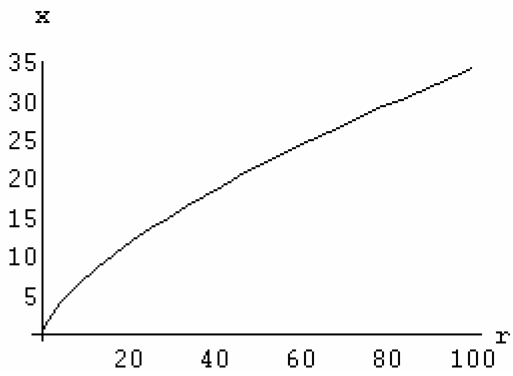
Analysis in Economics (I)

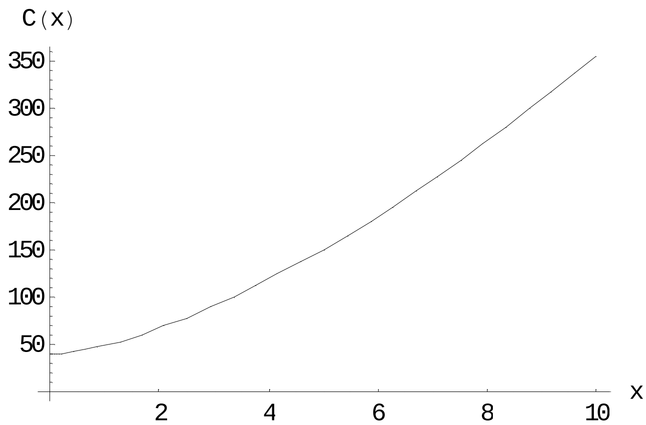
Solutions

1.

$$x = \sqrt[3]{4r^2}, \quad r \geq 0 \quad \Rightarrow \quad r = \frac{1}{2}\sqrt{x^3}$$



$$C(x) \; = 40 + 10\sqrt{x^3} \, .$$



2.
1.
$x - intercept$:

$$\left(-x^2 + 20x + 312 = 0 \;\wedge\; x \geq 0\right) \;\Rightarrow\; x = 30.29778313 \approx 30.30 \, ,$$

i. e. for an output of 20.30 units the company's profit will be equal to zero.

$y - intercept$:

$$x = 0 \;\Rightarrow\; P(x) = 312,$$

i. e. for an output of zero units the company's profit will be equal to 312.

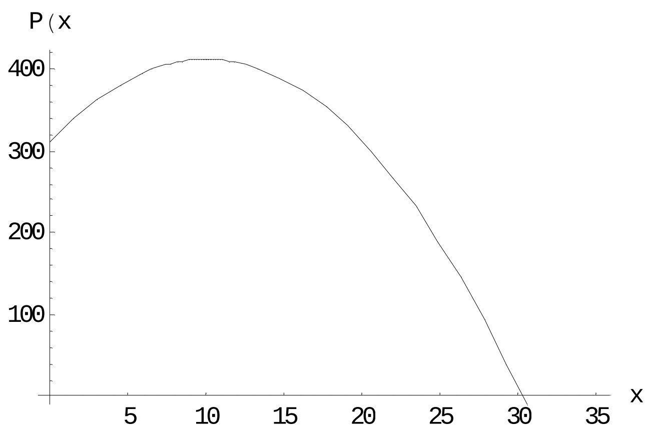
2.

$$P'(x) = -2x + 20\,, \quad P'(x) = 0 \;\Rightarrow\; -2x + 20 = 0 \;\Rightarrow\; x = 10\,,$$

$$P''(x) = -2 < 0\,.$$

Hence, $x = 10$ gives the maximum profit. This will be equal to $P(10) = 412$.

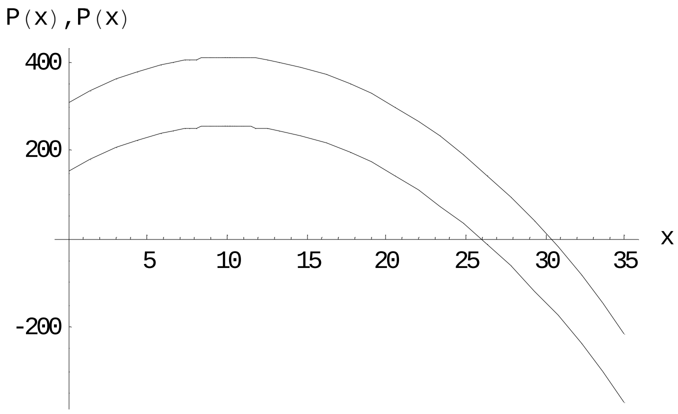
3.



4.
Changing the value of the constant simply moves the profit curve up or down and, in particular, it does not change the $x-$coordinate of the maximum. In this case, changing the original constant 312 to 156 means that the profit function is now

$$\tilde{P}(x) = -x^2 + 20x + 156\,,$$

and so the curve will move down by 312-156 = 156 which means that the maximum value will now be 412 – 156 = 256 and this will still occur when $x = 10$.
The effect of changing the constant in this way is illustrated in the following figure:



5.

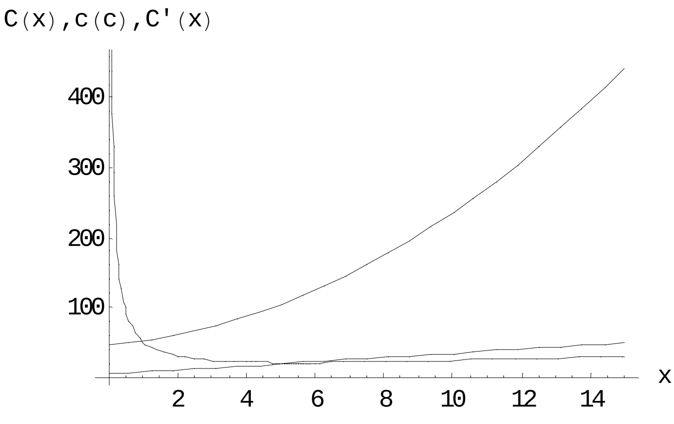
$$x = 4 \;\Rightarrow\; P(4) = 620, \qquad x = 8 \;\Rightarrow\; P(8) = 700,$$

$$C(x) - 620 = \frac{700 - 620}{8 - 4}\left(x - 4\right),$$

$$C(x) = 20x + 540$$

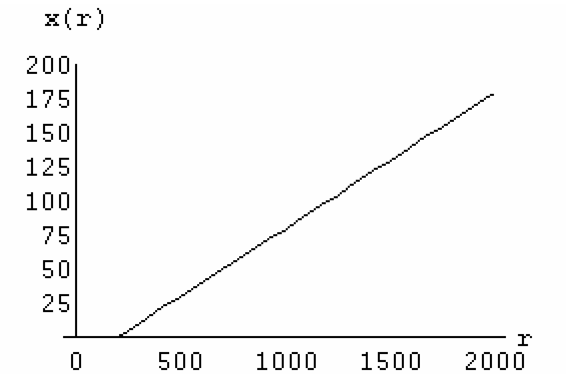
3.

$$C(x) = x \cdot c(x)$$
$$C(x) = 1.5x^2 + 4x + 46$$
$$C'(x) = 3x + 4$$



4.

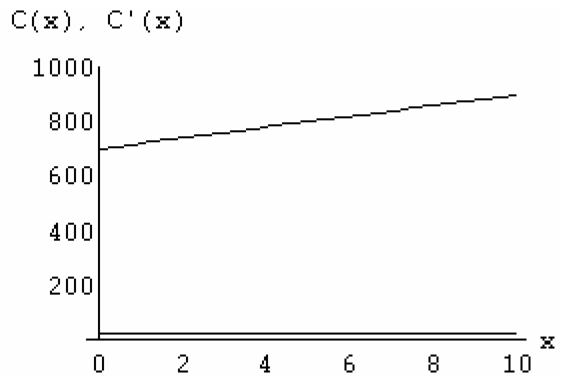
1.



$$x(r) = -20 + 0.1r \quad \Rightarrow \quad r(x) = 10x + 200$$

$$C(x) = C_{\text{var}} + C_{fix}$$
$$= (10x + 200) \cdot 2 + 300 = 20x + 700$$

$$C^{'}(x) = 20.$$

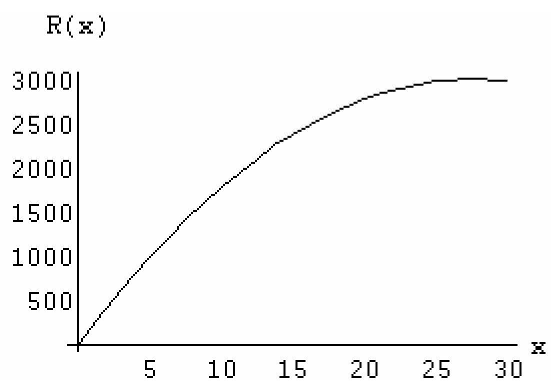


2.

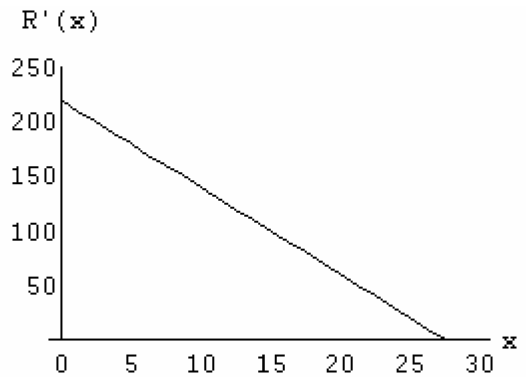
The cost function has because of $C^{'}(x) = 20 \neq 0$ no minimum.

3.

$$R(x) = p(x) \cdot x$$
$$= 220x - 4x^2,$$



$$R^{'}(x) = 220 - 8x.$$



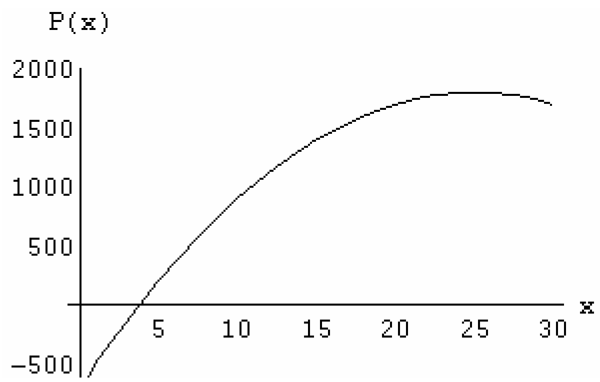
4.

$P(x) = R(x) - C(x)$, $\quad P'(x) = R'(x) - C'(x) = 220 - 8x - 20$.

$P'(x) = 0 \quad \Rightarrow \quad x = 25$,

$P''(x) = -8 < 0$, $\quad P(25) = 1800$ €.

The firm has, therefore, to produce 25 units in order to gain a maximum profit of 1800 €.



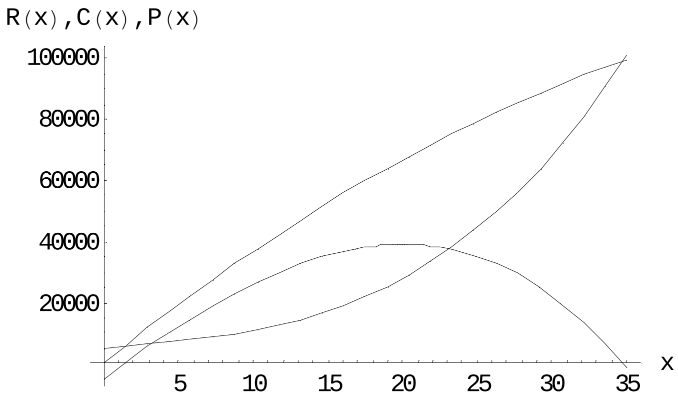
5.

$p(25) = 120$ €

5.

$$P(x) = R(x) - C(x)$$
$$= 4000x - 33x^2 - \left(2x^3 - 3x^2 + 400x + 5000\right)$$
$$P(x) = -2x^3 - 30x^2 + 3600x - 5000$$
$$\left(P'(x) = -6x^2 - 60x + 3600 \quad \wedge \quad x > 0\right) \Rightarrow x = 20$$
$$P''(x) = -12x - 60, \quad P''(20) = -300 < 0.$$

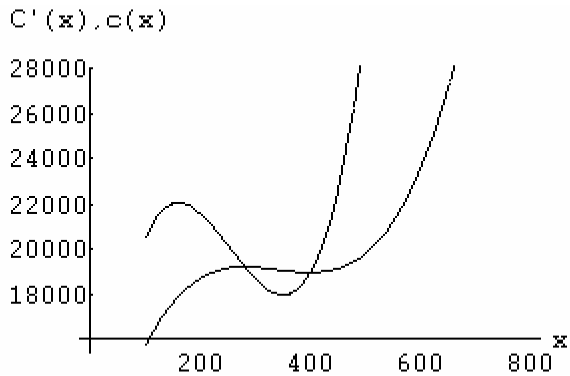
$\therefore$ Profit is maximised at $x = 20$ where $P(20) = 39000$.



6.

$$C'(x) = 8400 + 10^{-4}(2016000x - 9180x^2 + 12x^3)$$
$$c(x) = 8400 + 10^{-4}(1008000x - 3060x^2 + 3x^3)$$

$$C'(x) < c(x)$$
$$\Leftrightarrow$$
$$8400 + 10^{-4}(2016000x - 9180x^2 + 12x^3) < 8400 + 10^{-4}(1008000x - 3060x^2 + 3x^3)$$
$$\Leftrightarrow$$
$$x^3 - 680x^2 + 112000\text{x} < 0 \;\Leftrightarrow\; 280 < x < 400$$



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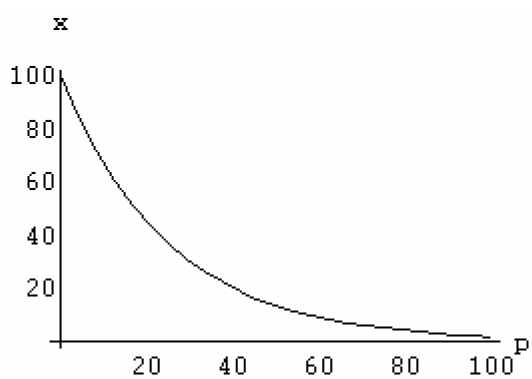
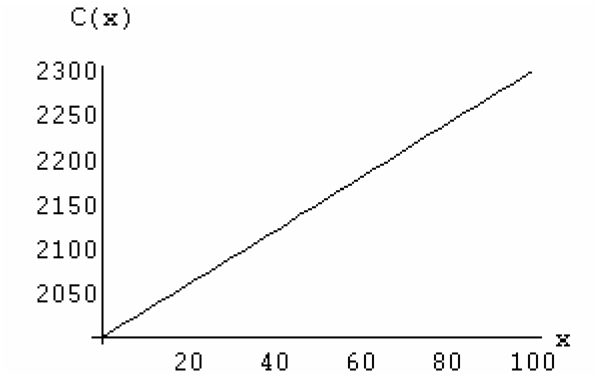
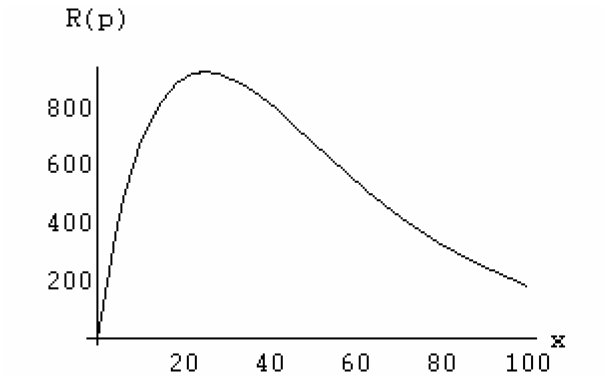




1.

$$R(p) = x(p) \cdot p$$
$$= 100pe^{-0.04p}, \qquad 0 \le p \le 100$$



$$P(p) = R(p) - C(p)$$
$$= 100pe^{-0.04p} - 2000 - 3000e^{-0.04p}, \qquad 0 \le p \le 100$$

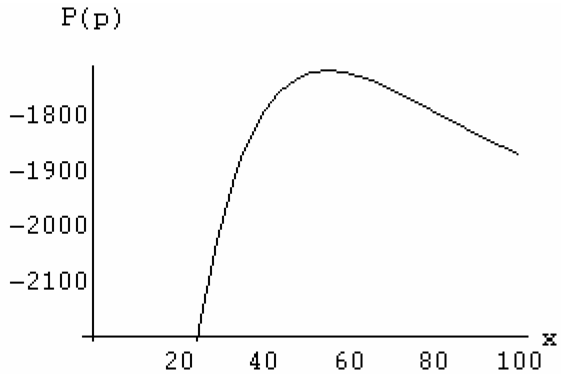
2.

$$P'(p) = e^{-0.04p}(220 - 4p), \qquad 0 < p < 100$$

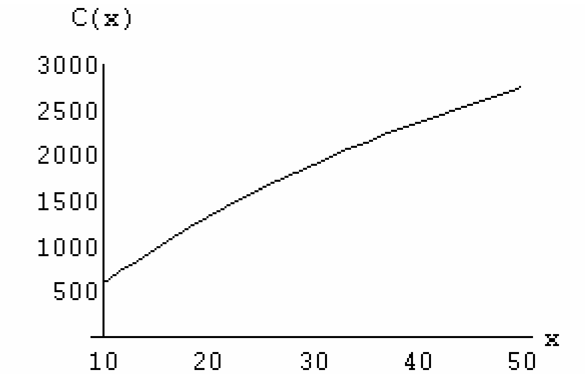
$$P''(p) := 0 \qquad \Rightarrow \qquad p = 55$$

$$P''(p) = -0.04e^{-0.04p}(220 - 4p) - 4e^{-0.04p} \qquad \Rightarrow \qquad P''(p) < 0\,.$$

The profit function $P(p)$ has because of $P(100) < P(55)$ its absolute maximum at $p = 55$



8.

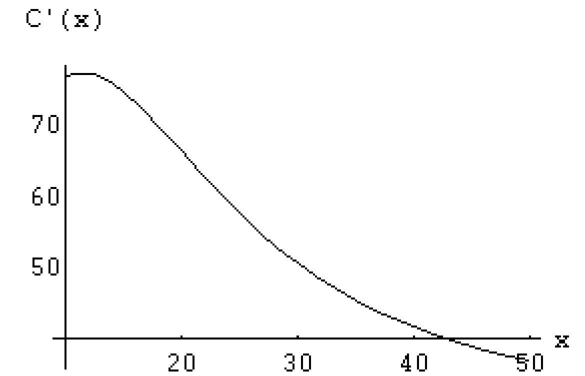


1.

$$C'(x) = 30 + \frac{1160000x}{(400+x^2)^2}, \qquad 10 < x < 50$$

$$C'(20) = 66.25\,€$$

An increase of the output by one unit leads to an approximate increase of the total costs by 66.25 €.



$$\begin{aligned} c(x) &:= \frac{C(x)}{x} \\ &= 30 + \frac{1450x}{400+x^2}, \qquad 10 \le x \le 50 \end{aligned}$$

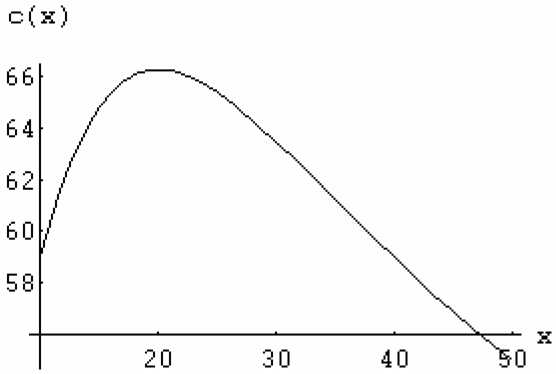
$$c'(x) = \frac{-1450x^2 + 580000}{(400+x^2)^2}, \qquad 10 < x < 50$$

$$c'(x) := 0 \qquad \Rightarrow \qquad x = 20$$

$$c''(x) = \frac{-2900x \cdot (400+x^2) - 4x \cdot (580000 - 1450x^2)}{(400+x^2)^3}, \qquad 10 < x < 50$$

$$c''(20) < 0$$

Consequently, at $x = 20$ the function $c(x)$ has over the interval $]10,\ 50[$ a relative maximum. Because of $c(10) = 59$, $c(50) = 55$ the function $c(x)$ has in $x = 50$ its absolute minimum.



3.
From 2. it follows that $c(x)$ increases on $[10,\ \ 20]$ and decreases on $[20,\ \ 50]$.

9.
1.
a) To maximise profits under price discrimination, the producer will set prices so that the marginal costs are equal to marginal revenues in each market. Thus

$$C'(x) = R_1'(x_1) = R_2'(x_2).$$

$C'(x) = 10$

$x_1 = 21 - 0.1p_1 \quad \Rightarrow \quad p_1 = 210 - 10x_1$

$R_1(x_1) = (210 - 10x_1)x_1 = 210x_1 - 10x_1^2, \qquad R_1'(x_1) = 210 - 20x_1$

$R_1'(x_1) = C'(x) \quad \Rightarrow \quad 210 - 20x_1 = 10 \quad \Rightarrow \quad x_1 = 10$

$p_1(10) = 110.$

$x_2 = 50 - 0.4p_2 \quad \Rightarrow \quad p_2 = 125 - 2.5x_2$

$R_2(x_2) = (125 - 2.5x_2)x_2 = 125x_2 - 2.5x_2^2, \quad R_2'(x_2) = 125 - 5x_2$

$R_2'(x_2) = C'(x) \quad \Rightarrow \quad 125 - 5x_2 = 10 \quad \Rightarrow \quad x_2 = 23$

$p_2(23) = 67.5.$

Thus, the discriminating producer charges a lower price in the foreign market.

b) If the producer does not discriminate, $p_1 = p_2$ and the two demand functions can simply be aggregated. Thus,

$x = x_1 + x_2 = 21 - 0.1p + 50 - 0.4p$

$$x = 71 - 0.5p \quad \Rightarrow \quad p = 142 - 2x$$

$$R(x) = (142 - 2x)x = 142x - 2x^2, \qquad R'(x) = 142 - 4x$$

$$R'(x) = C'(x) \quad \Rightarrow \quad 142 - 4x = 10 \quad \Rightarrow \quad x = 33$$

$$p(33) = 76.$$

Thus, when no discrimination takes place, the price falls somewhere between the relatively high price of the domestic market and the relatively low price of the foreign market. Notice however, that the quantity sold remains the same: At $p = 76,\ x_1 = 13.4,\ x_2 = 19.6,$ and $x = 33.$

2.
With discrimination,

$$R(x) = R_1(x_1) + R_2(x_2) = p_1 \cdot x_1 + p_2 \cdot x_2 = 110 \cdot 10 + 67.5 \cdot 23 = 2652.50$$

$$C(x) = 2000 + 10x = 2000 + 10(x_1 + x_2) = 2000 + 10(10 + 23) = 2330$$

$$P(x) = R(x) - C(x) = 2652.50 - 2330 = 322.50$$

Without discrimination,

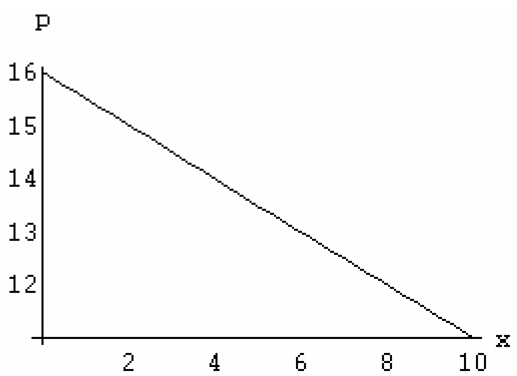
$$R(x) = p \cdot x = 76 \cdot 33 = 2508.$$

Total costs are equal to 2330 since costs do not change with or without discrimination. Thus

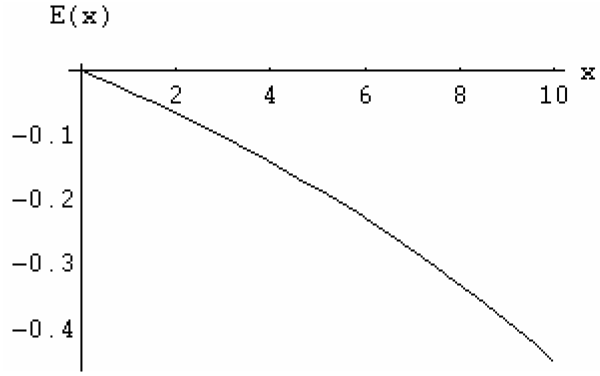
Profit = 2508 – 2330 = 178.

Therefore, profits are higher with discrimination than without discrimination.

10.
1.



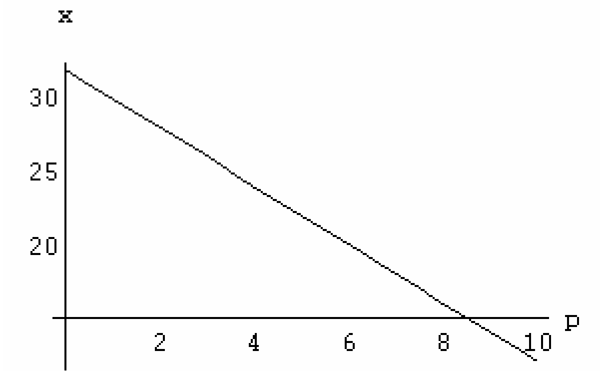
$$\varepsilon_{p,x}(x) = \frac{-0.5x}{16 - 0.5x}$$



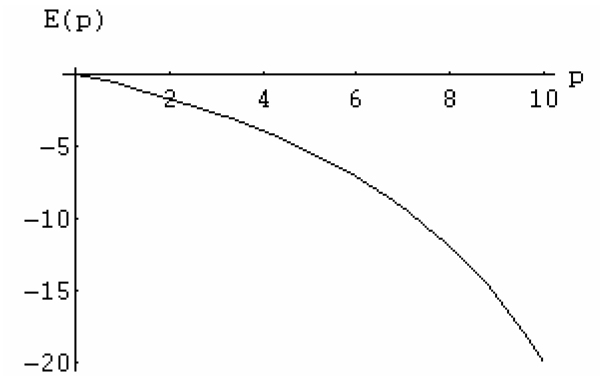
$$\varepsilon_{p,x}(8) = -\frac{1}{3}.$$

2.

$$p(x) = 16 - 0.5p \quad \Rightarrow \quad x(p) = 32 - 2p$$



$$\varepsilon_{x,p}(p) = \frac{-24p}{32 - 2p}$$



$p(8) = 12$

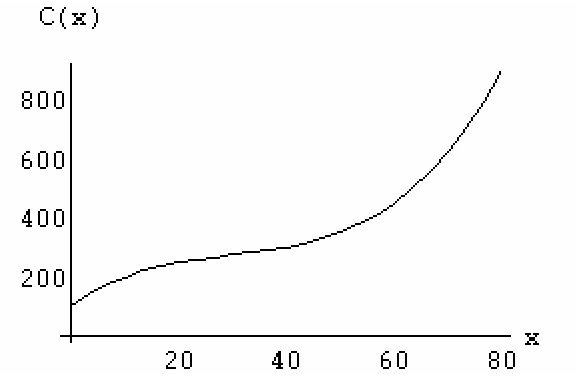
$\varepsilon_{x,p}(12) = -3$

11.

1.

$C(40)=300,\ C(50)=350, C(60)=450, C(70)=625$

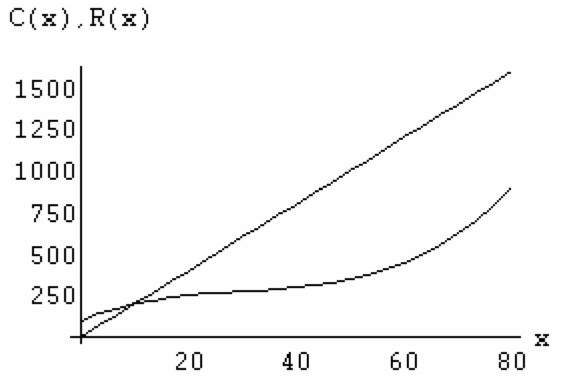
2.



3.

$R(x)=20x$

4.



5.

$C(x)=R(x)=10$ (See the curves above.)

6.

$$P(x)=R(x)-C(x)=-\frac{x^3}{240}+\frac{3x^2}{8}+\frac{20x}{3}-100$$

$$P'(x)=-\frac{x^2}{80}+\frac{3x}{4}+\frac{20}{3}=0$$

$x=67.86$

$$P''(x)=-\frac{x}{40}+\frac{3}{4}, \qquad P''(67.86)<0,$$

Maximum profit: $P(67.86)=777.21€$

7..

$$c_v(x) = \frac{C_v(x)}{x} = \frac{x^2}{240} - \frac{3x}{8} + \frac{40}{3}$$

$$c_v'(x) = \frac{x}{120} - \frac{3}{8} := 0 \quad \Rightarrow \quad x = 45, \qquad c_v''(x) = \frac{1}{120} > 0$$

8.

i)

$$C'(70) = 22.0833$$

ii)

$$C(71) - C(70) = 647.5875 - 625 = 22.5875$$

9.

i)

$$\varepsilon_{K,x}(x) = \frac{\left(\frac{x^2}{80} - \frac{3x}{4} + \frac{40}{3}\right)x}{\left(\frac{x^3}{240} - \frac{3x^2}{8} + \frac{40x}{3} + 100\right)}$$

$$\varepsilon_{K,x}(48) = 0.874$$

The cost function is inelastic at $x = 48$.

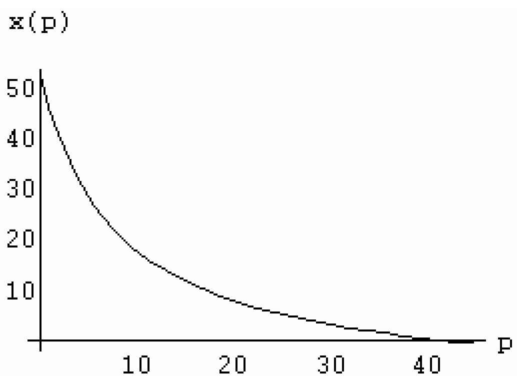
ii)

$$C(48) = 336.8$$

$$C(48 \cdot 1.01) = C(48.48) = 339.79$$

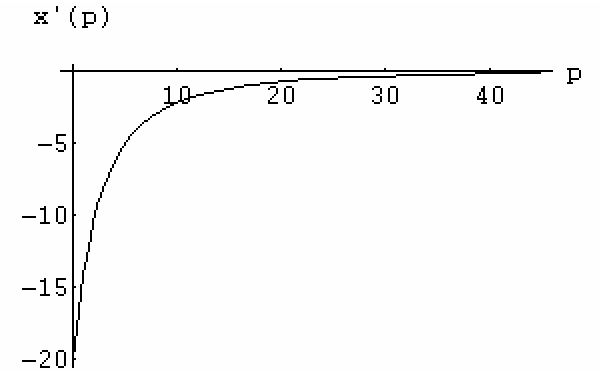
$$\frac{339.79}{336.8} \cdot 100 - 100 = 0.89$$

12.



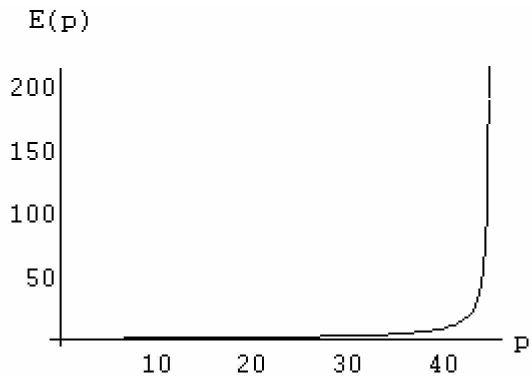
1.

$$x'(p) = \frac{-500}{(p+5)^2}$$



2.

$$\varepsilon_{x,p}(p) = \frac{\frac{500p}{(p+5)^2}}{\frac{500}{p+5} - 10} = -\frac{50p}{(5+p)(45-p)}$$



3.

$$10 = \frac{500}{p+5} - 10 \quad \Rightarrow \quad p = 20$$

$$40 = \frac{500}{p+5} - 10 \quad \Rightarrow \quad p = 5$$

$$\varepsilon_{x,p}(20) = -1.6, \qquad \varepsilon_{x,p}(5) = -0.625$$

4.

$$\frac{-50p}{(5+p)(45-p)} = -3 \qquad \Rightarrow \quad 3p^2 - 70p - 675 = 0 \quad \Rightarrow \quad p = 30.67\,€$$

$x(30.67) \approx 4.02$

5.

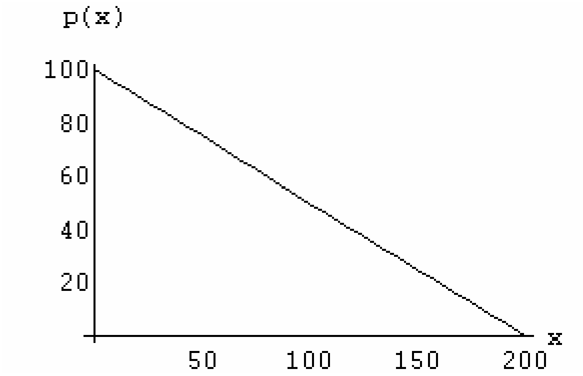
$$\frac{-50p}{(5+p)(45-p)} = -1 \quad \Rightarrow \quad p^2 + 10p - 225 = 0 \quad \Rightarrow \quad p \approx 10.81 \text{ €}$$

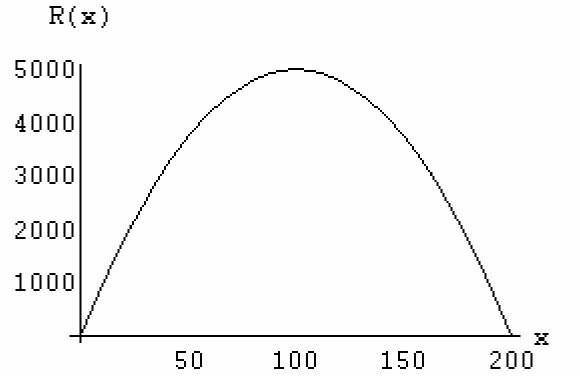
i. e. the demand function is elastic for $p > 10.81$ and inelastic for $p < 10.81$.

13.

1.

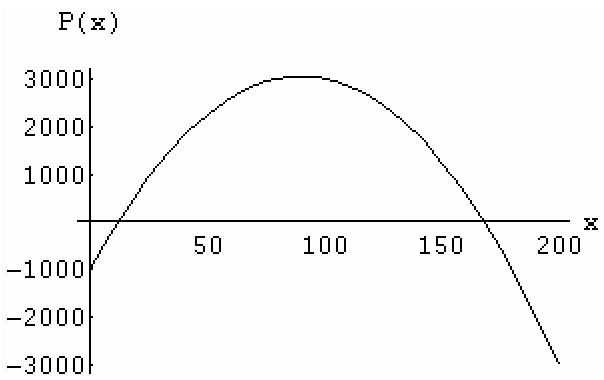
$$R(x) = p(x) \cdot x$$
$$= -0.5x^2 + 100x$$





2.

$$P(x) = R(x) - R(x)$$
$$= -0.5x^2 + 90x - 1000$$



3.

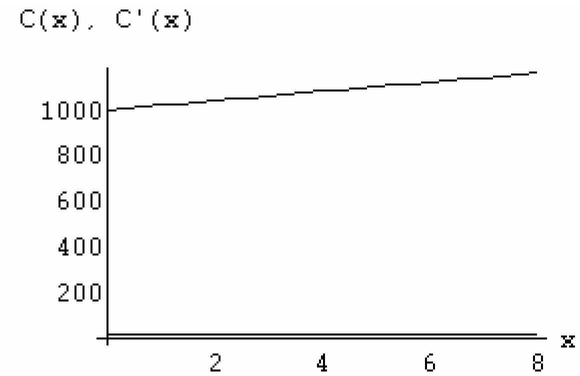
$$P'(x) = -x + 90 \quad , \quad P'(x) := 0 \quad \Rightarrow \quad x = 90$$

$$P''(x) = -1 < 0 .$$

There will, therefore, be a maximum profit of 3050 € for $x = 90$. The corresponding price will be $p(90) = 55$ €.

14.

1.

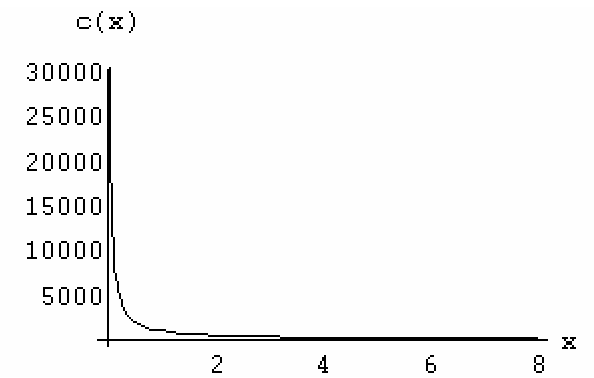


$C'(x) = 20$.

A change of output by one unit from any level x_0 will result in a change of total costs by 20 €.

2.

$$c(x) := \frac{C(x)}{x} = \frac{1000}{x} + 20, \qquad x > 0$$

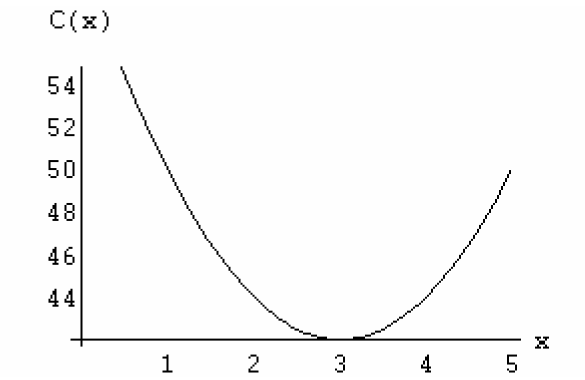


3.

$$\lim_{x\to+\infty} C(x) = \lim_{x\to+\infty} (1000 + 20x) = +\infty$$

$$\lim_{x\to+\infty} c(x) = \lim_{x\to+\infty} (\frac{1000}{x} + 20) = 20 = C'(x)$$

15.
1.



$$C'(x) = -12 + 4x$$

$$C'(x) := 0 \quad \Rightarrow \quad x = 3$$

$$C''(x) = 4 > 0.$$

Thus, the total costs function has its (absolute) minimum at $x = 3$ where $C(3) = 42$

2.

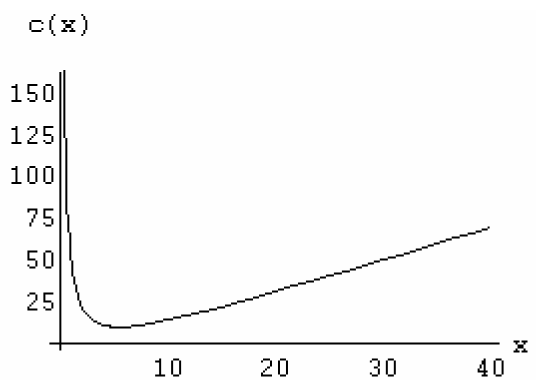
$$c(x) := \frac{C(x)}{x}$$
$$= \frac{60}{x} - 12 + 2x, \quad x > 0$$

$$c'(x) = -\frac{60}{x^2} + 2, \quad x > 0,$$

$$c'(x) := 0 \quad \Rightarrow \quad x = \sqrt{30},$$

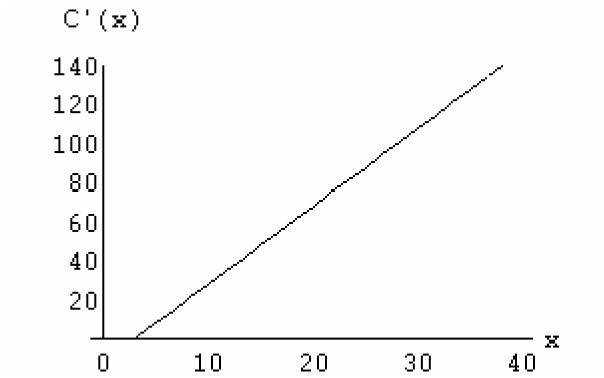
$$c''(x) = \frac{120}{x^3} > 0$$

Thus, the average costs function $c(x)$ has its (absolute) minimum at $x = \sqrt{30}$.



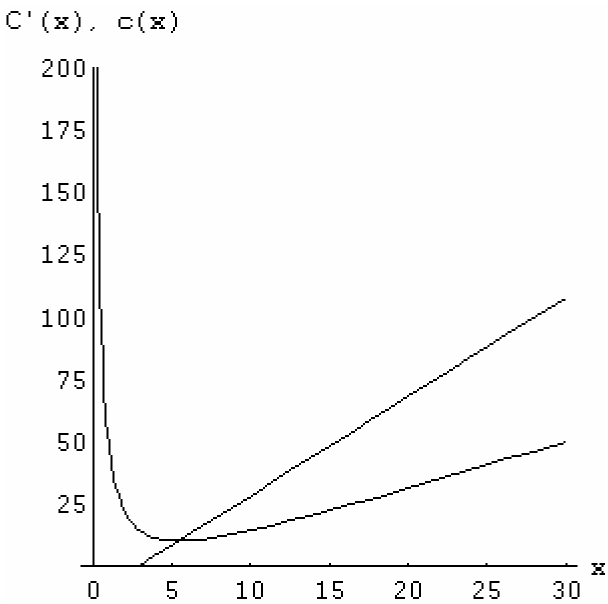
3.
Since

$$C'(x) = -12 + 4x, \qquad C''(x) = 4 \neq 0$$



4.

$$-12 + 4x = \frac{60}{x} - 12 + 2x \quad \Leftrightarrow \quad x = \sqrt{30}\ .$$

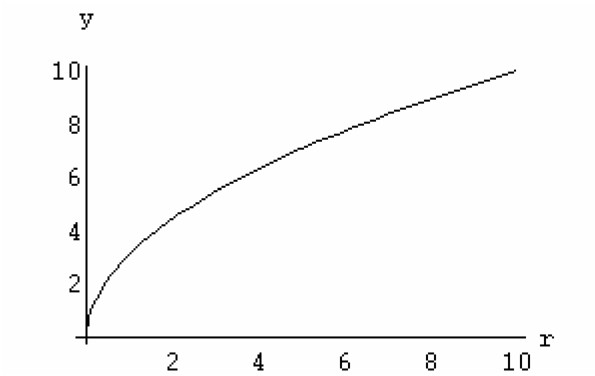


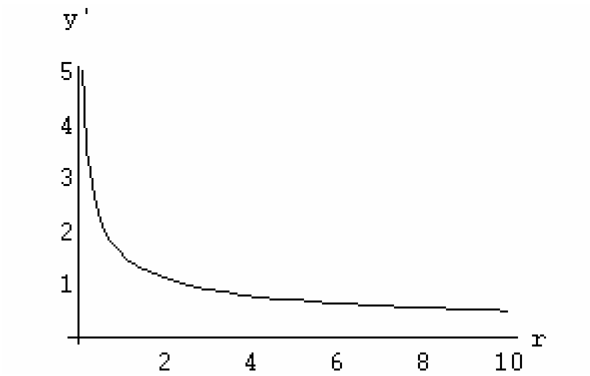
5.
$C'(x_0)$ gives the approximate change of costs after a change of the output x_0 by one unit.
$c(x_0)$ gives the costs per unit for an output x_0 .

16.
1.

$$y' = \frac{\sqrt{10}}{2\sqrt{r}}, \qquad y'' = -\frac{\sqrt{10}}{4\sqrt{r^3}} < 0\,,$$

Thus, the marginal production function is decreasing.



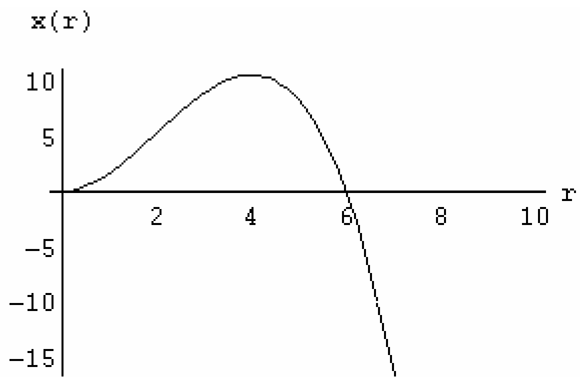


2.

$y = \sqrt{10r}\,, \qquad y^2 = 10r\,, \qquad r = 0.1y^2, \qquad K(r) = 2r$

$K(y) = 0.2y^2$

17.
1.



$$x'(r) = -r^2 + 4r$$
$$= r(4 - r)$$

$$x'(r) := 0 \;\; \Rightarrow \;\; r = 4$$

Because of

$$x''(r) = -2r + 4, \quad x''(4) < 0$$

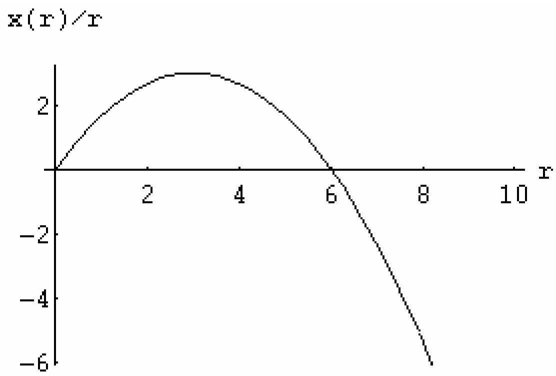
has $x(r)$ a relative (its absolute) maximum for $r = 4$.

2.

$$x(r) = -\frac{1}{3}r^3 + 2r^2 = r^2(2 - \frac{1}{3}r) < 0 \iff 2 - \frac{1}{3}r < 0 \iff r > 6$$

3.

$$\frac{x(r)}{r} = -\frac{1}{3}r^2 + 2r$$

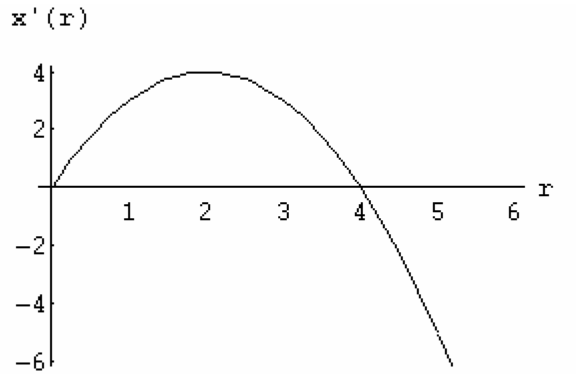


4.
Because of

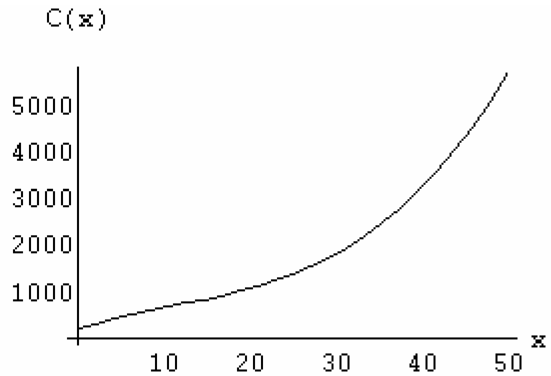
$$\left(\frac{x(r)}{r}\right)' = -\frac{2}{3}r + 2 := 0 \Rightarrow r = 3, \quad \left(\frac{x(r)}{r}\right)'' = -\frac{2}{3} < 0$$

the function $\frac{x(r)}{r}$ has a relative (its absolute) maximum at $r = 3$.

5.
$x'(r_0)$ means: the production will approximately change by $x'(r_0)$ units, if the input r_0 will be changed by one unit.



18.



1.

$$dC(x) = C'(x) \cdot dx$$
$$C'(x) = 0.18x^2 - 4x + 60$$

i) $dC(x = 10, dx = 2) = 76$ €

ii) $dC(x = 10, dx = -1) = -38$ €

2.

i) $\Delta C = C(12) - C(10) = 75.68$ €

ii) $\Delta C = C(9) - C(10) = -38.26$ €

iii) $\Delta C = C(12) - C(10) = 75.70$ €

iv) $\Delta C = C(9) - C(10) = -38.30$ €

19.

1.

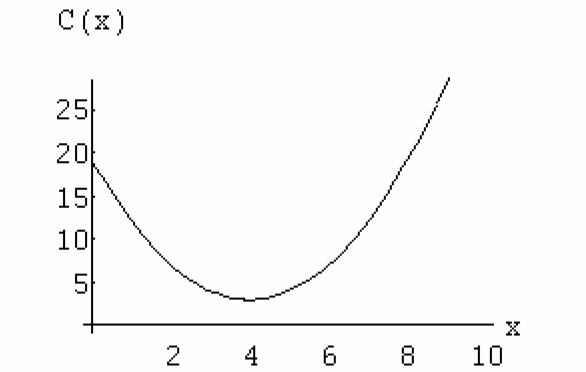
$$\lim_{x \to 0^+} C(x) = 16.$$

2.

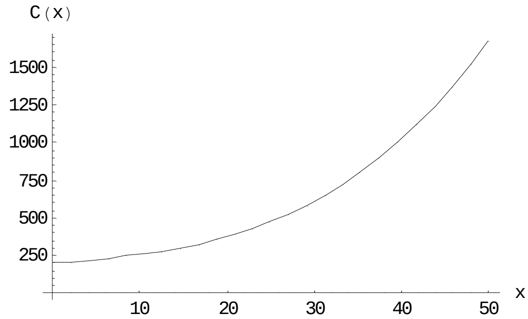
$$C(0) = 0.$$

3.

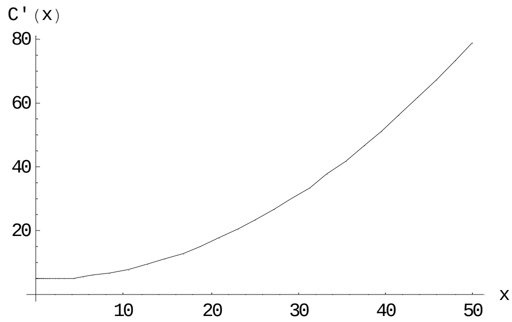
No, since $\lim_{x \to 0} C(0) \neq C(0)$.



20.
1.



$$C'(x) = 5 - 0.02x + 0.03x^2$$



An increase of output from 5 to 6 units will lead to an approximate increase of costs by 5.65 units.

2.

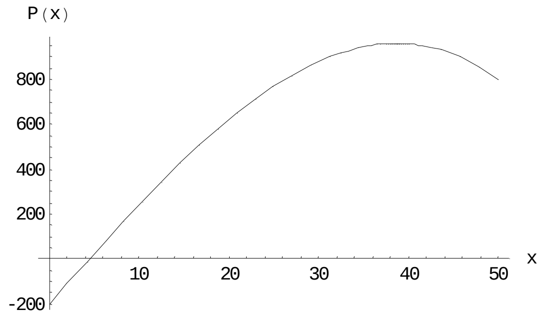
$$P(x) = R(x) - C(x)$$

$$= p(x) \cdot x - C(x)$$

$$= (50 - 0.01x) \cdot x - \left(200 + 5x - 0.01x^2 + 0.01x^3\right)$$

$$= -200 + 45x - 0.01x^3$$

21.



3.

$$P'(x) = 45 - 0.03x^2$$

$$P'(x) = 0 \ \wedge \ x \geq 0 \ \Rightarrow \ x = 38.73$$

$$P''(x) = -0.06x, \quad P''(38.73) < 0 .$$

$\therefore$ the firm will maximise its profit for an output of 38.73 units. The maximum profit will amount to $P(38.73x) = 961.90$.

4.

i)

$$\varepsilon_{P,x}(x) = \frac{x}{P(x)} \cdot P'(x)$$

$$\varepsilon_{P,x}(x) = \frac{x}{-0.01x^3 + 45x - 200} \cdot \left(-0.03x^2 + 45x\right)$$

$$\varepsilon_{P,x}(40) = \frac{40}{-0.01 \cdot 40^3 + 45 \cdot 40 - 200} \cdot \left(-0.03 \cdot 40^2 + 45 \cdot 40\right) \approx -0.125\,\%.$$

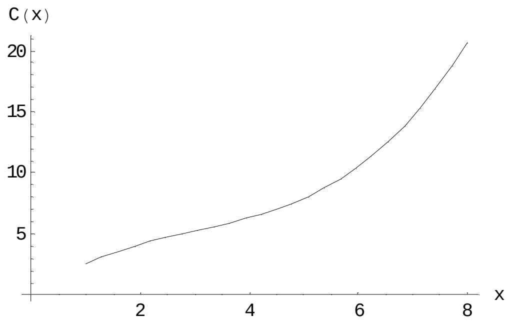
ii)

$$P(40 \cdot 1.01) = P(40.4) = 958.61, \qquad P(40) = 960.00 .$$

$$100 - \frac{958.61}{960.00} \cdot 100 = -0.145\,\%$$

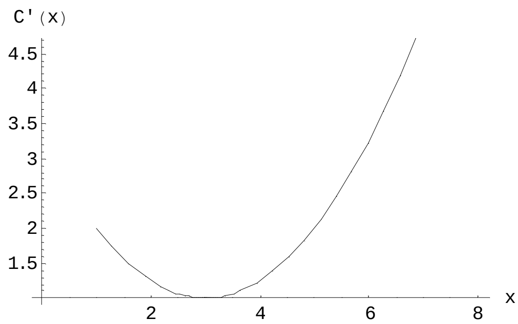
21.

$$C(x) = \frac{1}{12}x^3 - \frac{3}{4}x^2 + \frac{13}{4}x, \quad x \in [1,\ 8], \qquad (x\text{: output; } C\text{: costs}).$$



1.

$$C'(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4}, \quad x \in [1,\ 8]$$



$$C'(2) = 2$$

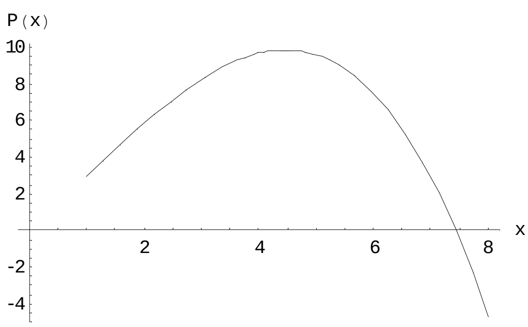
An increase of output from 5 to 6 units will lead to an approximate increase of costs by 2 units.

2.

$$P(x) = R(x) - C(x)$$

$$= \left(6 - \frac{x}{2}\right) \cdot x - \left(\frac{1}{12}x^3 - \frac{3}{4}x^2 + \frac{13}{4}x\right)$$

$$P(x) = -\frac{1}{12}x^3 + \frac{1}{4}x^2 + \frac{11}{4}x, \quad x \in [1,\ 8]$$



$$P'(x) = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4}, \quad x \in [1,\ 8]$$

$$P'(x) = 0, \quad -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4} = 0, \quad x \in [1,\ 8] \qquad \Rightarrow \qquad x \approx 4.46$$

$$P''(4.46) < 0.$$

$$P(4.46) \approx 9.85, \qquad P(1) \approx 2.92, \qquad P(8) \approx -4.67\ .$$

Hence, the firm's profit will be maximal for an output of $x \approx 4.46$. It will be approximately equal to 9.85.

3.
i)

$$\varepsilon_{P,x}(x) = \frac{x}{P(x)} \cdot P'(x)$$

$$= \frac{x}{-\frac{1}{12}x^3 + \frac{1}{4}x^2 + \frac{11}{4}x} \cdot \left(-\frac{1}{4}x^2 + \frac{1}{2}x + \frac{11}{4}\right).$$

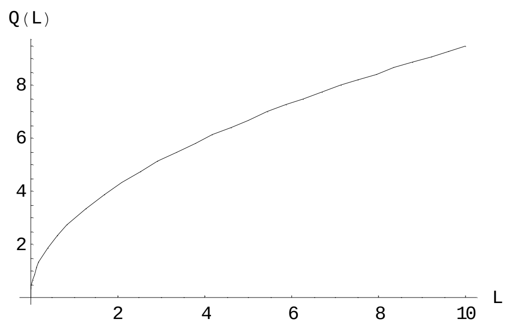
$$\varepsilon_{P,x}(3) = \frac{3}{8.25} \cdot 2 \approx 0.73$$

ii)

$$P(3) = 8.25, \qquad\qquad P(3.03) \approx 8.31$$

$$\frac{8.31}{8.25} \cdot 100 - 100 \approx 0.73$$

22.
1.



2.
Strictly concave.

23.
1.

$$C(x) - 1500 = \frac{1800 - 1500}{100 - 0} \cdot (x - 0)$$

$$C(x) = 3x + 1500$$

$$R(x) = 7x$$

$$P(x) = R(x) - C(x) = 4x - 1500 .$$

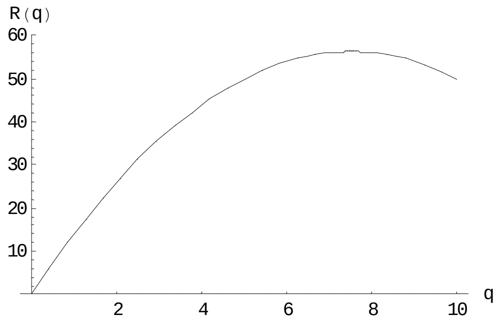
2.
The company will break even when $P(x) = 0$. Therefore,

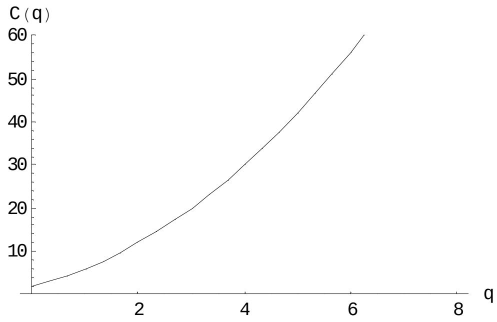
$$4x - 1500 = 0 , \qquad\qquad x = 375 .$$

So, by making and selling 375 mugs, the company will neither gain nor lose money.

24.

$$R(q) = 15q - q^2$$



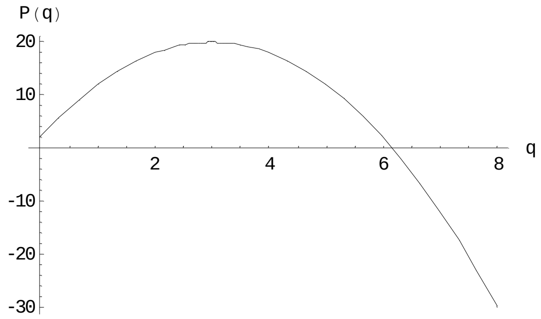


$$P(q) = R(q) - C(q)$$

$$= -2q^2 + 12q + 2$$

$$P'(q) = -4q + 12, \qquad -4q + 12 = 0, \qquad q = 3.$$

Because of $P''(q) = -4 < 0$, $q = 3$ maximises the firm's profit. The corresponding price level is $p(3) = 12$.



25.

1.

$$p = 10 - 0.5q \quad \Rightarrow \quad q = 20 - 2p$$

$$\varepsilon_{q,p}(p) = \frac{-2p}{20 - 2p}$$

2.

$$\frac{-2p}{20-2p} < -1, \qquad \frac{-p}{10-p} + 1 < 0, \qquad \frac{-2p+10}{10-p} < 0.$$

Case 1:

$$\begin{cases} -2p + 10 > 0 \\ -p + 10 < 0 \end{cases} \Rightarrow \quad p \in \varnothing$$

Case 2:

$$\begin{cases} -2p + 10 < 0 \\ -p + 10 > 0 \end{cases} \Rightarrow \quad 5 < p < 10.$$

The demand is inelastic for $p \in \left]5,\ 10\right[$ and elastic for $p \in \left]0,\ 5\right[\cup \left]10,\ +\infty\right[$.

26.

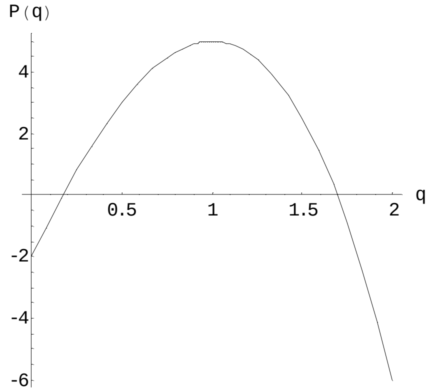
$$R(q) = p(q) \cdot q, \qquad R(q) = 15q - 6q^2,$$

$$P(q) = R(q) - C(q), \qquad P(q) = -2q^3 - 3q^2 + 12q - 2,$$

$$P'(q) = -6q^2 - 6q + 12$$

$$\left(P'(q) = 0 \ \wedge \ q \geq 0\right) \quad \Rightarrow \quad q = 1, \qquad P''(1) = -12 < 0,$$

$p(1) = 9, \quad P(1) = 5.$



27.

1.

$$Q'(L) = 6L - 0.3L^2 .$$

2.

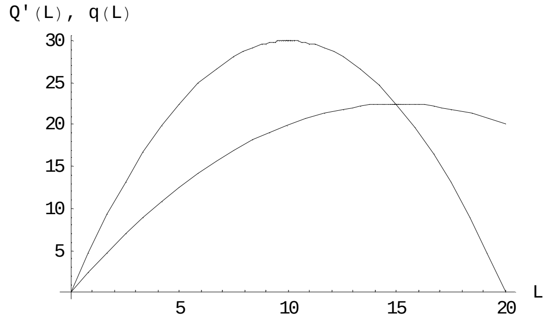
$$Q''(L) = 6 - 0.6L ,$$

$$Q''(L) = 0 \quad \Rightarrow \quad L = 10, \qquad Q'''(L) = -0.6 < 0$$

$$q(L) := \frac{3L^2 - 0.1L^3}{L} = 3L - 0.1L^2 ,$$
$$q'(L) = 3 - 0.2L, \qquad q'(L) = 0 \ \Rightarrow L = 15, \quad q''(L) = -0.2 < 0 .$$

3.

$$Q'(L) = q(L) \quad \Rightarrow \quad 6L - 0.3L^2 = 3L - 0.1L^2 \quad \Rightarrow \quad L = 15.$$



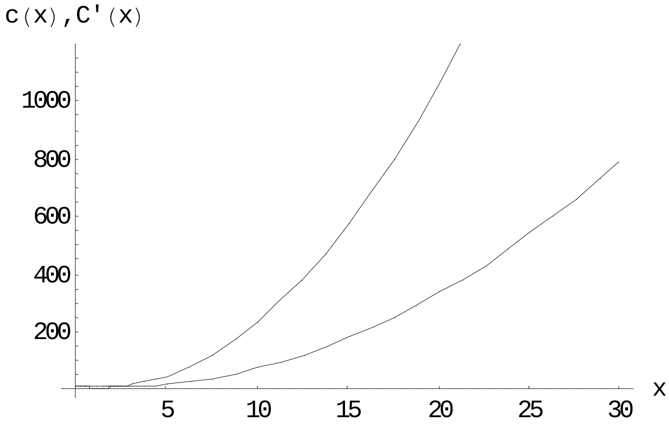
28.

1.

$$C(x) = 12x - 4x^2 + x^3$$

$$C'(x) = 12 - 8x + 3x^2$$

2.



3.

$$R(x) = 16x$$

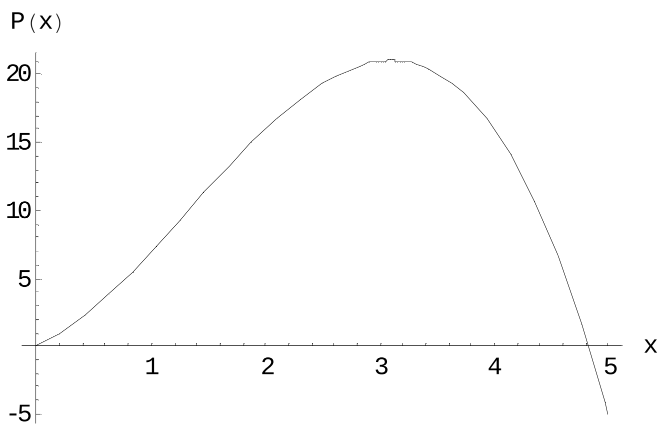
$$P(x) = R(x) - C(x) = 16x - \left(12x - 4x^2 + x^3\right)$$
$$P(x) = 4x + 4x^2 - x^3, \qquad P'(x) = 4 + 8x - 3x^2$$

$$\left(4 + 8x - 3x^2 = 0 \quad \wedge \quad x \geq 0\right) \quad \Rightarrow \quad x \approx 3.10,$$

Because of

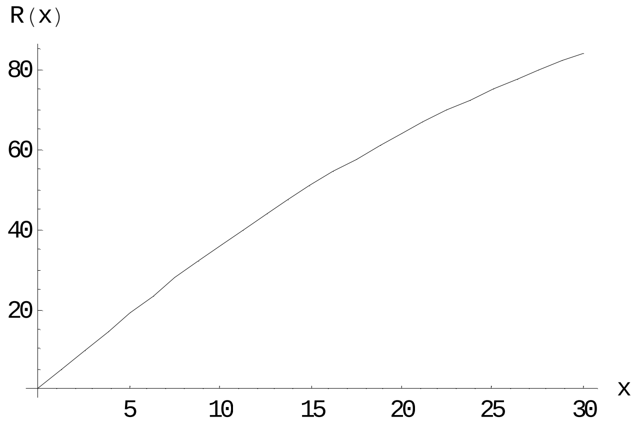
$$P''(x) = 8x - 6x, \quad P''(3.10) < 0$$

the firm should sell 3.10 units in order to maximise profit. The maximum profit will be $P(3.10) \approx 21.05$ €.



29.
1.

$$R(x) = p(x) \cdot x = 4x - \frac{x^2}{25}$$



$$P(x) = R(x) - C(x) = 4x - \frac{x^2}{25} - \left(\frac{x}{2500}\cdot(x-100)^2 + x\right)$$

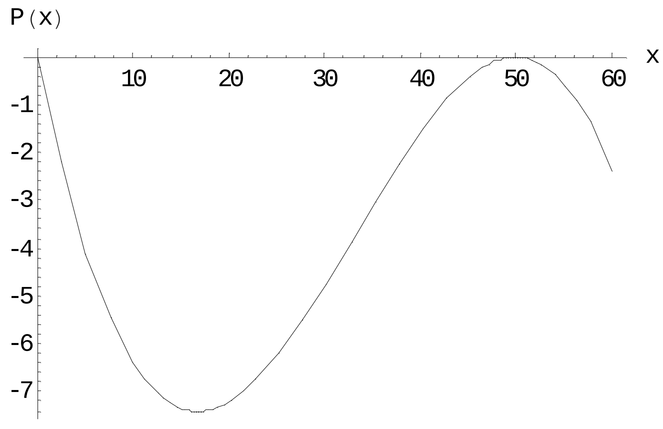
$$= 4x - \frac{x^2}{25} - \left(\frac{x^3}{2500} - \frac{2x^2}{25} + 5x\right)$$

$$= -\frac{x^3}{2500} + \frac{x^2}{25} - x,$$

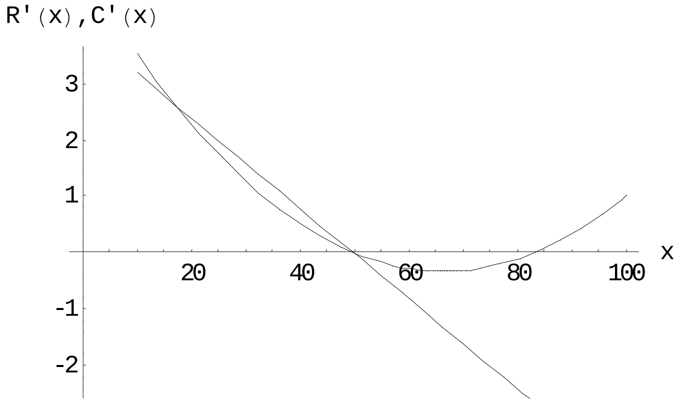
$$P'(x) = -\frac{3x^2}{2500} + \frac{2x}{25} - 1 = 0 \quad \Rightarrow \quad x_1 = 50, \quad x_2 = \frac{50}{3} \approx 16.67,$$

$$P''(x) = -\frac{6x}{2500} + \frac{2}{25}, \qquad P''(50) = -0.04 < 0, \quad P''(16,67) = 0.039992 > 0.$$

Therefore, the monopolist's profit will be maximised at $x = 50$, with $P(50) = 0.$ The corresponding price will be $p(50) = 2$.



2.



30.

1.

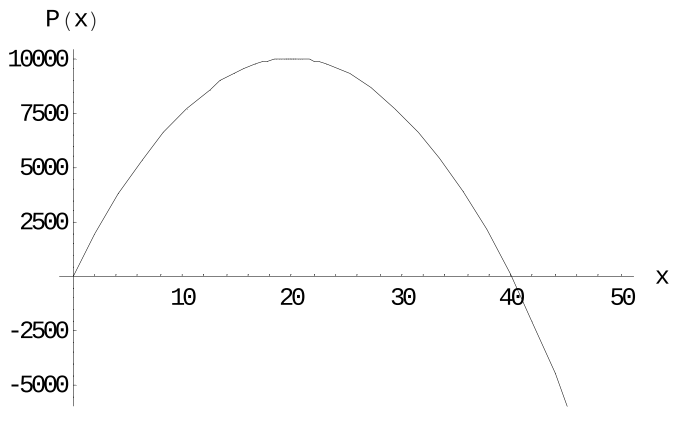
$$R(x) = p(x) \cdot x = 1200x - 10x^2$$

$$P(x) = R(x) - C(x) = 1200x - 10x^2 - 200x - 15x^2$$

$$P(x) = -25x^2 + 1000x$$
$$P'(x) = -50x + 1000$$

$$P'(x) = 0 \quad \Rightarrow \quad x = 20\,.$$

Because of $P''(x) = -50 < 0$ the output $x = 20$ maximises the monopolist's profit.



2.

$$p(20) = 1200 - 10 \cdot 20 = 1000.$$

3.

$$P(20) = 10000.$$

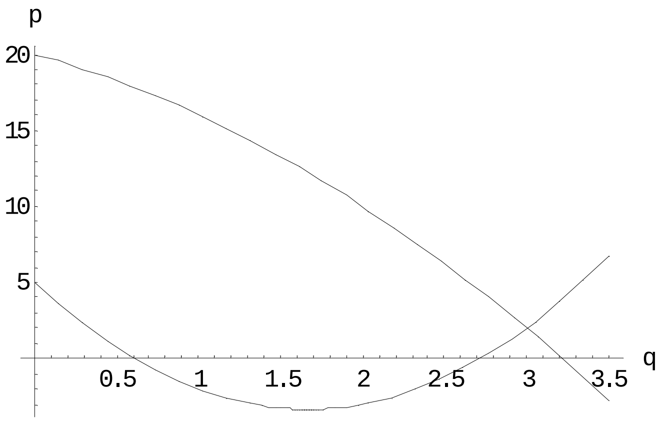
31.

$$p + q^2 + 3q - 20 = 0 \quad \Rightarrow \quad -q^2 - 3q + 20 = 0$$

$$p-3q^2+10q=5 \quad .\Rightarrow \quad 3q^2-10q+5=0$$

$$-q^2-3q+20=q^2-10q+5$$

$$\left\langle 4q^2-7q-15=0 \quad \wedge \quad q\ge 0\right\rangle \Rightarrow \quad q=3 \quad \Rightarrow \quad p=2$$



32

1.

$$R(x)=p(x)\cdot x=124x-10x^2$$

$$P(x)=R(x)-C(x)=124x-10x^2-\left(x^3-12x^2+60x+98\right)$$

$$P(x)=-x^3+2x^2+64x-98$$

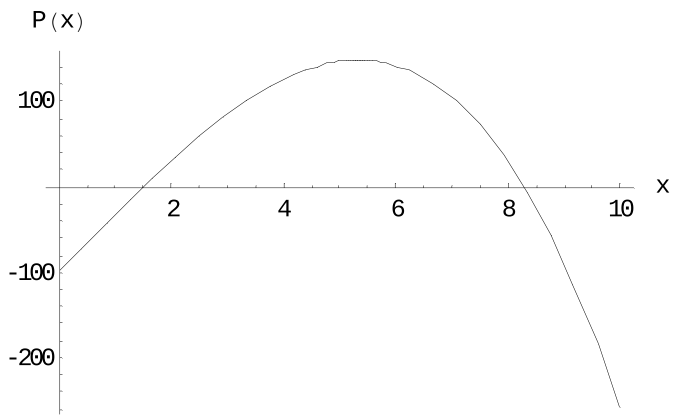
$$\left(\left(P'(x)=-3x^2+4x+64\right) \wedge x\ge 0\right) \Rightarrow 5.33$$

$$P''(x)=-6x+4, \quad P''(5.33)<0$$

The level of production $x=\dfrac{16}{3}$ will maximise the manufacturer's profit.

$$P(5.33)\approx 148.52$$

$$p(5.33)=70.7$$



2.

$$\varepsilon_{C,x}(x)=\frac{x}{C(x)}C'(x)=\frac{x\cdot\left(3x^2-24x+60\right)}{x^3-12x^2+60x+98}$$

$$\varepsilon_{C,x}(4)=0.2285714286\approx 0.23\%$$

An increase of output $x=4$ by 1% will lead to an approximate increase of costs by 0.23%. The cost function is at $x=4$ inelastic

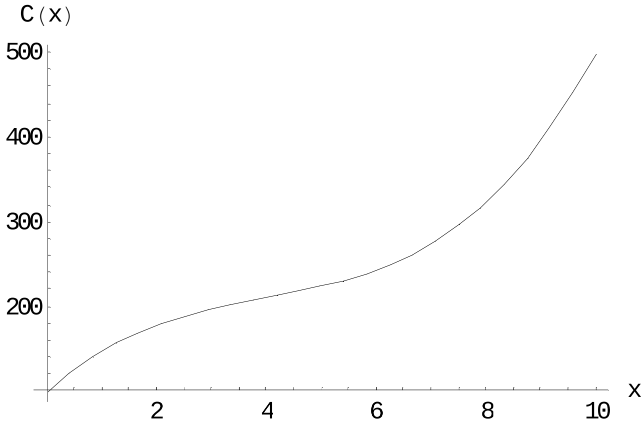
3

Because of $(-24)^2-4\cdot 3\cdot 60<0$ the equation $3x^2-24x+60=0$ has no real solution. Therefore, the cost function is monotone.

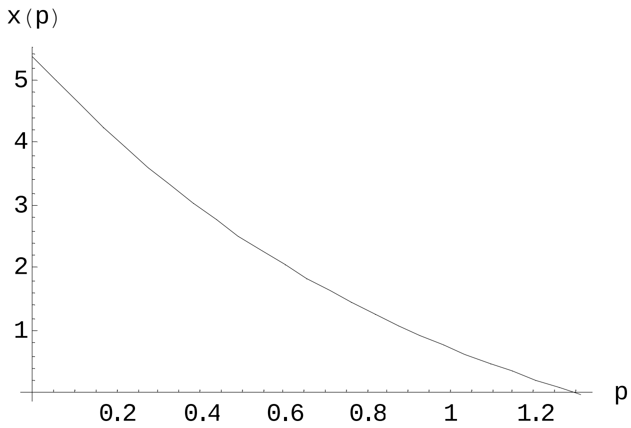
The cost function increases monotonically because of

$$C(0)=98<147=C(1).$$

Because of $C''(x)=6x-24<0,\ x<4$, the costs decrease degressively for $0<x<4$.



33.



$$x(p) = \frac{1}{e^{p-2}} - 2 = e^{2-p} - 2$$

$$t = 2 - p\,, \qquad \frac{dt}{dp} = -1$$

$$x(p) = e^t - 2\,, \qquad \frac{dx(p)}{dt} = e^t$$

$$x'(p) = -e^t = -e^{2-p} < 0\,.$$

Therefore, the demand function is strictly decreasing. This means that with an increase of prices the demand will decrease.

34.

1.

$$p(x) = 10 - 0.5x \quad \Rightarrow \quad x(p) = -2p + 20$$

$$\varepsilon_{x,p}(p) = \frac{-2p}{-2p + 20}$$

2.

$$\frac{-2p}{-2p+20} = -1 \quad \Rightarrow \quad p = 5 \qquad \Rightarrow \quad x(5) = 10$$

The demand function $x(p) = -2p + 20$

1. has a unitary elasticity in $p = 5$ ($x = 10$).

$$\varepsilon_{x,p}(4) = \frac{-8}{-8+20} = -0.4 > -1 \qquad \Rightarrow \quad x(4) = 12$$

2. is inelastic for $p \in \left]0,\ 5\right[$ ($x \in \left]0,\ 12\right[$)

$$\varepsilon_{x,p}(6) = \frac{-12}{-12+20} = -1.5 < -1 \qquad \Rightarrow \quad x(6) = 8$$

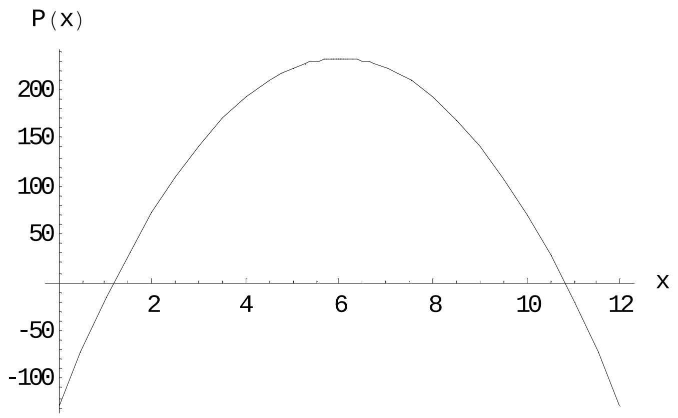
2. is elastic for $p \in \left]5,\ \infty\right[$ ($x \in \left]12,\ \infty\right[$).

35.

1.

$$P(x) = R(x) - C(x) = \left(180 - 2x\right)\cdot x - \left(128 + 60x + 8x^2\right)$$
$$P(x) = -10x^2 + 120x - 128$$
$$P'(x) = -20x + 120 := 0 \quad x = 6, \qquad P'(x) = -20 < 0.$$

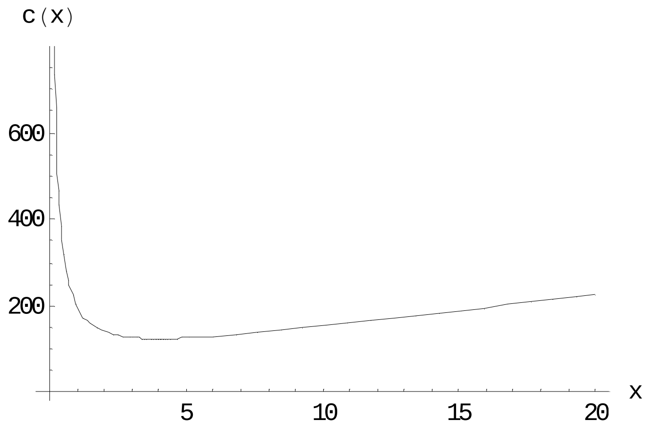
Therefore, at the level of $x = 6$ the will be a maximum profit equal to $P(6) = 232$.



2.

$$c(x) = \frac{128}{x} + 60 + 8x$$
$$c(x) = \frac{128}{x} + 60 + 8x, \quad x > 0$$
$$\left\langle c'(x) = -\frac{128}{x^2} + 8 := 0 \quad \wedge \ x > 0 \right\rangle \Rightarrow \quad x = 4$$

Because of $c''(x) = \dfrac{256}{x^3} > 0$ für $\forall x > 0$ the average cost function will assume its minimum at $x = 4$ with $c(4) = 124$.

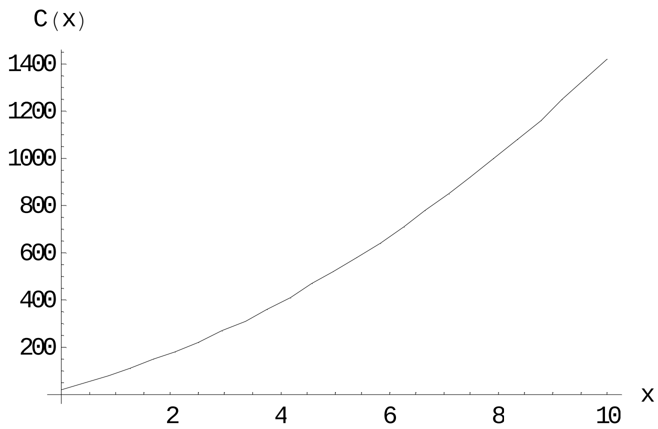


3.

$$C'(x) = 60 + 16x > 0, \quad \forall x \geq 0$$

$$C''(x) = 16 > 0, \quad \forall x \geq 0$$

Therefore, the cost function does not increase degressively for any x.



(Last updated: 22.08.2010)