Chapter III

Probability Algebra

Exercises

3. 1.
With reference to the following table what is the probability that a randomly chosen family will have household income

1. between at least 20000 € und less than 40000 €?
2. less than 40000 €?
3. at one of the two extremes of being either less than 20000 € or at least 100000 €?

Annual Household Income for 500 Families

Category	Income range	Number of families
1	[0 20000[	60
2	[20000 40000[	100
3	[40000 60000[	160
4	[60000 100000[	140
5	100000 and above	40
		Total 500

3. 2.
A consumer survey reveals that the probability of a computer owner shopping on the Internet was 0.17, while the probability of a computer owner downloading software was 0.33. Further, the probability of a computer owner doing both was 0.14.
Find the probability of the following events:

1. that a computer owner does not shop on the Internet
2. that a computer owner will either shop on the Internet or download software
3. that the computer owner will neither shop on the Internet nor download software.

3. 3.
If 85% of people have a bowl of cereal for breakfast, 60% of people have some toast for breakfast and 50% of people have both cereal and toast for breakfast, what proportion of people have neither cereal nor toast for breakfast?

3. 4.
According to official statistics, 33% of U. S. adults 20 years of age and over are overweight. If two people in the U. S. are selected at random, what is the probability that both are overweight?

3. 5.
In a sample survey, 1800 senior citizens were asked whether or not they have ever been victimised by a dishonest telemarketer. The following table gives the responses by age group:

Age in Years	Victimised	Not Victimised
60- 69 (A)	106	698
70 – 79 (B)	145	447
80 or over (C)	61	343

Suppose one person is randomly selected from these senior citizens.
Find the probability for the following events:

1. A person has been victimised or belongs to group B
2. A person has never been victimised or belongs to group C.

3. 6.
According to The Digest of Education Statistics 1996, 78.0% of U.S. 13-year-olds were able to perform numerical operations and beginning problem solving.
If two 13- year-olds are randomly selected, what is the probability that none of them can perform numerical operations and beginning problem solving?

3. 7.
Three coins are fairly tossed, and we define:

E_1 : „First two coins are heads“,
E_2 : „Last coin is heads“,
E_3 : „All three coins are heads“.

1. Are E_1 and E_2 independent?
2. Are E_1 and E_3 independent?

3. 8.
A single card is drawn from a standard deck, and we define:

E : „It is an ace. “,
F : „It is a heart. “

Are E and F independent, when we use

1. An ordinary 52-card deck?
2. An ordinary deck, with all the spades deleted?
3. An ordinary deck with all the spades from 2 to 9 deleted?

3. 9.
According to the Labor Force Statistics from the Current Population Survey, in 1996, 52.8% of families in the USA had the husband and wife employed. If 3 families were randomly selected, what would have been the probability that they all had both spouses employed?

3. 10.
In the United States, 43% of people wear a seat belt while driving. If two people are chosen at random, what is the probability that both of them wear a seat belt?

3. 11.
A survey found that 47% of teenagers have a part time job. The same survey found that 78% plan to attend college.
If a teenager is chosen at random, what is the probability that the teenager has a part time job and plans to attend college?

3. 12.
A student goes to the library. The probability that she checks out

(a) a work of fiction is 0.40,
(b) a work of non-fiction is 0.30, and
(c) both fiction and non-fiction is 0.20.

What is the probability that the student checks out a work of fiction, non-fiction, or both?

3. 13.
Marie is getting married tomorrow, at an outdoor ceremony. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% at the time. When it does not rain, he incorrectly forecasts rain 10% at the time.
What is the probability that it will rain on the day of Mary's wedding?

3. 14.
Let A and B be two random events with the probabilities:

$$P(A) = 0.3, \quad P(B) = 0.5, \quad P(A \cap B) = 0.2.$$

Find the probabilities for the following events:

a) $\bar{A}$, b) $\bar{B}$, c) $A \cup B$, d) $\bar{A} \cap \bar{B}$.

3. 15.
The machines $M_i, i = 1, 2, 3,$ produce 20%, 45% and 35% respectively of the total production of a certain article in a firm. Experience shows that 2%, 5% and 3% of the articles produced on these machines are defective.
Find the probability that

1. a defective article has <u>not</u> been produced on the second machine .
2. a non-defective article has been produced either on the second or on the third machine.

3. 16.
The machines $M_i, i = 1, 2, 3,$ produce 30%, 45% and 25% respectively of the total production of a certain article in a firm. Experience shows that 95%, 92% and 98% of the articles produced on these machines are of the highest quality.
Find the probability that

1. an article not being of the highest quality has <u>not</u> been produced on the first machine .
2. an article of highest quality has been produced either on the first or on the third machine.

3. 17.
In throwing three darts at a board, a professional dart player has a probability of 0.9 hitting his targeted region on the first throw. On his second throw, his probability of hitting the targeted region increases by 0.05 if he did hit the targeted region in the first throw but decreases by 0.05 if he missed it.

1. What is his probability of hitting his targeted region on two successive throws?
2. What is the probability that the dart player hits his targeted region on exactly one of his two throws.
3. What is the probability that he misses twice in a row?

3. 18.
Suppose a survey classified the population as male or female, and as favouring or opposing the death penalty. Suppose, the proportions in each category were:

	Death	not Death
Male	0.459	0.441
Female	0.051	0.049

Find the probability of an individual favouring the death penalty, conditional on being male.

3. 19.
Suppose there is a certain defect randomly found in 0.5% of the products of a firm. A quality control is *positive* (shows the presence of this certain defect) in 99% of all cases. But it yields a *negative* result (indicates the presence of this type of defect where there is actually no such defect) with a probability of 5%.

Find the probability that

1. the defect will not be present in any particular product.
2. the quality control will yield a negative result if the defect is present.
3. the quality control will yield a negative result if the defect is not present.
4. the quality control will yield a positive result, irrespective of whether the defect is present or not.
5. the quality control will yield a negative result, irrespective of whether the defect is present or not.
6. the defect is present if the quality control result is positive.
7. the defect is not present if the quality control result is positive.
8. the defect is absent if the quality control result is negative.
9. the defect is present if the quality control result is negative.

3. 20.
Suppose that you are given two drawers. You cannot see the contents of the drawers, but you are told that one drawer contains two gold coins and the other drawer contains one gold coin and one silver coin.
If somebody pulls a coin at random out of drawer A and it turns out to be the gold, what is the probability that drawer A is the drawer with two gold coins?

3. 21.
If there is an increase in capital investment next year, the probability that structural steel will increase in price is 0.90. If there is no increase in such investment, the probability of an

increase is 0.40. Overall, we estimate that there is a 60 percent chance that capital investment will increase next year.

1. What is the probability that structural steel prices will not increase even though there is an increase in capital investment?
2. What is the overall probability of an increase in structural steel prices next year?
3. Suppose that during the next year structural steel prices in fact increase. What is the probability that there was an increase in capital investment?

3. 22.
A factory has three types of machines producing an item. Probabilities that the item is of high quality if it is produced on i-th machine $(i = 1, 2, 3)$ are given in the following table:

Machine	Probability of High Quality
1	0.8
2	0.7
3	0.9

The total production is done 30% on type 1 machine, 50% on type 2, and 20% on type 3.

One item is selected at random from the production.

1. What is the probability that it is of high quality?
2. What is the probability that it is not of high quality?
3. If it is of high quality, what is the probability that it was produced on machine 1?
4. If it is of high quality, what is the probability that it was produced on machine 3?
5. If it is not of high quality, what is the probability that it was not produced on machine 3?
6. If it is not of high quality, what is the probability that it was not produced on machine 2?

3. 23.
In a certain community approximately 20% of senior citizens (65 or older) spent their holidays abroad each year. However, about 30% of people under 65 spent their holidays abroad each year. Also, approximately 25% of the community is composed of senior citizens.

1. What is the probability that a person selected at random from the community is a senior citizen who will spend his holidays abroad this year?
2. What is the probability that a person selected at random from the community is a person under 65 who will spend his holidays abroad this year?
3. What is the probability that a person selected at random from the community will spend his holidays abroad this year?
4. Are the events an individual is a senior citizen and the event an individual will spend his holidays abroad this year independent? Are they mutually exclusive?
5. What is the probability that a person selected at random from the community will spend his holidays abroad this year or is a senior citizen?

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