

Lineare Algebra in der Ökonomie

Input-Output-Analyse

Lösungen

3. 1.

1.

$$A = \begin{pmatrix} \frac{200}{1000} & \frac{300}{1500} & \frac{150}{1700} \\ \frac{100}{1000} & \frac{420}{1500} & \frac{480}{1700} \\ \frac{50}{1000} & \frac{200}{1500} & \frac{900}{1700} \end{pmatrix} \approx \begin{pmatrix} 0.2000 & 0.2000 & 0.0882 \\ 0.1000 & 0.2800 & 0.2824 \\ 0.0500 & 0.1333 & 0.5294 \end{pmatrix}.$$

2.

$$E - A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.2000 & 0.2000 & 0.0882 \\ 0.1000 & 0.2800 & 0.2824 \\ 0.0500 & 0.1333 & 0.5294 \end{pmatrix} = \begin{pmatrix} 0.8000 & -0.2000 & -0.0882 \\ -0.1000 & 0.7200 & -0.2824 \\ -0.0500 & -0.1333 & 0.4706 \end{pmatrix}$$

$$B := (E - A)^{-1} \approx \begin{pmatrix} 1.3426 & 0.4719 & 0.5348 \\ 0.2727 & 1.6583 & 1.0462 \\ 0.2199 & 0.5199 & 2.4781 \end{pmatrix}.$$

3.

$$B - (E + A) = \begin{pmatrix} 1.3426 & 0.4719 & 0.5348 \\ 0.2727 & 1.6583 & 1.0462 \\ 0.2199 & 0.5199 & 2.4781 \end{pmatrix} - \begin{pmatrix} 1.2000 & 0.2000 & 0.0882 \\ 0.1000 & 1.2800 & 0.2824 \\ 0.0500 & 0.1333 & 1.5294 \end{pmatrix}$$
$$= \begin{pmatrix} 0.1426 & 0.2719 & 0.4466 \\ 0.1727 & 0.3783 & 0.7638 \\ 0.1699 & 0.3866 & 0.9487 \end{pmatrix}.$$

4.

$$y = (E - A)x = \begin{pmatrix} 0.8000 & -0.2000 & -0.0882 \\ -0.1000 & 0.7200 & -0.2824 \\ -0.0500 & -0.1333 & 0.4706 \end{pmatrix} \begin{pmatrix} 1200 \\ 1800 \\ 1600 \end{pmatrix} = \begin{pmatrix} 458.88 \\ 724.16 \\ 453.02 \end{pmatrix}.$$

5.

$$x = (E - A)^{-1} y = \begin{pmatrix} 1.3426 & 0.4719 & 0.5348 \\ 0.2727 & 1.6583 & 1.0462 \\ 0.2199 & 0.5199 & 2.4781 \end{pmatrix} \begin{pmatrix} 458.88 \\ 724.16 \\ 453.02 \end{pmatrix} \approx \begin{pmatrix} 1200.10 \\ 1799.96 \\ 1600.03 \end{pmatrix}$$

6.

$$\begin{pmatrix} y_1 \\ 600 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.8000 & -0.2000 & -0.0882 \\ -0.1000 & 0.72000 & -0.2824 \\ -0.0500 & -0.1333 & 0.4706 \end{pmatrix} \begin{pmatrix} 1200 \\ x_2 \\ 2000 \end{pmatrix}$$

$$600 = -0.1000 \cdot 1200 + 0.7200x_2 - 0.2824 \cdot 2000 \Rightarrow x_2 \approx 1784.44$$

$$y_1 = 0.8000 \cdot 1200 - 0.2000 \cdot 1784.44 - 0.0882 \cdot 2000 \approx 426.71$$

$$y_3 = -0.0500 \cdot 1200 - 0.1333 \cdot 1784.44 + 0.4706 \cdot 2000 \approx 643.33$$

7.

$$\tilde{A} = \begin{pmatrix} \frac{200}{1000} & \frac{400}{1500} & \frac{100}{1700} \\ \frac{450}{1000} & \frac{180}{1500} & \frac{70}{1700} \end{pmatrix} \approx \begin{pmatrix} 0.2000 & 0.2667 & 0.0588 \\ 0.4500 & 0.1200 & 0.0412 \end{pmatrix}.$$

8.

$$\tilde{B} = \begin{pmatrix} 0.2000 & 0.2667 & 0.0588 \\ 0.4500 & 0.1200 & 0.0412 \end{pmatrix} \begin{pmatrix} 1.3426 & 0.4719 & 0.5348 \\ 0.2727 & 1.6583 & 1.0462 \\ 0.2199 & 0.5199 & 2.4781 \end{pmatrix} = \begin{pmatrix} 0.3541 & 0.5672 & 0.6101 \\ 0.6360 & 0.4328 & 0.4683 \end{pmatrix}.$$

9.

$$\begin{pmatrix} 0.3541 & 0.5672 & 0.6101 \\ 0.6360 & 0.4328 & 0.4683 \end{pmatrix} \begin{pmatrix} 458.88 \\ 724.16 \\ 453.02 \end{pmatrix} = \begin{pmatrix} 849.62 \\ 817.41 \end{pmatrix}.$$

(Letzte Änderung: 25.04.2010)