Chapter VI

Parameters of a Random Variable

Solutions

6. 1.

a)

$$F(x) = \begin{cases} 0 & -\infty < x \le -3 \\ 0.10 & -3 < x \le 0 \\ 0.25 & 0 < x \le 1 \\ 0.35 & 1 < x \le 2 \\ 0.60 & 2 < x \le 3 \\ 1 & 3 < x < +\infty \end{cases}$$

b)

$$P(X > 0) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$= 0.10 + 0.25 + 0.40 = 0.75$$

c)

$$E(X) = -3 \cdot 0.1 + 0 \cdot 0.15 + 1 \cdot 0.1 + 2 \cdot 0.25 + 3 \cdot 0.4 = 1.5$$

$$D^2(X) = (-3)^2 \cdot 0.1 + 0^2 \cdot 0.15 + 1^2 \cdot 0.1 + 2^2 \cdot 0.25 + 3^2 \cdot 0.4 - (1.5)^2 = 3.35 .$$

6. 2.

Let

X **: „**Number of elements that will fall out“.

$$E(X) = \sum_{i=1}^{3} p_i \cdot x_i = 0 \cdot (1 - p_1) \cdot (1 - p_2) \cdot (1 - p_3)$$

$$+ 1 \cdot [p_1 \cdot (1 - p_2) \cdot (1 - p_3) + p_2 \cdot (1 - p_1) \cdot (1 - p_3) + p_3 \cdot (1 - p_1) \cdot (1 - p_2)]$$

$$+ 2 \cdot [p_1 \cdot p_2 \cdot (1 - p_3) + p_1 \cdot p_3 \cdot (1 - p_2) + p_2 \cdot p_3 \cdot (1 - p_1)]$$

$$+ p_1 \cdot p_2 \cdot p_3$$

$$= p_1 + p_2 + p_3 .$$

6. 3.

a)

$$\int_{-\infty}^{+\infty} f(x)dx = \int_0^1 \alpha \cdot x^2 \cdot (1-x)\, dx = 1,$$

$$\alpha \cdot \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 1, \qquad \alpha \cdot \left(\frac{1}{3} - \frac{1}{4}\right) = 1, \qquad \alpha = 12.$$

b)

$$F(x) = \begin{cases} 0 & x \le 0 \\ 12\left(\frac{x^3}{3} - \frac{x^4}{4}\right) & 0 < x \le 1, \\ 1 & 1 < x \end{cases}$$

$$E(X) = \int_0^1 x \cdot f(x)dx = \int_0^1 12 \cdot \left(x^3 - x^4\right)dx = 12 \cdot \left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 12 \cdot \left(\frac{1}{4} - \frac{1}{5}\right) = \frac{3}{5}.$$

$$D^2(X) = \int_0^1 x^2 \cdot f(x)dx - \left(E\left(X\right)\right)^2 = 12 \cdot \int_0^1 \left(x^4 - x^5\right)dx - \frac{9}{25} = 12 \cdot \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1 - \frac{9}{25} = \frac{1}{25}.$$

c)

$$P\left(X < \frac{1}{2}\right) = F\left(\frac{1}{2}\right) = 12 \cdot \left[\frac{\frac{1}{8}}{3} - \frac{\frac{1}{16}}{4}\right] = 0.3125.$$

$$P\left(X < E(X)\right) = P\left(X < \frac{3}{5}\right) = F\left(\frac{3}{5}\right) = 12 \cdot \left[\frac{\left(\frac{3}{5}\right)^3}{3} - \frac{\left(\frac{3}{5}\right)^4}{4}\right] = 0.4752.$$

6. 4.

Number	Probability	Prize (€)
1	$\frac{1}{1000}$	500
5	$\frac{5}{1000}$	50
20	$\frac{20}{1000}$	10
974	$\frac{974}{1000}$	0

1.

$$ER = 500 \cdot \frac{1}{1000} + 50 \cdot \frac{5}{1000} + 10 \cdot \frac{20}{1000} + 0 \cdot \frac{974}{1000} = 0.95.$$

2.

$$EE = (500-1) \cdot \frac{1}{1000} + (50-1) \cdot \frac{5}{1000} + (10-1) \cdot \frac{20}{1000} + (0-1) \cdot \frac{974}{1000} = -0.05.$$

Note that this lottery is not a profitable game; you lose 0.05 € on every ticket.

6. 5.

1.

$$E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1.$$

2.

$$D^2(X) = (0-1)^2 \cdot \frac{1}{4} + (1-1)^2 \cdot \frac{1}{2} + (2-1)^2 \cdot \frac{1}{4} = \frac{1}{2}.$$

$$D(X) = \sqrt{\frac{1}{2}} = 0.707106781.$$

6. 6.

1.

$E(X) = 0 \cdot 0.05 + 1 \cdot 0.10 + 2 \cdot 0.15 + 3 \cdot 0.25 + 4 \cdot 0.30 + 5 \cdot 0.10 + 6 \cdot 0.05 = 3.15$ trucks.

2.

$D^2(X) = 0^2 \cdot 0.05 + 1^2 \cdot 0.10 + 2^2 \cdot 0.15 + 3^2 \cdot 0.25 + 4^2 \cdot 0.30 + 5^2 \cdot 0.10 + 6^2 \cdot 0.05 - (3.15)^2 = 2.1275$

.

3.

$D(X) = \sqrt{2.1275} \approx 1.46$ trucks.

6. 7.

1.

$E(X) = 4 \cdot 0.01 + 5 \cdot 0.08 + 6 \cdot 0.29 + 7 \cdot 0.42 + 8 \cdot 0.14 + 9 \cdot 0.06 = 6.78$ periods.

2.

$D^2(X) = 4^2 \cdot 0.01 + 5^2 \cdot 0.08 + 6^2 \cdot 0.29 + 7^2 \cdot 0.42 + 8^2 \cdot 0.14 + 9^2 \cdot 0.06 - (6.78)^2 = 1.0316.$

3.

$D(X) = \sqrt{1.0316} \approx 1.02$ periods.

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