Chapter V

Discrete and Continuous Random Variables

Solutions

5. 1.
1.

$$p_3 = 1 - (0.70 + 0.10 + 0.05 + 0.05 + 0.03) = 1 - 0.93 = 0.07$$

2.

$$P(X \le 3) = 0.70 + 0.10 + 0.05 + 0.05 = 0.90$$

3.

$$F(x) = \begin{cases} 0.00 & \text{when} \quad -\infty < x \le 0 \\ 0.70 & \text{when} \quad 0 < x \le 1 \\ 0.80 & \text{when} \quad 1 < x \le 2 \\ 0.85 & \text{when} \quad 2 < x \le 3 \\ 0.90 & \text{when} \quad 3 < x \le 4 \\ 0.93 & \text{when} \quad 4 < x \le 5 \\ 1.00 & \text{when} \quad 5 < x < +\infty \end{cases}$$

4.
i)

$$F(3.8) = P(X < 3.8) = 0.9$$

The probability that the number of broken bricks is less than 3.8 is equal to 90%.

ii)

$$F(4.7) - F(1.8) = 0.93 - 0.80 = 0.13 = P(1.8 \le X < 1.8)$$

The probability that the number of broken bricks is at least 1.8 and less than 4.7 is equal to 13%.

5. 2.
1.

$$p_2 = 1 - (0.70 + 0.10 + 0.05 + 0.05) = 1 - 0.90 = 0.10$$

2.

$$P(X \geq 2) = 0.10 + 0.05 + 0.05 = 0.20$$

3.

$$F(x) = \begin{cases} 0.00 & \text{when} \quad -\infty < x \leq 0 \\ 0.70 & \text{when} \quad 0 < x \leq 1 \\ 0.80 & \text{when} \quad 1 < x \leq 2 \\ 0.90 & \text{when} \quad 2 < x \leq 3 \\ 0.95 & \text{when} \quad 3 < x \leq 4 \\ 1.00 & \text{when} \quad 4 < x < +\infty \end{cases}$$

4.
i)

$$F(2.06) = 0.9$$

The probability that the number of passengers is less than 2.06 is equal to 90%.

ii)

$$F(3.9) - F(0.05) = 0.95 - 0.70 = 0.25 = P(0.05 \leq X < 3.9)$$

The probability that the number of passengers is at least 0.05 and less than 3.9 is equal to 25 %.

5. 3.
1.

$$F(t) = P(T < t) \approx P(T \leq t) = 1 - P(T > t) = 1 - e^{-\frac{t}{10}}$$

$$P(5 \leq T < 10) = \left(1 - e^{-1}\right) - (1 - e^{-0.5}) = 0.23865122$$

2.

$$f(t) = F'(t) = \frac{1}{10} e^{-\frac{t}{10}}, \quad t > 0$$

5. 4.
1.

$$P(X > 2) = 0.05 + 0.05 = 0.10$$

2.

$$P\left(X \geq 2\right) = 0.10 + 0.05 + 0.05 = 0.20\,.$$

3.

$$P(X = 0) = 0.7$$

4.

$$k = 3.$$

5.

0, 0.7, 0.7, 0.9, 1.

5. 5.

$$P\left(0 < X < \frac{1}{3}\right) = F\left(\frac{1}{3}\right) - F(0) = \left(\frac{3}{4}\cdot\frac{1}{3} + \frac{3}{4}\right) - \frac{3}{4} = \frac{1}{4}\,.$$

5. 6.

a)

$$\int_{-\infty}^{+\infty} f(x)dx = 1;$$

$$\int_{-3}^{0} a\cdot(3+x)dx + \int_{0}^{3} a\cdot(3-x)dx = \left[3ax + a\cdot\frac{x^2}{2}\right]_{-3}^{0} + \left[3ax - a\cdot\frac{x^2}{2}\right] = 1;$$

$$-\left(3a\cdot(-3) + a\cdot\frac{9}{2}\right) + \left(9a - 9\cdot\frac{a}{2}\right) = 1;$$

$$a = \frac{1}{9}.$$

b)

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$F(x) = \begin{cases} 0 & -\infty < x \leq -3 \\ \frac{1}{18}x^2 + \frac{1}{3}x + \frac{1}{2} & -3 < x \leq 0 \\ -\frac{1}{18}x^2 + \frac{1}{3}x + \frac{1}{2} & 0 < x \leq 3 \\ 1 & 3 < x < +\infty \end{cases};$$

c)

$$P\left(\frac{1}{2} \leq X \leq 1\right) = F(1) - F(0.5) = \frac{1}{9}\cdot\int_{0.5}^{1}(3-x)dx = \frac{1}{8}\,.$$

5. 7.

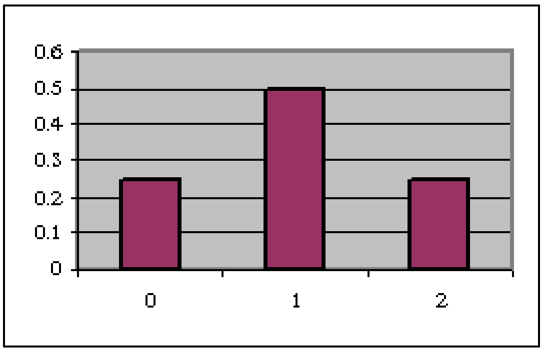
Let H_i and T_i denote the observation of a head and a tail, respectively, on the i^{th} toss for $i = 1,\ 2$ ·

The four simple events and the associated values of H_i are shown in the following table:

Event	**Probability**	**Number of Heads**
(H_1, H_2)	0.25	2
(H_1, T_2)	0.25	1
(T_1, H_2)	0.25	1
(T_1, T_2)	0.25	0

The Probability Distribution Function

x	0	1	2
$P(X = x)$	0.25	0.50	0.25



5. 8.

1.

$$\int_{\frac{3}{2}}^{3} k(t-\frac{3}{2})(3-t)\,dt = 1,$$

$$\int_{\frac{3}{2}}^{3} k(t-\frac{3}{2})(3-t)\,dt = k\int_{\frac{3}{2}}^{3}(t-\frac{3}{2})(3-t)dt = k\int_{\frac{3}{2}}^{3}\left(-t^2+\frac{9}{2}t-\frac{9}{2}\right)dt$$

$$= k\left[-\frac{t^3}{3}+\frac{9t^2}{4}-\frac{9t}{2}\right]_{\frac{3}{3}}^{3} = 1,$$

$$k\left[\left(-\frac{27}{3}+\frac{9}{4}\cdot 9-\frac{9}{2}\cdot 3\right)-\left(-\frac{\frac{27}{8}}{3}+\frac{9}{4}\cdot\frac{9}{4}-\frac{9}{2}\cdot\frac{3}{2}\right)\right]=\frac{9}{16}\cdot k=1,$$

$$k=\frac{16}{9}.$$

2.

$$\frac{16}{9}\int_{\frac{5}{2}}^{3}(t-\frac{3}{2})\left(3-t\right)dt=\frac{16}{9}\left[-\frac{t^3}{3}+\frac{9t^2}{4}-\frac{9t}{2}\right]_{\frac{5}{2}}^{3}$$

$$=\frac{16}{9}\cdot\frac{7}{48}=\frac{7}{27}\approx 0.259259259$$

(Last revised: 05.06.07)