Chapter 2

Linear Optimization (Introduction)

Exercises

2. 1.
A firm can produce a product using three production processes. Each process uses labour (L), capital (C) and materials and supplies (M).
The variable capital stands for various types of machinery and equipment. The use of labour and capital is measured in hours. M is an index number for the quantities of materials and supplies used.
The availability of inputs of labour, capital, and materials and supplies, and the amount of each input used to produce one unit of product by each production activity, are categorised by the coefficients of the following table:

	Production Process			
	1	2	3	Availability
L	25	32	43	850
C	35	25	20	800
M	30	36	40	980

The first column of the table means that 25 hours of labour (L), 35 hours of capital (C), and 30 units of materials and supplies (M) are used by process 1 to produce one unit of the product produced by the activity 1. The first row of the table means that 25, 32, and 43 units of labour are used to produce one unit of the product produced by the activities 1, 2 and 3, respectively. Furthermore, the total hours of labour used can not exceed 850 hours. The other columns and rows read similarly.

The unit costs of L, C, and M are:

L: 14, C: 34, M: 12.

Consumers' tastes are reflected in the prices of one unit of the product produced by the activities 1, 2 and 3:

1: 2610, 2: 2400, M: 2250.

The firm would like to maximise its profit.
Formulate the problem as a linear optimisation model.

2. 2.
Suppose we have been asked to commence the preliminary design of a small complex of apartments. Our studies show that 1-bedroom and 2-bedroom apartments are the most desirable in this area. Our client has pointed out that there is, at most, demand for only three

3-bedroom and six 2-bedroom apartments and, consequently, has requested that no more than this number be built. Our objective is to find the number of 1-bedroom, 2-bedroom, and 3-bedroom apartments that will maximise the financial return.
The following costs and profits have been ascertained:

Apartment Type	Capital Cost (in $1,000s)	Profit (in $1,000s)
1-bedroom	90	20
2-bedroom	180	24
3-bedroom	220	27

The available capital is $1,800,000. If maximising profit were the only criterion, clearly, it would be most advantageous if we built all 1-bedroom apartments. However, the local planning authority has a planning code that discourages small apartments by placing penalties against them. For this site, the maximum number of penalty points a development can accrue is 960. The following penalties are incurred for each unit:

Apartment Type	Penalty Points
1-bedroom	120
2-bedroom	60
3-bedroom	20

Formulate the problem as a model of linear optimisation.

2. 3.
The liquid part of a diet is to provide at least 300 calories, 36 units of vitamin A, and 90 units of vitamin C daily. A cup of dietary drink X provides 60 calories, 12 units of vitamin A, and 10 units of vitamin C. A cup of dietary drink Y provides 60 calories, 6 units of vitamin A, and 30 units of vitamin C. Suppose that dietary drink X costs 0.12 € per cup and drink Y costs 0.15 € per cup.

How many cups of each drink should be consumed each day to minimise the cost and still meet the stated daily requirements?
Formulate the problem as a model of linear optimisation.

2. 4.
A fruit grower has 150 acres of land available to raise two crops, A and B. It takes one day to trim an acre of crop A and two days to trim an acre of crop B, and there are 240 days per year available for trimming. It takes 0.3 day to pick an acre of crop A and 0.1 day to pick an acre of crop B, and there are 30 days per year available for picking.
The fruit grower would like to know the number of acres of each fruit that should be planted in order to maximise his profit, assuming that the profit is 140 € per acre of crop A and 235 € per acre of crop B.
Formulate the problem as a model of linear optimisation.

2. 5. (*Product Mix Problem*)
A manufacturing process requires three different input viz., A, B and C. A sandal soap of the first type requires 30 gm of A, 20 gm of B and 6 gm of C, while this data for the second type of soap is 25, 5 and 15, respectively. The maximum availability of A, B and C are 6000, 3000 and 3000 gm, respectively. The selling price of the sandal soap of the first and second type are 14 € and 15, respectively. The profit is proportional to the amounts of soaps manufactured. The manufacturer would like to know how many soaps of the first and second kinds should be produced to maximise his profit.
Formulate the problem as a model of linear optimisation.

2. 6. (*Bus Scheduling Problem*)
A company runs buses during the time period 5 AM to 1 AM. Each bus can operate for 8 hours successively, and then it is directed to workshop for maintenance and fuel. The minimum number of buses required fluctuates with the time interval. The desired numbers of buses during different time intervals are given in the following table:

Time Intervals	Minimum No. of Buses Required
5 AM – 9 AM	5
9 AM – 1 PM	13
1 PM – 5 PM	11
5 PM – 9 PM	14
9 PM – 1 AM	4

The company keeps in view the reduction of air pollution and smog problem. It is required to determine the number of buses to operate during different shifts that will meet the minimum requirement while minimising the total number of daily buses in operation.
Formulate the problem as a model of linear optimisation.

2. 7. (*The Warehousing Problem*)
A warehouse has a capacity of 2000 units. The manager of the warehouse buys and sells the stock of potatoes over a period of 6 weeks to make profit. Assume that in the j – th week the same unit price p_j holds for both purchase and sale. In addition, there is unit cost 15 € as weekly expense for holding stock. The warehouse is empty at the beginning and is required to be empty after the sixth week. The question is: How should the manager operate to maximise his profit?
Formulate the problem as a model of linear optimisation.

2. 8. (*Caterer Problem*)
A festival organiser has to organise its annual cultural festival continuously for next five years. There is an arrangement of dinner for every invited team. The requirement of napkins during these five days is

Days	1	2	3	4	5
Napkins required	80	50	100	80	150

Accordingly, a caterer has been requested to supply the napkins according to the above schedule. After the festival is over the caterer has no use of napkins. A new napkin costs 2 €. The washing charges for a used napkin is 0.50 € by ordinary services and 1 €, if express service is used. A napkin given for washing by ordinary service is returned third day, while under express service is returned next day.
The question is: How should the caterer meet the requirement of the festival organiser so that the total cost is minimised?
Formulate the problem as a model of linear optimisation.

2. 9. (*Trim-Loss Problem*)
Paper cutting machines are available to cut standard news print rolls into the subrolls. Each standard roll is of 180 cm width and a number of them must be cut to produce smaller subrolls at the current orders for 30 of width 70 cm, 60 of width 50 cm and 40 of width 30 cm. Formulate the problem as to minimise the amount of wastes. Ignoring the recycling or other use for the trim, assume that the length of each required subroll is the same as that of the standard roll.

2. 10.
Two alloys, A and B are made from four different metals, I, II, II, and IV, according to the following specifications:

Alloy	Specifications	Selling price (€)/ton
A	at most 80% of I at least 30% of II at least 50% of IV	200
B	between 40% and 60% of II at least 30% of II at most 70% of IV	300

The four metals, in turn are extracted from three different ores with the following data:

Ore	Max. Quantity	Constituents (%)					Purchase
	(tons)	I	II	III	IV	others	Price (€)/ton
1	1000	20	10	30	30	10	30
2	2000	10	20	30	30	10	40
3	3000	5	5	70	20	0	50

How much of each alloy should be produced to maximise the profit? Formulate the problem as a linear optimisation model.

2. 11. (*An Agricultural Allocation Model*)
A farmer owns 1000 acres of more or less homogeneous farmland. His options are to breed cattle, or plant wheat, corn, or tomatoes. It takes four acres to support one head of cattle. Annually, 12000 hours of labour are available. (For simplicity, we assume here that these 12000 hours could be used at any time during the year, i.e., through hiring casual labour during seasons of high need, e.g., for harvesting).
The following table provides information regarding the profit, yield, and labour needs for the four economic activities:

	Cattle	Wheat	Corn	Tomatoes
Profit	$ 1600/head	$5/bushel	$6/bushel	50¢/lb
Yield per Acre	¼ heads/acre	50 bushels	80 bushels	1000 lbs
Annual Labour Requirement	40 hrs/head	10 hrs/acre	12 hrs/acre	25 hrs/acre

Furthermore, it is required that at least 20%of the farmland that is cultivated in the process must be used for the purpose of cattle breeding, at most 30% of the available farmland can be use for growing tomatoes, and the ratio between the amount of farmland assigned to growing wheat and that left uncultivated should not exceed 20 to 1.
The farmer would like to maximise his profit.
Formulate the problem as a linear optimisation model.

2. 12. (*A Portfolio Selection Problem*)
An investor has $1 million to invest in any combination of bonds, stocks, term deposits, a saving account, real estate, and gold. The anticipated (or known) interest rates, the risk factors (where a high number indicates a high risk) and the expected increase in the value of the investments are shown in the following table:

Types of investment	Interest (in % annually)	Risk factor	Expected annual Increase in value [%]
Bonds	5	3	0
Stocks	2	10	7
Term deposits	4	2	0
Saving accounts	3	1	0
Real estate	0	5	7
Gold	0	20	11

The objective is to maximise the amount that is expected to be available in a year's time subject to the following restrictions:

- Of the total amount of money invested, at least 30% must be invested in bonds, not more than 10% in stocks, and at least 10% in term deposits and/or saving accounts.
- Up to 50% of the total money invested in real estate may be borrowed against in the form of a mortgage at an interest rate of 6%. The amount borrowed cannot exceed $150000.
- The average risk factor of the investment cannot exceed 4.5.
- The average annual interest should be at least 2.5%.
- The amount of money invested in gold cannot exceed $100000 or 8% of the total money available, whichever is smaller.

Formulate the problem as a linear optimisation model.

2. 13. (*An Inventory Problem*)
A company wants to plan its production for one of its products for the next four months. The following table shows the anticipated demand, the production capacities, and the unit production costs for the individual months, as well as the inventory holding costs that are incurred carrying over one unit from one month to the next.

	Periods			
	Month 1	Month 2	Month 3	Month 4
Demand	50	120	150	160
Production capacity	100	100	160	150
Unit production cost	\$1	\$1.1	\$1.2	\$1.2
Inventory cost	\$0.3	\$0.2	\$0.2	

At present, no units are in stock and after the four months, it is not desired to have any stock left.
Total costs are to be minimised.
Formulate the problem as a linear optimisation model.

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