

## Chapter 6

### *Linear Optimization (Duality)*

### Exercises

#### 6. 1.

An oil company has two refineries. Each day, refinery  $R_1$  produces 200 barrels of high-grade oil, 300 barrels of medium-grade oil, and 200 barrels of low-grade oil and costs \$12000 to operate. Each day, refinery  $R_2$  produces 100 barrels of high-grade oil, 100 barrels of medium-grade oil, and 200 barrels of low-grade oil and costs \$10000 to operate. The company must produce at least 800 barrels of high-grade oil, 900 barrels of medium-grade oil, and 1,000 barrels of low-grade oil.

The problem is to find the number of days that each refinery should be operated to lead to minimum cost for the oil company

1. Formulate the problem as a model of linear optimisation.
2. Solve the problem by the simplex method.
3. Give and interpret the solutions of both the primal and the dual problems.

#### 6. 2.

A nutritionist is planning a menu consisting of two main foods  $F_1$  and  $F_2$ . Each ounce of  $F_1$  contains 2 units of fat, 1 unit of carbohydrates, and 4 units of protein. Each unit of  $F_2$  contains 3 units of fat, 3 units of carbohydrates, and 3 units of protein. The nutritionist wants the meal to provide at least 18 units of fat, at least 12 units of carbohydrates, and at least 24 units of protein. An ounce of  $F_1$  costs 20 cents and an ounce of  $F_2$  costs 25 cents.

The question is how many ounces of each food should be bought to minimise the cost of the meal and yet satisfy the nutritionist's requirements.

1. Formulate the problem as a model of linear optimisation.
2. Solve the problem by the simplex method.
3. Give and interpret the solutions of both the primal and the dual problems.