Chapter V

Discrete and Continuous Random Variables

Exercises

5. 1.
A builder orders a shipment of bricks. The random variable X , the number of broken bricks per lot, is estimated by suppliers to have the following probability function:

x_i	0	1	2	3	4	≥ 5
$P(X = x_i)$	0.7	0.1	0.05	0.05	0.03	p_5

1. Find p_5.
2. What is the probability that the number of broken bricks is at most 3?
3. Determine and sketch the distribution function $F(x)$ of the random variable X .
4. Find and interpret

   i) $F(3.8)$
   ii) $F(4.7) - F(1.8)$

5. 2.
The random variable X giving the number of passengers (excluding the driver) per car in rush hour traffic has the following probability function:

x_i	0	1	2	3	4
$P(X = x_i)$	0.7	p_2	0.1	0.05	0.05

1. Find p_2.
2. What is the probability that the number of passengers is at least 2?
3. Determine and sketch the distribution function $F(x)$ of the random variable X .
4. Find and interpret

   a. $F(2.06)$
   b. $F(3.9) - F(0.05)$

5. 3.
After the start of observation on a given summer evening, the time T , in minutes until the first shooting star is observed, follows an exponential distribution for which

$$P(T > t) = e^{-\frac{1}{10} \cdot t}$$

where $t > 0$.

1. Determine the probability that it takes between five and ten minutes for the first shooting star to be observed.
2. Determine the probability density function of T .

5. 4. (See Example 5. 2.)
A car pooling study shows that the number of passengers, X , in a car (excluding the driver) is likely to assume the values 0, 1, 2, 3, and 4 with probabilities given by the table

x_i	0	1	2	3	4
$P(X = x_i)$	0.7	p_2	0.1	0.05	0.05

1. Determine $P(X > 2)$.
2. Determine $P(X \geq 2)$.
3. What is the probability that a car will have no passengers?
4. Determine the smallest value of k so that $P(X < k) > 0.85$.
5. Evaluate $P(X \leq k)$ for

$$k = -12,\ 0,\ 0.5,\ 2.4,\ 103 .$$

5. 5.
Let the random variable X have the following distribution function:

$$F(x) = \begin{cases} 0 & \text{when} \quad x \leq -1 \\ \frac{3}{4}x + \frac{3}{4} & \text{when} \quad -1 < x \leq \frac{1}{3}. \\ 1 & \text{when} \quad \frac{1}{3} < x. \end{cases}$$

What is the probability that that X lies in the interval $\left]0,\ \frac{1}{3}\right[$?

5. 6.
Consider the function

$$f(x) = \begin{cases} a(3+x) & \text{when} \quad -3 \leq x \leq 0 \\ a(3-x) & \text{when} \quad 0 < x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

a) For what value of a will $f(x)$ be the density function of the random variable X ?
b) Determine the distribution function of X .
c) Find the probability that X lies in the interval $\left[\frac{1}{2},\ 1\right]$.

5. 7.

A balanced coin is tossed twice and the number X of heads is observed. Find the probability distribution for X.

5. 8.

Suppose that the Time T (in minutes) for individuals to complete a task has the following probability density function:

$$f(t) = \begin{cases} k(t - \frac{3}{2})(3 - t) & \frac{3}{2} \le t \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

a) Show that $k = \frac{16}{9}$.

b) Suppose that a failure is said to occur when an individual takes longer than two and a half minutes to complete the task. What is the probability of a failure?

(Last revised: 05.06.07)