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HTW Berlin

Exam
Applied Statistics

Problem 1	**20 Points**

$\mu = 3.02, \quad \sigma = 0.39, \quad n = 20.$

$$\sigma_{\bar{x}} = \frac{0.39}{\sqrt{20}} = 0.087206651 \approx 0.0872.$$

1.

$$P(\bar{x} \geq 3.10) = 1 - P(\bar{x} < 3.10)$$

$$= 1 - \Phi\left(\frac{3.10 - 3.02}{0.0872}\right) = 1 - \Phi\left(0.92\right) = 1 - 0.8212 = 0.1788.$$

2.

$$P(\bar{x} \leq 2.90) \approx P(\bar{x} < 2.90) = \Phi\left(\frac{2.90 - 3.02}{0.0872}\right) = \Phi\left(-1.38\right)$$

$$= 1 - \Phi\left(1.38\right) = 1 - 0.9162 = 0.0838.$$

3.

$$P(2.95 \leq \bar{x} < 3.11) = F\left(3.11\right) - F\left(2.95\right)$$

$$= \Phi\left(\frac{3.11 - 3.02}{0.0872}\right) - \Phi\left(\frac{2.95 - 3.02}{0.0872}\right)$$

$$= \Phi\left(1.032\right) - \Phi\left(-0.803\right) = \Phi\left(1.032\right) - (1 - \Phi(0.803))$$

$$= 0.8485 - 1 + 0.7881 = 0.6366.$$

Problem 2 — 20 Points

We have

$$\bar{x} = 24.57\ €,\ s = 6.60\ €, \quad s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{6.60}{\sqrt{100}} = 0.66.$$

1.

$$\mu \in [24.57 - 1.96 \cdot 0.66,\ 24.57 + 1.96 \cdot 0.66\ ] = [23.28,\ 25.86].$$

2.

The confidence interval for the Euro amount of purchases is simply the total number of customers in the population multiplied by the confidence limits for the mean purchase amount per customer:

$$[23.28 \cdot 4000,\ 25.86 \cdot 4000] = [93120,\ 103440].$$

Problem 3 — 30 Points

$$n = 40, \quad \bar{x} = 1950, \quad \sigma = 500, \quad \alpha = 0.05$$

1.

$H_0 :\ \mu \le 1800; \quad \mu > 1800$

2.

Normal distribution.

3.

$z_{crit} = 1.645$

4.

$$z_{stat} = \frac{1950 - 1800}{\frac{500}{\sqrt{40}}} = 1.897366596 \approx 1.897.$$

5.

$1.897 > 1.645.$

$\therefore$ We reject H_0.

Problem 4 **30 Points**

1.

Working Table

x_i	y_i	y_i^*	$(y_i^* - \bar{y})^2$	$(y_i - y_i^*)^2$
2.6	3300	3301.36	121549.85	1.84960000
3.4	3600	3766.24	13511.7376	27635.7376
3.6	4000	3882.46	54037.6516	13815.6516
3.2	3500	3650.02	0.00040000	22506.0004
3.5	3900	3824.35	30397.9225	5722.9225
2.9	3600	3475.69	30383.9761	15452.9761
	21900		249881.138	85135.1378

$$\bar{y} = \frac{21900}{6} = 3650$$

$$SSR = 249881.138, \quad SSE = 85135.1378, \quad SST = SSR + SSE = 335016.2758.$$

2.

$$r^2 = \frac{249881.138}{335016.2758} = 0.7458776067 \approx 0.746$$

About 74.6% of the monthly salary is explained by the GPA. It is a relatively good fit.

$$r = +\sqrt{0.7458776067} = 0.8636420594 \approx 0.8636.$$

There is a direct relationship between the GPA and the monthly salary.

3.

Step 1:

$$H_0: \beta_1 = 0, \qquad H_1: \beta_1 \neq 0$$

Step 2:

$$s = \sqrt{\frac{85135.1378}{6-2}} = 145.8896311, \qquad s_{b_1} = \frac{145.8896311}{\sqrt{0.74}} = 169.5932513$$

$$t_{stat} = \frac{581.1}{169.5932513} = 3.426433514$$

Step 3:
Because of

$$t_{stat} = 3.426433514 > 2.776 = t_{4;0.05},$$

we reject H_0.