Chapter III

Probability Algebra

Solutions

3. 1.

1.

$$P(2)=\frac{100}{500}=0.20.$$

2.

$$P(1\text{ or }2)=\frac{60}{500}+\frac{100}{500}=0.32.$$

3.

$$P(1\text{ or }5)=\frac{60}{500}+\frac{40}{500}=0.20.$$

3. 2.

Denote by

I: "A computer owner shops on the Internet",
D: "A computer owner downloads software",

1.

$$P(\bar{I})=1-0.17=0.83.$$

2.

$$P(I\cup D)=P(I)+P(D)-P(I\cap D)=0.17+0.33-0.14=0.36.$$

3.

$$P(\bar{I}\cap\bar{D})=1-(I\cup D)=1-0.36=0.64.$$

3. 3.

Denote by

C: "People have a bowl of cereal for breakfast."
T: "People have some toast for breakfast."

$$P(\bar{C}\cap\bar{T})=1-P(C\cup T)=1-(P(C)+P(T)-P(C\cap T)$$

$$=1-(0.85+0.60-0.50)=0.05$$

3. 4.

$$P=\frac{1}{3}\cdot\frac{1}{3}\approx 0.11$$

3. 5.
Denote by

V : „A senior citizen has been victimised“

1.

$$P\left(V \cup B\right) = P(V) + P(B) - P(V \cap B)$$

$$P\left(V \cup B\right) = \frac{106+145+61}{1800} + \frac{145+447}{1800} - \frac{145}{1800} = \frac{759}{1800} = 0.421666666 \approx 0.42 .$$

2.

$$P(\bar{V} \cup C) = P(\bar{V}) + P(C) - P(\bar{V} \cap C)$$

$$P(\bar{V} \cup C) = \frac{698+447+343}{1800} + \frac{61+343}{1800} - \frac{343}{1800} = \frac{1549}{1800} = 0.860555555 \approx 0.86..$$

3. 6.

$$P = \left(1 - 0.78\right)^2 = 0.22^2 = 0.0484 .$$

3. 7.
List of all possible cases:

H	H	H
H	H	T
H	T	H
T	H	H
H	T	T
T	H	T
T	T	H
T	T	T

$E_1 = \{(H, H, H), \{H, H, T\}\}$

$E_2 = \{(H, H, H), (H, T, H), (T, H, H), (T, T, H)\}$

$E_3 = \{(H, H, H)\}$

1.

$$P(E_1) = \frac{1}{4} = P(E_1 / E_2) ,$$

$\therefore$ E_1 and E_2 are independent.

2.

$$P(E_1) = \frac{1}{4} \neq 1 = P(E_1 / E_3)$$

$\therefore$ E_1 and E_3 are dependent.

3. 8.

1.

$$P(E)=\frac{4}{52}=\frac{1}{13}=P(E/F)$$

$\therefore$ E and F are independent.

2.

$$P(E)=\frac{3}{39}=\frac{1}{13}=P(E/F)$$

$\therefore$ E and F are independent.

$$P(E)=\frac{4}{44}=\frac{1}{11}\neq\frac{1}{13}P(E/F)$$

$\therefore$ E and F are dependent.

3. 9.

$$P=0.528^3\approx 0.15$$

3. 10.

$$P=0.43^2=0.1849$$

3. 11.

Denote by

J: "A teenager has a part time job."
C: "A teenager plans to attend college."

$$P(J\cap C)=P(C)\cdot P(J)=0.47\cdot 0.78=0.3666$$

(Note: It will be assumed that two events are independent.)

3. 12.

$$P=0.40+0.30-0.20=0.50$$

3. 13.

Let

A: „the weatherman predicts rain. "

B_1: „it rains on Mary's wedding. "

B_2: „it does not rain on Mary' wedding. "

In terms of probabilities, we know the following:

$$P(B_1)=\frac{5}{365}=0.0136986 \quad , \quad P(B_2)=\frac{360}{365}=0.9863014$$

$$P(A/B_1)=0.9 \quad , \quad P(A/B_2)=0.1\,.$$

$$P(B_1/A)=\frac{P(A/B_1).P(B_1)}{\sum_{i=1}^{2}P(A/B_i)\cdot P(B_i)}$$

$$=\frac{\frac{5}{365}\cdot 0.9}{\frac{5}{365}\cdot 0.9+\frac{360}{365}\cdot 0.1}=\frac{0.0123288}{0.1109589}=0.1111114\,.$$

Therefore, despite the weatherman’s gloomy prediction, there is a good chance that Marie will not get rained at her wedding.

3. 14.

a)

$$P(\bar{A})=1-P(A)=1-0.3=0.7$$

b)

$$P(\bar{B})=1-P(B)=1-0.5=0.5$$

c)

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.3+0.5-0.2=0.6$$

d)

$$P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)=1-0.6=0.4$$

3. 15.

Let

A: „The article is defective.”

B_i: “the article has been produced on machine M_i, $i=1,2,3$.”

We have:

$$P(B_1)=0.20, \qquad P(B_2)=0.45, \qquad P(B_3)=0.35\,,$$

$$P(A/B_1)=0.02, \qquad P(A/B_2)=0.05, \qquad P(A/B_3)=0.03\,.$$

1.

$$P(\bar{B}_2/A)=1-P(B_2/A)$$

$$= 1 - \frac{0.45 \cdot 0.05}{0.20 \cdot 0.02 + 0.45 \cdot 0.05 + 0.35 \cdot 0.03}$$

$$= 1 - \frac{0.0225}{0.037} \approx 0.39$$

2.

$$P(\bar{B}_3 / \bar{A}) = 1 - P(B_3 / \bar{A})$$

$$= 1 - \frac{0.20 \cdot 0.98}{0.20 \cdot 0.98 + 0.45 \cdot 0.95 + 0.35 \cdot 0.93}$$

$$= 1 - \frac{0.1960}{0.949} \approx 0.79 .$$

3. 16.
Let

A: „The article is of the highest quality."

B_i: "The article has been produced on machine M_i, $i = 1, 2, 3$."

We have:

$P(B_1) = 0.30,\qquad P(B_2) = 0.45,\qquad P(B_3) = 0.25$,

$P(A / B_1) = 0.95,\qquad P(A / B_2) = 0.92,\qquad P(A / B_3) = 0.98$.

1.

$$P(\bar{B}_1 / \bar{A}) = 1 - P(B_1 / \bar{A})$$

$$= 1 - \frac{0.30 \cdot 0.05}{0.30 \cdot 0.05 + 0.45 \cdot 0.08 + 0.25 \cdot 0.02}$$

$$= 1 - \frac{0.015}{0.056} \approx 0.73$$

2.

$$P(\bar{B}_2 / A) = 1 - P(B_2 / A)$$

$$= 1 - \frac{0.45 \cdot 0.92}{0.30 \cdot 0.95 + 0.45 \cdot 0.92 + 0.25 \cdot 0.98}$$

$$= 1 - \frac{0.414}{0.944} \approx 0.56$$

3. 17.

1.

Let H_i stand for the event "the player hits his targeted region on the i^{th} throw." Then

$$P(H_1) = 0.90, \quad P(H_2 / H_1) = 0.95.$$

Thus

$$P(H_1 \cap H_2) = P(H_1) \cdot P(H_2 / H_1) = 0.90 \cdot 0.95 = 0.885.$$

2.

$$P(\text{"exactly one hit"}) = P\left[\left(H_1 \cap \bar{H}_2\right) \cup \left(\bar{H}_1 \cap H_2\right)\right]$$

$$= P(H_1) \cdot P(\bar{H}_2 / H_1) + P(H_2) \cdot P(H_2 / \bar{H}_1)$$

$$= 0.90 \cdot 0.05 + 0.10 \cdot 0.85 = 0.130.$$

3.

$$P(\text{"missing in a row"}) = P\left(\bar{H}_1\right) \cdot P\left(\bar{H}_2 / \bar{H}_1\right) = 0.10 \cdot 0.15 = 0.015.$$

3. 18.

Definition of events:

D: „An individual favours death penalty“,
M: „An individual is a man. “

$$P(D / M) = \frac{P(D \cap M)}{P(M)}$$
$$= \frac{0.459}{0.459 + 0.441} = 0.51$$

3. 19.

Definition of events:

A: „ The defect will not be present in any particular product”
B: „The quality control will yield a positive result”.

Therefore, we have:

$$P(A) = 0.005, \quad , \quad P(B / A) = 0.99, \quad P(B / \bar{A}) = 0.05.$$

1.

$$P(\bar{A}) = 1 - P(A) = 1 - 0.005 = 0995\,.$$

2.

$$P(\bar{B}/A) = 1 - (B/A) = 1 - 0.99 = 0.01\,.$$

3.

$$P(\bar{B}/\bar{A}) = 1 - P(B/\bar{A}) = 1 - 0.05 = 0.95\,.$$

4.

$$P(B) = P(B/A)\cdot P(A) + P(B/\bar{A})\cdot P(\bar{A})$$

$$= 0.99\cdot 0.005 + 0.05\cdot 0.995 = 0.0547\,.$$

5.

$$P(\bar{B}) = P(\bar{B}/A)\cdot P(A) + P(\bar{B}/\bar{A})\cdot P(\bar{A})$$

$$= 0.01\cdot 0.005 + 0.95\cdot 0.995 = 0.9453$$

$(= 1 - P(B))$.

6.

$$P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)} = \frac{0.99\cdot 0.005}{0.0547} = 0.0905\,.$$

7.

$$P(\bar{A}/B) = \frac{P(B/\bar{A})\cdot P(\bar{A})}{P(B)} = \frac{0.05\cdot 0.995}{0.0547} = 0.9095 \quad (= 1 - P(A/B))$$

8.

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{B}/\bar{A})\cdot P(\bar{A})}{P(\bar{B})} = \frac{0.95\cdot 0.995}{0.9453} = 0.99995$$

9.

$$P(A/\bar{B}) = \frac{P(\bar{B}/A)\cdot P(A)}{P(\bar{B})} = \frac{0.01\cdot 0.005}{0.9453} = 0.00005 \; (= 1 - P(\bar{A}/\bar{B})).$$

3. 20.
Definition of events:

A: „Drawer A has two gold coins”.
B: „Person chooses a gold coin out of the four coins”.

$$P(A) = 0.5\,, \quad P(B) = 0.75\,, \quad P(B/A) = 1\,.$$

Using Bayes’ Theorem, we have

$$P(A/B)=\frac{P(B/A)\cdot P(A)}{P(B)}=\frac{1\cdot 0.5}{0.75}=\frac{2}{3}.$$

3. 21.
Denote by

I: "Increase of capital investment."
R: "Rise of structural steel prices."

1.

$$P(\bar{R}/I)=0.10.$$

2.

$$P(R)=P\left(I\cap R\right)+P(\bar{I}\cap R)$$

$$=P(I)\cdot P(R/I)+P(\bar{I}).P(R(R/\bar{I})$$

$$=0.60\cdot 0.90+0.40\cdot 0.40=0.70.$$

3.

$$P(R/I)=\frac{P(I\cap R)}{P(R)}=\frac{P(I)\cdot P(R/I)}{P(I)\cdot P(R/I)+P(\bar{I}).P(R(R/\bar{I})}$$

$$=\frac{0.60\cdot 0.90}{0.60\cdot 0.90+0.40\cdot 0.40}=\frac{0.54}{0.70}\approx 0.77.$$

3. 22.
Denote by

A: „An item is of high quality",

B_i $(i=1,2,3)$: „An item is produced on machine i".

We have:

$P(B_1)=0.30$, $P(B_2)=0.50$, $P(B_3)=0.20$,

$P(A/B_1)=0.80$, $P(A/B_2)=0.70$, $P(A/B_3)=0.90$.

1.

$$P(A)=\sum_{i=1}^{3}P(B_i)\cdot P(A/B_i)=0.30\cdot 0.80+0.50\cdot 0.70+0.20\cdot 0.90=0.77.$$

2.

$$P(\bar{A})=1-P(A)=1-0.77=0.23.$$

3.

$$P(B_1 / A) = \frac{P(B_1) \cdot P(A / B_1)}{P(A)} = \frac{0.30 \cdot 0.8}{0.77} = 0.311688311 \approx 0.31.$$

4.

$$P(B_3 / A) = \frac{P(B_3) \cdot P(A / B_3)}{P(A)} = \frac{0.20 \cdot 0.9}{0.77} = 0.233766233 \approx 0.23.$$

5.

$$P(\bar{B}_3 / \bar{A}) = 1 - P(B_3 / \bar{A}) = 1 - \frac{P(\bar{B}_3) \cdot P(\bar{A} / B_3)}{P(\bar{A})} = 1 - \frac{0.20 \cdot 0.10}{0.23} = 0.913043478 \approx 0.91$$

6.

$$P(\bar{B}_2 / \bar{A}) = 1 - P(B_2 / \bar{A}) = 1 - \frac{P(\bar{B}_2) \cdot P(\bar{A} / B_2)}{P(\bar{A})} = 1 - \frac{0.50 \cdot 0.30}{0.23} = 0.347826087 \approx 0.35.$$

3. 23.
Denote by

A: "A person spends his holidays abroad"

S: "A person is a senior."

1.

$$P\left(A \cap S\right) = P(S) \cdot P(A / S) = 0.25 \cdot 0.20 = 0.05.$$

2.

$$P(A \cap \bar{S}) = P(\bar{S}) \cdot P(A / \bar{S}) = 0.75 \cdot 0.30 = 0.225.$$

3.

$$P(A) = P\left(A \cap S\right) + P(A \cap \bar{S}) = 0.05 + 0.225 = 0.275.$$

4.

$$P(A) = 0.275 \neq 0.200 = P(A / S)\,.$$

The two events are, therefore, dependent.

$$P(A \cap S) = 0.05 \neq 0\,.$$

The two events are, therefore, not mutually exclusive.

5.

$$P(A \cup S) = P(A) + P(S) - P(A \cap S) = 0.275 + 0.250 - 0.05 = 0.475.$$

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