250
	75	175			
W_2	25	13	5	11	130
		55	75		
W_3	15	15	7	17	235
			165	70	
Demand	75	230	240	70	615

$$z_0 = 6940$$

To → From ↓	P_1	P_2	P_3	P_4	Supply	u_i
W_1	15	20	16	21	250	0
	75	175	12	22		
W_2	25	13	5	11	130	-7
	8	55	75	15		
W_3	15	15	7	17	235	-5
	10	15	165	70		
Demand	75	230	240	70	615	
v_j	15	20	12	22		

From ↓ To →	P_1	P_2	P_3	P_4	Supply	u_i
W_1	15 75	20 175	12	16 18	21 250	7
W_2	8 25	13 55	5	5 70	11 130	0
W_3	10 15	15	235	7 13	17 235	2
Demand	75	230	240	70	615	
v_j	8	13	5	11		

From ↓ To →	P_1	P_2	P_3	P_4	Supply	u_i
W_1	15 75	20 175	12	16 18	21 250	7
W_2	8 25	13 55	5	5 70	11 130	0
W_3	10 15	15	235	7 13	17 235	2
Demand	75	230	240	70	615	
v_j	8	13	5	11		

From ↓ To →	P_1	P_2	P_3	P_4	Supply
W_1	15 75	20 175		16 21	250
W_2	25	13	60	5 70	130
W_3	15	15 55	7 180	17	235
Demand	75	230	240	70	615

$$z^* = 7780$$

$$X^* = \alpha \begin{pmatrix} 75 & 175 & 0 & 0 \\ 0 & 55 & 75 & 0 \\ 0 & 0 & 165 & 70 \end{pmatrix} + (1-\alpha) \begin{pmatrix} 75 & 175 & 0 & 0 \\ 0 & 0 & 60 & 70 \\ 0 & 55 & 180 & 0 \end{pmatrix}, \quad 0 \leq \alpha \leq 1$$

7. 2.

1.

We use Vogel's method to find an initial solution:

From ↓ To →	C_1	C_2	C_3	Supply	d_i^r
P_1	32 20	60	200	20	28
P_2	40 10	68 20	80	30	28
P_3	120	104 15	60 30	45	97
Demand	30	35	30		
d_j^c	8	8	73		

$$z_0 = 4170$$

2.

From ↓ To →	C_1	C_2	C_3	Supply	u_i
P_1	32 20	60	200	20	-8
P_2	40 10	68 -	80 +	30	0
P_3	120	104 +	M -	45	36
	76	15	30		
Demand	30	35	30		
v_j	40	68	M-36		

Diagram illustrating the Vogel's method steps in the second table:

- A closed loop is shown with arrows indicating the flow of units: from P_2 to C_2 (20 units), from C_2 to P_3 (20 units), from P_3 to C_3 (20 units), and from C_3 to P_2 (20 units).
- Labels $M-44$ and $M-36$ are placed near the C_2 and C_3 columns respectively, indicating the remaining supply/demand after adjustments.

To → From ↓	C_1	C_2	C_3	Supply	u_i
P_1	32	60	200	20	-8
	20	180-M	72		
P_2	40	68	80	30	0
	10	184-M	20		
P_3	120	104	M	45	M-80
	M-40	35	10		
Demand	30	35	30		
v_j	40	184-M	80		

To → From ↓	C_1	C_2	C_3	Supply	u_i
P_1	32	60	200	20	-88
	20	16	72		
P_2	40	68	80	30	-80
	0	24	30		
P_3	120	104	M	45	0
	10	35	160		
Demand	30	35	30		
v_j	120	104	160		

$$X^* = \begin{pmatrix} 20 & 0 & 0 \\ 0 & 0 & 30 \\ 10 & 35 & 0 \end{pmatrix}, \quad z^* = 7880$$

(Last updated: 10.09.2015)