Chapter 5

Linear Optimization (Simplex Method)

Solutions

5. 1.

1.

Let

x_1 : number of pairs of men's shoes

x_2 : number of pairs of women's shoes

The model

$$z = x_1 + 1.20x_2 \to \max!$$
$$2x_1 + 3x_2 \le 960$$
$$2x_1 + x_2 \le 480$$
$$x_1, x_2 \ge 0: \text{ integer}$$

2.

The standard form

$$z = x_1 + 1.20x_2 \to \max!$$
$$2x_1 + 3x_2 + x_3 = 960$$
$$2x_1 + x_2 + x_4 = 480$$
$$x_i \ge 0, \; i = 1, 2, 3, 4$$

Simplex Tableau

BV	x_1	x_2	x_3	x_4	x_0
x_3	2	3	1	0	960
x_4	2	1	0	1	480
z	-1	-1.2	0	0	0
x_2	$\frac{2}{3}$	1	$\frac{1}{3}$	0	320
x_4	$\frac{4}{3}$	0	$-\frac{1}{3}$	1	160
z	$-\frac{1}{5}$	0	$\frac{2}{5}$	0	384
x_2	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	240
x_1	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	120
z	0	0	$\frac{7}{20}$	$\frac{3}{20}$	408

$$x^* = \begin{pmatrix}240 & 120 & 0 & 0\end{pmatrix}^*, \qquad z^* = 408$$

5. 2.

1.

Denote by

x_1: sales amount of exterior paint

x_2: sales amount of interior paint.

<u>*The model*</u>:

$$z(x_1, x_2) = 3000x_1 + 2000x_2 \to \text{max!}$$

$$x_1 + 2x_2 \le 6$$

$$2x_1 + x_2 \le 8$$

$$-x_1 + x_2 \le 1$$

$$x_2 \le 2$$

$$x_1, x_2 \ge 0.$$

2.
<u>Standard form</u>:

$$z(x_1, x_2) = 3000x_1 + 2000x_2 \to \max!$$
$$x_1 + 2x_2 + x_3 \qquad = 6$$
$$2x_1 \quad + x_2 \quad + x_4 \qquad = 8$$
$$-x_1 \quad x_2 \qquad + x_5 \quad = 1$$
$$x_2 \qquad + x_6 = 2$$
$$x_i \geq 0,\ i = 1, 2, \ldots, 6.$$

Simplex Tableau

BV	x_1	x_2	x_3	x_4	x_5	x_6	x_0
x_3	1	2	1	0	0	0	6
x_4	**2**	1	0	1	0	0	8
x_5	-1	1	0	0	1	0	1
x_6	0	1	0	0	0	1	2
z	-3000	-2000	0	0	0	0	0
x_3	0	$\frac{3}{2}$	1	$-\frac{1}{2}$	0	0	2
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	4
x_5	0	$\frac{3}{2}$	0	$\frac{1}{2}$	1	0	5
x_6	0	1	0	0	0	1	2
z	0	-500	0	1500	0	**0**	12000
x_2	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	0	0	$\frac{4}{3}$
x_1	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	0	$\frac{10}{3}$
x_5	0	0	-1	1	1	0	3
x_6	0	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	1	$\frac{2}{3}$
z	0	0	$\frac{1000}{3}$	$\frac{4000}{3}$	0	0	$\frac{38000}{3}$

$$x^* = \begin{pmatrix} \frac{10}{3} & \frac{4}{3} & 0 & 0 & 3 & \frac{2}{3} \end{pmatrix}^T, \qquad z^* = \frac{38000}{3}.$$

5. 3.

1.

Denote by

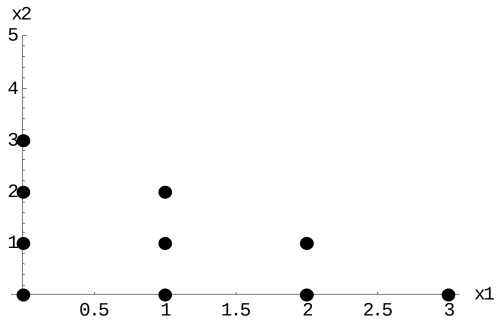
x_1: number of model I screens to be produced,

x_2: number of model II screens to be produced.

<u>*The model*</u>:

$$P(x_1, x_2) = 120x_1 + 80x_2 \rightarrow \max!$$
$$2x_1 + x_2 \le 6$$
$$7x_1 + 8x_2 \le 28$$
$$x_1, x_2 \ge 0: \text{ integer.}$$

2.



3.

<u>*Standard form*</u>:

$$P(x_1, x_2) = 120x_1 + 80x_2 \rightarrow \max!$$
$$2x_1 + x_2 + x_3 = 6$$
$$7x_1 + 8x_2 + x_4 = 28$$
$$x_1, x_2, x_3, x_4 \ge 0$$
$$x_1, x_2: \text{ integer.}$$

Simplex Tableau

BV	x_1	x_2	x_3	x_4	x_0
x_3	**2**	1	1	0	6
x_4	7	8	0	1	28
z	**-120**	-80	0	0	0
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	3
x_4	0	$\frac{9}{2}$	$-\frac{7}{2}$	1	7
z	0	**-20**	60	0	360
x_1	1	0	$\frac{8}{9}$	$-\frac{1}{9}$	$\frac{20}{9}$
x_2	0	1	$-\frac{7}{9}$	$\frac{2}{9}$	$\frac{14}{9}$
z	0	0	$\frac{400}{9}$	$\frac{40}{9}$	$\frac{3520}{9}$

$$x^* = \begin{pmatrix} \frac{20}{9} & \frac{14}{9} & 0 & 0 \end{pmatrix}^*, \qquad z^* = \frac{3520}{9}$$

The simplex method *does not* yield an optimal *integer* solution. Based upon the graphical solution of the problem, we have the following feasible points:

(0, 0), (1, 0), (2, 0), (3, 0), (0, 1), (1, 1), (2, 1), (0, 2), (1, 2), 0, 3)

The corresponding values of the objective function are:

0, 120, 240, 360, 80, 200, 320, 160, 280, 240.

Therefore, we have:

$$x^* = \begin{pmatrix} 3 & 0 & 0 & 0 \end{pmatrix}^*, \qquad z^* = 360.$$

5. 4.

Denote by

x_1 : amount of chicken

x_2 : amount of grain.

The model:

$$
\begin{aligned}
z = 10x_1 + x_2 &\rightarrow \text{min!}\\
10x_1 + 2x_2 &\geq 200\\
5x_1 + 2x_2 &\geq 150\\
x_1, x_2 &\geq 0.
\end{aligned}
$$

Standard form:

$$
\begin{aligned}
-10x_1 - x_2 &\rightarrow \text{max!}\\
10x_1 + 2x_2 - x_3 \qquad &= 200\\
5x_1 + 2x_2 \qquad - x_4 &= 150\\
x_1, x_2, x_3, x_4 &\geq 0.
\end{aligned}
$$

A basic feasible solution is not immediately available.

$$
\begin{aligned}
-10x_1 - x_2 &\rightarrow \text{max!}\\
10x_1 + 2x_2 - x_3 \qquad + x_5 \qquad &= 200\\
5x_1 + 2x_2 \qquad - x_4 \qquad + x_6 &= 150\\
x_i \geq 0,\ i &= 1, 2, \ldots, 6.
\end{aligned}
$$

$$
\begin{aligned}
x_5 &= 200 - 10x_1 - 2x_2 + x_3\\
x_6 &= 150 \ - 5x_1 - 2x_2 + x_4\\
x_5 + x_6 &= 350 - 15x_1 - 4x_2 + x_3 + x_4
\end{aligned}
$$

$$
\tilde{z} = -\left(x_5 + x_6\right) = -350 + 15x_1 + 4x_2 - x_3 - x_4 \rightarrow \text{max!}
$$

s.t.

$$
\begin{aligned}
10x_1 + 2x_2 - x_3 \qquad + x_5 \qquad &= 200\\
5x_1 + 2x_2 \qquad - x_4 \qquad + x_6 &= 150\\
x_i \geq 0,\ i &= 1, 2, \ldots, 6.
\end{aligned}
$$

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BV	x_1	x_2	x_3	x_4	x_5	x_6	x_0
x_5	10	2	-1	0	1	0	200
x_6	5	2	0	-1	0	1	150
z	10	1	0	0	0	0	(0)
$\tilde{z}$	-15	-4	1	1	0	0	-350
x_1	1	$\frac{1}{5}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	0	20
x_6	0	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	1	50
z	0	-1	1	0	-1	0	(-200)
$\tilde{z}$	0	-1	$-\frac{1}{2}$	1	$\frac{3}{2}$	0	-50
x_1	1	0	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$-\frac{1}{5}$	10
x_2	0	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	1	50
z	0	0	$\frac{3}{2}$	-1	$-\frac{3}{2}$	1	-150
$\tilde{z}$	0	0	0	0	1	1	0
x_4	5	0	-1	1			50
x_2	5	1	$-\frac{1}{2}$	0			100
z	5	0	$\frac{1}{2}$	0			-100

$$x^* = (0 \quad 100 \quad 0 \quad 50)^T, \qquad z^* = 100 \text{ cents}$$

(Last updated: 19.11.2012)