Analysis in Economics (II)

Exercises

1.
A local music hall is selling ticket for a certain concert. The matinee show costs 15 € per person and the evening show costs 25 € per person. If x_1 is the number of matinee tickets purchased and x_2 the number of evening tickets for the concert, find the revenue function $R(x_1, x_2)$ for the concert and evaluate and interpret $R(125, 200)$

2.
Suppose a metal manufacturing company has a Cobb-Douglas production function

$$f(L,K) = 10 \cdot L^{0.25} K^{0.75}$$

where L is the number of hours of labour and K is the Euro amount of capital invested.
If the company uses 2000 hours of labour and 1500 € in capital, how many units of metal will be produced?

3.
The production function of a firm is given by

$$y(L,C) = 2L^{0.4}C^{0.6}, \quad L, C > 0$$

Here are:

C: capital
L: labour
y: production .

Give the partial marginal productivity functions of labour and capital.

4.
The production function of a firm is given by

$$y(L,C) = 90L^{0.8}C^{0.2}, \; L, C > 0$$

Here are:

C: capital
L: labour
y: production .

Find and interpret the partial marginal productivity functions of labour and capital

i) for $L = 1000, \quad \mathrm{C} = 200$
ii) If each capital unit can be substituted by 8 units of labour.

5.
The production function of a firm is given by

$$y(L,C) = L^{0.8}C^{0.2}, \quad L,C > 0$$

Here are:

C: capital
L: labour
y: production .

Find and interpret the partial derivatives of first and second order of this function with respect to labour and capital.

6.
A firm has the production function

$$Q = \sqrt{C \cdot L}\ .$$

Here are:

C: quantity of capital
L: quantity of labour
Q: quantity of production .

Sketch and label the level curves for $Q = 1,\ 2,\ 3$ on a single diagram. What is the economic term for these curves? How can they be interpreted?

7.
A firm's production Q depending on two input amounts r_1 and r_2 is given by the following function:

$$Q(r_1, r_2) = 440 + 4r_1 + 10r_2 - r_1^2 + 3r_1 \cdot r_2 - 2.5r_2^2\ .$$

Determine the factor combination for which the production will be maximal. How much will it be for this combination?

8.
Two producers Pr_1 and Pr_2 produce the same product. Denote by

$p_i,\ i = 1,\ 2$: the price dictated by Pr_i,
$x_i,\ i = 1,\ 2$: the amount of the product supplied by Pr_i

It has been estimated that the following relations hold between p_i and $x_i,\ i = 1,\ 2$.

$$x_1 = 100 - 2p_1 - \ \ p_2$$
$$x_2 = 120 - \ \ p_1 - 3p_2$$

Further, we have the following cost functions of the producers:

$$C_1(x_1) = 120 + 2x_1$$
$$C_2(x_2) = 120 + 2x_2 .$$

1. Find the profit functions of each producer as well as their common profit function depending on the prices p_1, p_2.
2. Determine the prices such as they will guarantee a maximum total profit. Calculate the maximum profit?
3. Following a „price war“ producer Pr_2 decides to set his price at the level $p_2 = 16$. Determine the level of p_1 maximising the profit of producer Pr_1.
4. Is it advantageous for the consumers if the two producers put an end to their „price war“?

9.
A company produces two types of scooters $S_1,\ S_2$. The demand for the two types is given by the following function:

$$p_1\left(x_1, x_2\right) = 35 - 3x_1 + 4x_2 ,$$
$$p_2\left(x_1, x_2\right) = 20 - 2x_1 + x_2 .$$

($p_i,\ i = 1,\ 2$: price of S_i ; $x_i,\ i = 1,\ 2$: demand for S_i)

The company's cost function, in hundreds of €, is given by

$$C\left(x_1, x_2\right) = 8.5x_1 + 6x_2 + 400 .$$

1. Find the revenue function $R\left(x_1, x_2\right)$ of the company and evaluate $R(10,\ 18)$.
2. Determine the company's profit function $P\left(x_1, x_2\right)$ and evaluate $P(10,\ 18)$.
3. Find and interpret the first partial derivatives of the profit function for $x_1 = 10,\ x_2 = 18$.
4. Determine the partial elasticities of the revenue function for $x_1 = 10,\ x_2 = 18$ and interpret them.

10.
An online fitness company sells two types of shoes, aerobic and running. The store pays 30 € for a pair of aerobic shoes and 45 € for a pair of running shoes. The daily demand equations for each type of shoe are as follows:

$x_1 = 850 - 36p + 15q$: demand equation for aerobic shoes
$x_2 = 1075 + 20p - 25q$: demand equation for running shoes

where p is the selling price for each pair of aerobic shoes and q is the selling price for each pair of running shoes.
What price should the store charge for each model of shoes if it wants to maximise its profit.

11.
Draw the isoquants for the production function

$$x(r_1, r_2) = r_1^2 + r_2^2 .$$

12.
Given the profit function

$$P(x_1, x_2) = -3x_1^2 - 2x_2^2 - 2x_1x_2 + 160x_1 + 120x_2 - 18$$

for a firm producing two products, maximise profit.

13.
In monopolistic competition producers must determine the price that will maximise their profit. Assume that a producer offers two different brands of products, for which the demand functions are:

$$x_1 = 14 - 0.25p_1$$
$$x_2 = 24 - 0.5p_2$$

and the joint cost function

$$C(x_1, x_2) = x_1^2 + 5x_1x_2 + x_2^2 .$$

Determine the profit-maximising level of output, the price that should be charged for each brand of the product, and the profit.

14.
A monopolistic firm has the following demand functions for each of its products P_1 and P_2:

$$x_1 = 72 - 0.5p_1 ,$$
$$x_2 = 120 - p_2 .$$

The combined cost function is

$$C(x_1, x_2) = x_1^2 + x_2^2 + x_1x_2 + 35 ,$$

The total production is to be equal to 40.

Find the profit-maximising level of

(1) output
(2) price
(3) profit.

15.
Given a budget constraint

$$4L + 3K = 108 ,$$

optimise the generalised Cobb-Douglas production function

$$Q(L,K) = L^{0.5} \cdot K^{0.4}.$$

16.

1. Minimise costs for a firm with the cost function

$$C(x_1, x_2) = 5x_1^2 + 2x_1x_2 + 3x_2^2 + 800$$

   subject to the production quota

$$x_1 + x_2 = 39.$$

2. Estimate additional costs if the production quota is increased to 40

17.

A rancher faces the profit function

$$P(x_1, x_2) = 110x_1 - 3x_1^2 - 2x_1x_2 - 2x_2^2 + 140x_2.$$

where

x_1 : sides of beef

x_2 : hides.

The output must be in the proportion

$$x_1 = 2x_2.$$

Using the *method of Lagrange multiplier*, determine the level of output that will maximise the rancher's profit.

18.

Maximise the following utility function

$$u(x_1, x_2) = x_1^{0.60} \cdot x_2^{0.25}$$

subject to the budget constraint

$$8x_1 + 5x_2 = 680,$$

using the method of Lagrange multipliers. Interpret your results.

19.

1. Maximise the utility function

$$u(x_1, x_2) = x_1^2 \cdot x_2^2$$

   subject to the budget constraint

$$x_1 + 2x_2 = 20 .$$

2. Estimate the effect of changing the right-hand side of the budget constraint by 1 unit on the objective function.

(Last updated: 09.08.2010