Dr. sc. Jilla Siassi
September 2023
HTW Berlin

Exam
Applied Statistics

Problem 1 — 20 Points

$$n = 20, \quad \bar{x} = 46, \quad s = 3$$

$$s_{\bar{x}} = \frac{3}{\sqrt{20}} \approx 0.6708$$

1.

$$P(\bar{x} > 45) = 1 - P(\bar{x} \le 45) \approx 1 - P((\bar{x} < 45) = 1 - F(45)$$
$$= 1 - \Phi\left(\frac{45-46}{0.6708}\right) = 1 - \Phi(-1.49) = 1 - (1 - \Phi(1.49) = 0.9319\,.$$

2.

$$P(\bar{x} < 45.5) = F(45.5) = \Phi\left(\frac{45.5-46}{0.6708}\right)$$
$$= \Phi(-075) = 1 - \Phi(0.75) = 1 - 0.7734 = 0.2266\,.$$

3.

$$P(44.5 \le \bar{x} < 47.0) = F(47.0) - F(44.5) = \Phi\left(\frac{47.0-46.0}{0.6708}.\right) - \Phi\left(\frac{44.5-46.0}{0.6708}\right)$$

$$= \Phi(1.49) - \Phi(-2.24) = \Phi(1.49) - (1 - \Phi(2.24))$$

$$= 0.9319 - 1 + 0.9875 = 0.9194\,.$$

Problem 2 — 20 Points

$$\bar{x} = \frac{12.25 + 12.37 + 12.68 + 12.84 + 12.90 + 12.97 + 13.02 + 13.35}{8} = \frac{102.38}{8} = 12.7975$$

$$s = \sqrt{\frac{(2.25 - 12.7975)^2 + (12.37 - 2.7975)^2 + \ldots + (13.35 - 12.7975)^2}{7}} = 0.3572$$

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0.3572}{\sqrt{8}} = 0.1263$$

$$\mu \in [12.7957 - 2.365 \cdot 0.1263,\ 12.7957 + 2.365 \cdot 0.1263] = [12.4970,\ 13.0944]$$

Problem 3	**30 Points**

$n = 55, \quad \bar{x} = 780, \quad \sigma = 230, \quad \alpha = 0.01 \quad (\alpha = 0.025)$

1.

$H_0 : \mu = 850; \quad H_1 : \mu < 850$

2.

t distribution.

3.

$\alpha = 0.01 \quad \Rightarrow \quad t_{crit} = -2.397$

$\alpha = 0.025 \quad \Rightarrow \quad t_{crit} = -2.005$

4.

$$t_{stat} = \frac{780 - 850}{\frac{230}{\sqrt{55}}} \approx -2.257.$$

5.

$\alpha = 0.01 \quad \Rightarrow \quad -2.397 < -2.257 \quad \Rightarrow$ Reject H_0

$\alpha = 0.025 \quad \Rightarrow \quad$ -2.005 $> -2.257 \quad \Rightarrow$ Don't reject H_0

Problem 4	**30 Points**

1.*Working Table*

x_i	y_i	y_i^*	$(y_i - \bar{y})^2$	$(y_i - y^*)^2$	$(x_i - \bar{x})^2$
2.90	38	39.236490	2.46938776	1.52890752	0.00148775
3.81	48	47.132651	71.0408163	0.75229429	0.75938777
3.20	38	41.839620	2.46938776	14.7426817	0.06834490
2.42	35	35.071482	20.8979592	0.00510968	0.26891632
3.94	50	48.260674	108.755102	3.02525493	1.00285920
2.05	31	31.860955	73.4693878	0.74124351	0.78955917
2.25	37	33.596375	6.6122449	11.5846631	0.47413060
20.57	277		285.714286	32.3801548	3.36468571

$\bar{y} = \frac{277}{7} = 39.571428, \quad \bar{x} = \frac{20.57}{7} = 2.93857142$

$SSE = 32.3801548, \quad SST = 285.714286, \quad SSR = SST - SSE = 285.714286 - 285.714286 = 253.3341312$

2.

$$r^2 = \frac{253.3341312}{285.714286} = 0.8966694583 \approx 0.8967$$

About 89.67% of the starting salary is explained by the GPA. It is a relatively good fit.

$$r = +\sqrt{0.8966694583} \;=\; 0.9416312751 \approx 0.9416.$$

There is a direct relationship between the GPA and the starting salary.

3.

Step 1:

$$H_0: \beta_1 = 0, \qquad H_1: \beta_1 \neq 0$$

Step 2:

$$s = \sqrt{\frac{32.3801548}{5}} = 2.5448047. \qquad s_{b_1} = \frac{2.5448047}{\sqrt{3.36468571}} = 1.38733776$$

$$t_{stat} = \frac{8.6771}{1.38733776} = 6.25449710$$

Step 3:
Because of

$$t_{stat} = 6.25449710 > 2.571 = t_{5;0.05}.$$

we reject H_0.