Chapter 6

Linear Optimization (Duality)

Solutions

6. 1.

1.

Denote by

$x_i,\ i = 1,2:$ number of days operated in R_i

$$
\begin{aligned}
z &= 12000x_1 + 10000x_2 \to \min!\\
200x_1 + 100x_2 &\geq 800\\
300x_1 + 100x_2 &\geq 900\\
200x_1 + 200x_2 &\geq 1000\\
x_1, x_2 &\geq 0 : \text{integer}
\end{aligned}
$$

2.

The dual:

$$
\begin{aligned}
Z &= 800\lambda_1 + 900\lambda_2 + 1000\lambda_3 \to \max!\\
200\lambda_1 + 300\lambda_2 + 200\lambda_3 &\leq 12000\\
100\lambda_1 + 100\lambda_2 + 200\lambda_3 &\leq 10000\\
\lambda_1, \lambda_2, \lambda_3 &\geq 0
\end{aligned}
$$

<u>Standard form:</u>

$$
\begin{aligned}
Z &= 800\lambda_1 + 900\lambda_2 + 1000\lambda_3 \to \max!\\
200\lambda_1 + 300\lambda_2 + 200\lambda_3 + \lambda_4 \qquad &= 12000\\
100\lambda_1 + 100\lambda_2 + 200\lambda_3 \qquad + \lambda_5 &= 10000\\
\lambda_i \geq 0,\ i &= 1,2,\ldots,5.
\end{aligned}
$$

Simplex Tableau

BV	λ_1	λ_2	λ_3	λ_4	λ_5	λ_0
λ_4	200	300	200	1	0	12000
λ_5	100	100	200	0	1	10000
Z	-800	-900	-1000	0	0	0
λ_1	1	$\frac{5}{2}$	1	$\frac{1}{100}$	0	60
λ_5	0	-50	100	$-\frac{1}{2}$	1	40000
Z	0	300	-200	4	0	48000
λ_1	1	2	0	$\frac{1}{100}$	$-\frac{1}{100}$	20
λ_3	0	$-\frac{1}{2}$	1	$-\frac{1}{100}$	$\frac{1}{100}$	40
Z	0	200	0	3	2	56000

Solution of the primal:

$$x^* = \begin{pmatrix} 3 & 2 & 0 & 200 & 0 \end{pmatrix}^T, \qquad z^* = 56000$$

Solution of the dual:

$$\lambda^* = \begin{pmatrix} 20 & 0 & 40 & 0 & 0 \end{pmatrix}^T, \qquad Z^* = 56000 .$$

Increasing the right-hand side of only the first constraint in the primal by one unit will lead to an increase of the value of the objective function by 20 units.

Increasing the right-hand side of only the second constraint in the primal by one unit will have no influence on the value of the objective function.

Increasing the right-hand side of only the third constraints in the primal by one unit will lead to an increase of the value of the objective function by 40 units.

6. 2.

1.

Denote by

$x_i,\ i = 1,2$: amount of F_i

$$
\begin{aligned}
z = 20x_1 + 25x_2 &\to \min \\
2x_1 + 3x_2 &\geq 18 \\
x_1 + 3x_2 &\geq 12 \\
4x_1 + 3x_2 &\geq 24 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

2.

The dual:

$$
\begin{aligned}
Z = 18\lambda_1 + 12\lambda_2 + 24\lambda_2 &\to \max \\
2\lambda_1 + \lambda_2 + 4\lambda_3 &\leq 20 \\
3\lambda_1 + 3\lambda_2 + 3\lambda_3 &\leq 25 \\
\lambda_1, \lambda_2, \lambda_3 &\geq 0
\end{aligned}
$$

<u>Standard form</u>:

$$
\begin{aligned}
Z = 18\lambda_1 + 12\lambda_2 + 24\lambda_2 &\to \max \\
2\lambda_1 + \lambda_2 + 4\lambda_3 + \lambda_4 \qquad &= 20 \\
3\lambda_1 + 3\lambda_2 + 3\lambda_3 \qquad + \lambda_5 &= 25 \\
\lambda_i \geq 0,\ i = 1,2,\ldots,5.&
\end{aligned}
$$

Simplex Tableau

BV	λ_1	λ_2	λ_3	λ_4	λ_5	λ_0
λ_4	2	1	4	1	0	20
λ_5	3	3	3	0	1	25
Z	-18	-12	-24	0	0	0
λ_3	$\frac{1}{2}$	$\frac{1}{4}$	1	$\frac{1}{4}$	0	5
λ_5	$\frac{3}{2}$	$\frac{9}{4}$	0	$-\frac{3}{4}$	1	10
Z	-6	-6	0	6	0	120
λ_3	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{5}{3}$
λ_1	1	$\frac{3}{2}$	0	$-\frac{1}{2}$	$\frac{2}{3}$	$\frac{20}{3}$
Z	0	3	0	3	4	160

Solution of the primal:

$$x^* = \begin{pmatrix} 0 & 3 & 4 & 0 & 3 \end{pmatrix}^T, \qquad z^* = 160$$

Solution of the dual:

$$\lambda^* = \begin{pmatrix} \frac{20}{3} & 0 & \frac{5}{3} & 0 & 0 \end{pmatrix}^T, \qquad Z^* = 160.$$

Increasing the right-hand side of the only the first constraint in the primal by one unit will lead to an increase of the value of the objective function by $20/3$ units.

Increasing the right-hand side of only the second constraint in the primal by one unit will have no influence on the value of the objective function.

Increasing the right-hand side of only the third constraint in the primal by one unit will lead to an increase of the value of the objective function by $5/3$ units.

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